Japan-Korea Joint Seminar on Number Theory and Related Topics 2014

Chida, Masataka
Title: Regulators and special values of Rankin-Selberg L-functions
Abstract: This is a joint work with François Brunault. In this talk, we will introduce an explicit formula for the regulator of an element in the motivic cohomology of the product of two Kuga-Sato varieties. This result is related to Beilinson's conjecture on non-critical values of Rankin-Selberg L-functions for modular forms.

Choi, Dohoon
Title: Quantum modular forms and Eichler-Shimura cohomology theory
Abstract: Quantum modular forms are new modular objects introduced by Zagier. In this talk, I will talk about some properties of quantum modular forms related to Eichler-Shimura cohomology thoery.

Ha, Junsoo
Title: Smooth Polynomial solutions to $X + Y = Z$ over finite fields
Abstract: We say an integer is y-smooth if all of its prime factors are less than or equal to y . We consider the Diophantine equation $a + b = c$ where all variables are y-smooth and $(a, b, c) = 1$. A recent work of Lagarias and Soundararajan showed that this equation has at least $\exp(y^{1/\kappa})$ solutions for $\kappa > 8$ when y is large. In this talk, I will describe some recent progress in this problem and an analogous theorem for the polynomial rings over finite fields.

Hirose, Minoru
Title: The partial derivatives of abelian L-functions at s=0 and refinement of Stark conjecture.
Abstract: Shintani's method give a way to define the several variable version of abelian L-functions of number fields. We formulate a refinement of stark conjecture by using partial derivatives of this function.

Jeong, Sangtae
Title: Criteria of measure-preservation for 1-Lipschitz functions on $\mathbb{F}_q[[T]]$ in terms of van der Put basis and its applications
Abstract: We characterize the measure-preservation of 1-Lipshcitz functions on $\mathbb{F}q[[T]]$ in terms of the van der Put expansion. We use these results to give sufficient conditions for expanding measure-preserving 1-Lipshcitz functions using three well-known bases: Carlitz polynomials, digit derivatives, and digit shifts. We show that these conditions are also necessary for $\mathbb{F}_2[[T]]$. Moreover, we give an alternate, unified proof of the ergodicity criteria of $\mathbb{F}_2[[T]]$ in terms of the aforementioned bases.

Jun, Byungheup
Title: Todd series for nonreduced cones, Q-invariance and its application to Fukuhara-Yui conjecture

Abstract: S. Fukuhara and N. Yui introduced a generalization of Dedekind sums to higher degree and dimensions, as certain higher dimensional cotangent sums twisted by Hurwitz zeta values, namely Apostol-Zagier sums. They discovered a generating function for the sums and, using analytic property of the generating function, computed their reciprocity law, which directly generalizes that of the cotangent sums of Zagier. In this talk, we understand these generalized Dedekind sums in the frame of Todd series associated to lattice cones. The classical Todd series appear in defining the Todd genus of a manifold inside the Hirzebruch-Riemann-Roch formula. Its higher dimensional generalization was made in around 90's in the context of equivariant Riemann-Roch, geometric quantization and Euler-Maclaurin summation formula for lattice polytopes related to toric varieties. After small modification, Todd series defines a cocycle for singular complex of lattice cones. It has been shown, thru diverse works, that the coefficients are closely related to (generalized) Dedekind sums and their reciprocity fits better in the cocycle property of Todd. We extend the definition of the Todd series of lattice cones to nonreduced case, so we extend the notion of generalized Dedekind sums to non-primitive lattice vectors. The Todd series defines a germ of holomorphic functions on the complexification of the underlying vector space of the cone. As a function, Todd function is determined not by the Z-structure but on the Q-structure of the cone. Using this invariance, we show that the Todd coefficients as well as the generalized Dedekind sums have certain homogeneity coming from the independence of the lattice structure. Application of iterated residue on certain boundary of degenerated cone yields a reciprocity formula of very general form. As a particular case, we recover the reciprocity formula of Apostol-Zagier sums as given by Fukuhara and Yui. In this line, we answer a conjecture of Fukuhara-Yui on the completeness of their reciprocity by suggesting an amelioration of their reciprocity formula. Finally an algorithm to compute the generalized Dedekind sums will be suggested.

This is a joint work with Hi-joon Chae and Jungyun Lee.

Lee, Dong Uk
Title: Nonemptiness of Newton strata of Shimura varieties of Hodge type
Abstract: For a Shimura variety of Hodge type with hyperspecial level at a prime p, the Newton stratification on its special fiber at p is a stratification defined in terms of the slope sequence of the Frobenius action on the Dieudonne module of parameterized abelian varieties ("F -isocrystals with G-structure"). There has been a conjectural group-theoretic description of the F-isocrystals that are expected to show up in that special fiber. We confirm this conjecture by two different methods. More precisely, for any F-isocrystal withG-structure that is expected to appear (in a precise sense), first we construct a special point which has good reduction and whose reduction has associated F-isocrystal equal to given one. Secondly, we produce a Kottwtiz triple (with trivial Kottwitz invariant) with the F-isocrystal component being the given one. According to a recent result of Kisin which establishes the Langlands-Rapoport conjecture, such Kottwitz triple arises from a point in the reduction.

Lee, Jun Ho
Title: On the unit groups of some orders generated by units
Abstract: We are interested in the unit groups of some orders generated by a unit(or

units). We introduce known results in the case that the rank of the unit group is 1. When the rank of the unit group is 2, we will share our results and recent progress. This is a joint work with Stéphane R. Louboutin.

Lee, Yoonbok

Title: Simple zeros of primitive Dirichlet L-functions and the asymptotic large sieve (Joint work with Chandee, Liu and Radziwill)

Abstract: Assuming the Generalized Riemann Hypothesis (GRH), we show using the asymptotic large sieve that 91% of the zeros of primitive Dirichlet L-functions are simple. This improves on earlier work of Özlük which gives a proportion of at most 86%. We further compute q-analogue of the Pair Correlation Function $F(\alpha)$ averaged over all primitive Dirichlet L-functions in the range $|\alpha| < 2$. Previously such a result was available only when the average included all the characters χ.

Lee, Jungyun

Title: Indivisibility of class numbers of real quadratic function fields

Abstract: In this talk, we will prove that a density of real quadratic function fields whose class number is not divisible by a given prime l is positive and find an explicit lower bound of the density (This is a jont work with Yoonjin Lee.).

Miura, Takashi

Title: On the Fitting ideals of the ideal class groups of non-cyclic abelian CM-extensions

Abstract: Let K be a CM-field which is abelian over a totally real field k, and G a Galois group of K/k. The Fitting ideal of the ideal class group Cl_K over the group ring $\mathbb{Z}[G]$ is defined to be an ideal of $\mathbb{Z}[G]$ generated by all the minors of the matrix corresponding to some finite presentation $\mathbb{Z}[G]^{\oplus m} \to \mathbb{Z}[G]^{\oplus n} \twoheadrightarrow Cl_K$, which can be regarded as a generalization of the characteristic ideal of an Iwasawa module. In this talk, we will explicitly determine the Fitting ideal of the p-part of Cl_K in terms of Stickelberger elements under certain assumptions when the p-Sylow subgroup of G is isomorphic to $\mathbb{Z}/p\mathbb{Z} \oplus \mathbb{Z}/p\mathbb{Z}$.

Moon, Hyunsuk

Title: On the invariant $M(A_{/K}, n)$ of Chen-Kuan for Galois representations

Abstracts: Let X be a finite set with a continuous action of the absolute Galois group of a global field K. We suppose that X is unramified outside a finite set S of places of K. For a place $\mathfrak{p} \notin S$, let $N_{X,\mathfrak{p}}$ be the number of fixed points of X by the Frobenius element at $\mathfrak{p}$. We define the average value $M(X)$ of $N_{X,\mathfrak{p}}$ where $\mathfrak{p}$ runs through the non-archimedean places in K. This generalize the invariant of Chen-Kuan and we apply this for Galois representations. Our results show that there is a certain relationship between $M(X)$ and the size of the image of Galois representations.

Morisawa, Takayuki

Title: Class Numbers in $\mathbb{Z}_S$-extensions of $\mathbb{Q}(\sqrt{-1})$ with restricted ramification.

Let $S_{\mathbb{Q}}$ be a finite set of distinct prime numbers which are congruent to 1 modulo 4

and S a set of prime ideals of $\mathbb{Q}(\sqrt{-1})$ consisting of a choice of primes above p for each p in $S_{\mathbb{Q}}$. In this talk, we shall discuss the ℓ-indivisibility of class numbers of intermediate fields in the unique $\mathbb{Z}_S$-extension of $\mathbb{Q}(\sqrt{-1})$ unramified outside S.
This is a joint work with J. Lamplugh.

Narita, Hiro-aki

Title: Lifting to an inner form of GL(4) and counterexamples of the Ramanujan conjecture

Abstract: In this talk we carry out an explicit construction of cusp forms on the 5-dimensional real hyperbolic space by lifting from Maass cusp forms for $\Gamma_0(2)$. We show that automorphic representations generated by them provide counterexamples of the Ramanujan conjecture for GL(2) over a division quaternion algebra, which is an inner form of GL(4). This is a joint work with Masanori Muto and Ameya Pitale.

Onishi, Yoshihiro

Title: New addition formulae for Weierstrass elliptic functions and for higher genus Abelian functions

Abstract: For the Weierstrass functions $\sigma(u)$ and $\wp(u)$ we know the formula $\sigma(u + v)\sigma(u - v)/\sigma(u)^2\sigma(v)^2 = \wp(v) - \wp(u)$ and three or more variable generalization of this. This formula corresponds to canonical involution $v \mapsto -v$. Through my recent researches, I realized that this involution should be regarded as the exchange of the two points v and $-v$ which appear as the inverse image of the function $v \mapsto \wp(v)$. This means that it is natural to ask what happens if we replace the function $\wp(u)$ by another function. Investigating from this point of view and treating a function of any order, we get some new addition formulae. I will talk about higher genus case also from this point of view. This is a joint work with J.C. Eilbeck and M. England.

Otsubo, Noriyuki

Title: CM regulators, values of L-functions and hypergeometric functions.

Abstract: The conjectures of Beilinson-Bloch-Kato describe special values of L-functions in terms of regulators. When an abelian variety has complex multiplication by an abelian field, its period is written in terms of the gamma function. Hence so is the central L-value due to the Birch-Swinnerton-Dyer conjecture. In this talk, we give examples of CM motives whose regulators are written in terms of generalized hypergeometric functions, and provide evidence for the Beilinson conjecture.

Park, Jeehoon

Title: Batalin-Vilkoviski algebra for hypersurfaces and period integrals

Abstract: The goal of this talk is to reveal hidden structures on the Betti cohomology and the period integral of a smooth projective hypersurface X in terms of BV(Batalin-Vilkovisky) algebras and homotopy Lie theory. We construct an explicit BV algebra which is quasi-isomorphic to the middle-dimensional primitive cohomology of X and enhance the Griffiths period integral to a BV algebra morphism. Moreover, we lift the Hodge filtration and the cult product polarization to the BV algebra, As an application, we provide an explicit algorithm to compute the period matrix of a deformed hypersurface and the

Gauss-Manin connection. This is a joint work with Jae-Suk Park.

Sano, Takamichi
Title: On arithmetic properties of Rubin-Stark elements
Abstract: This is a joint work with David Burns and Masato Kurihara. The Rubin-Stark conjecture predicts the existence of certain integral elements, called Rubin-Stark elements, which are related to the values of Artin L-functions at $s = 0$. In this talk, we propose a conjecture which relates Rubin-Stark elements with higher Fitting ideals of certain Weil-etale cohomology groups. Our conjecture generalizes and refines the Brumer conjecture concerning the annihilation of ideal class groups. We explain that our conjecture is a natural consequence of the equivariant Tamagawa number conjecture for a particular Tate motive.

Sato, Nobuo
Title: A refinement of Zagier's conjecture for imaginary quadratic fields.
Abstract: Zagier formulated a conjecture on special values of Hecke L-function at non-positive integers using polylogarithm functions. We formulate a refinement of Zagier's conjecture for imaginary quadratic fields based on Shintani's method.

Shin, Donghwa
Title: Generation of class fields by using the Weber function
Abstract: Let K be an imaginary quadratic field and $\mathcal{O}_K$ be its ring of integers. Let h_E be the Weber function on a certain elliptic curve E with complex multiplication by $\mathcal{O}_K$. We show that if N (> 1) is an integer prime to 6, then the function h_E alone generates the ray class field modulo $N\mathcal{O}_K$ over K when evaluated at some N-torsion point of E.

Yamamoto, Shuji
Title: On Kawashima's functions
Abstract: G. Kawashima introduced a family of functions as Newton series interpolating finite multiple harmonic sums, and used them to obtain a class of relations among multiple zeta values. These functions can also be regarded as multiple generalizations of the digamma function. In this talk, I discuss how classical formulas for the digamma function generalize to Kawashima's functions.

Yamana, Shunsuke
Title: Poles of the twisted exterior cube L-functions for GL(6)
Abstract: I determine the irreducible cuspidal automorphic representations of GL(6) whose twisted exterior cube L-functions have poles. The proof is very short and uses quadratic base change.