Matching Extension in Graphs Embedded in Surfaces ♠

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Let $G$ be a graph with a perfect matching. The graph $G$ is said to be $1$-extendable (or matching-covered) if every edge of $G$ extends to (i.e. is a subset of) a perfect matching in $G$. Such graphs are basic to the study of the problem of determining the number of perfect matchings in a graph. This problem has significance in certain areas of organic chemistry. (Cf. [11], [12], [13].) In their chemical context, perfect matchings are called Kekulé structures.

A natural generalization of $1$-extendability is the concept of an $n$-extendable graph. (Cf. [5].) Again, let $G$ be a graph with a perfect matching and let $n$ be a non-negative integer such that $n \leq (|V(G)| - 2)/2$. The graph $G$ is said to be $n$-extendable if every matching of size $n$ extends to (i.e., is a subset of) a perfect matching in $G$.

Somewhat later, the $n$-extendable property was generalized in the following way by Aldred and Porteous (Cf. [7].) Let $m$, $n$ and $p$ be non-negative integers such that $p \geq 2(m+n+1)$. Then a graph $G$ with $p$ vertices is said to have the property $E(m, n)$ (or simply “$G$ is $E(m, n)$”) if for each pair of disjoint matchings $M$ and $N$ of sizes $m$ and $n$, respectively, there exists a perfect matching $F$ in $G$ such that $M \subseteq F$ and $F \cap N = \emptyset$. Clearly, a graph $G$ is $E(m, 0)$ if and only if it is $m$-extendable.

In a different direction and even more recently, investigation has begun on “distance-restricted” matchings. (Cf. ([1, 2, 3]).) The idea here is that although a matching $M$ of size $k$ might not extend to a perfect matching in $G$, if one requires that the edges of $M$ be suitably mutually distant from each other, then $M$ does extend.

The study of extendability as applied to embedded graphs had its beginning in a 1989 paper by the author [6] in which it was proved that no planar graph is $3$-extendable.

Subsequently, many papers have appeared which deal with all of the above concepts when restricted to graphs embedded in surfaces. In this area, topological properties such as genus and face-width are taken into account. (Cf. [8].)

This talk briefly reviews some of the work done in this area, concentrating primarily on the newest results, some of which have appeared in the last two or three years and some of which have not yet been published. Specifically, a recently obtained complete characterization of $2$-extendable and $3$-extendable $4$-connected quadrangulations of the torus will be discussed. (Cf. [9], [10].)

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References


