Resolvent problem for the complex Ginzburg-Landau equation with non-homogeneous term in L^2

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Let Ω be a bounded domain in $\mathbb{R}^N$ with C^2-boundary $\partial\Omega$. In $L^2(\Omega)$ we consider the following *resolvent* problem:

$$\text{(RCGL)} \qquad \begin{cases} (\xi + i\eta)u - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^{q-2}u = f & \text{on} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where $i = \sqrt{-1}$, $\xi, \lambda, \kappa \in \mathbb{R}_+ := (0, \infty)$, $\eta, \alpha, \beta \in \mathbb{R}$ and $q > 2$ are constants and u is a complex-valued unknown function. (RCGL) is associated with the initial-boundary value problem for the complex Ginzburg-Landau equation (CGL). For (CGL) see [3]. In particular, if $\lambda \geq \sqrt{\kappa^2 + \beta^2}$, then there exists a unique strong solution to (RCGL) for "$f \in H_0^1(\Omega)$" (see [1]); for the case where "$f \in L^q(\Omega)$" see [2]. In this paper we improve the result of [1]. Namely, we establish the existence and uniqueness of strong solutions to (RCGL) for "$f \in L^2(\Omega)$" without any restriction on λ, κ and β under the restriction on q:

$$(*) \qquad \begin{cases} 2 < q < 2 + \dfrac{4}{N} & (N \geq 2), \\ 2 < q < 4 & (N = 1). \end{cases}$$

Definition. A function u is said to be a *strong solution* to (RCGL) if
(a) $u \in H^2(\Omega) \cap H_0^1(\Omega) \cap L^{2(q-1)}(\Omega)$;
(b) u satisfies the equation in (RCGL) formulated in $L^2(\Omega)$.

Main Theorem. *Let $N \in \mathbb{N}$, $\xi, \lambda, \kappa \in \mathbb{R}_+$ and $\eta, \alpha, \beta \in \mathbb{R}$. Assume that condition* $(*)$ *is satisfied. Then for any $f \in L^2(\Omega)$ there exists a unique strong solution u to* (RCGL) *with ξ satisfying $\xi > (C\delta R^\theta)^{\frac{1}{1+\theta}}$, where $R := \|f\|_{L^2(\Omega)}$, $C = C(q, N, \lambda, \kappa, \beta) > 0$, $\delta = \delta(q, \kappa, \beta) > 0$ and $\theta = \theta(q, N) > 0$ are constants.*

To prove this theorem we establish a new type of inequality instead of those used in [1].

References

[1] 小山　雅賢, Stationary problem for the complex Ginzburg-Landau equation, 修士論文, 2005.

[2] N. Okazawa, Semilinear elliptic problems associated with the complex Ginzburg-Landau equation, *Partial Differential Equations and Functional Analysis*, 169–187, Oper. Theory Adv. Appl. vol. 168, Birkhäuser, Basel, 2006.

[3] N. Okazawa and T. Yokota, Non-contraction semigroups generated by the complex Ginzburg-Landau equation, *Nonlinear Partial Differential Equations and Their Applications* (Shanghai, 2003), 490–504, GAKUTO Internat. Ser. Math. Sci. Appl. vol. 20, Gakkōtosho, Tokyo, 2004.