Continuous dependence on modelling parameters for the complex Ginzburg-Landau equation with inhomogeneous boundary condition

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Let Ω be a star-shaped (with respect to the origin) bounded domain in $\mathbb{R}^d$ $(d \geq 2)$ with smooth boundary $\partial\Omega$. Then we consider the following initial-boundary value problem for the complex Ginzburg-Landau equation with "inhomogeneous" Dirichlet boundary condition:

$$(\mathbf{CGL})_{\mu,\nu} \quad \begin{cases} \dfrac{\partial u}{\partial t} = \gamma u - (\mu + i\nu)|u|^2 u + (\alpha + i\beta)\Delta_x u & \text{in } \Omega \times (0,T), \\ u = u_B & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases}$$

where $i = \sqrt{-1}$, $\mu > 0$, $\alpha > 0$, γ, ν, $\beta \in \mathbb{R}$, $\Delta_x u := \sum_{k=1}^d \partial^2 u/\partial x_k^2$ and u is a complex-valued unknown function. Here Ω and $\mu + i\nu$ satisfy the following additional conditions:
(**A1**) $\min\{x \cdot n(x); x \in \partial\Omega\} > 0$, where $n(x)$ is the unit outward normal vector at $x \in \partial\Omega$;
(**A2**) $|\nu| \leq \sqrt{3}\mu$,
where (**A2**) is determined by the Liskevich-Perelmuter inequality (see e.g., [**1**, Lemma 1.2]).

Yang-Gao [**2**] showed that the solution to $(\mathbf{CGL})_{\mu,\nu}$ depends continuously on the modelling parameter $\mu + i\nu$ under the various cases. Their results and ours are summarized as follows.

	[**2**, Th. 2.2]	[**2**, Th. 2.4]	[**2**, Th. 3.1]	**Main Theorem**
spatial dimension	$2 \leq d \leq 4$	$d \geq 2$	$2 \leq d \leq 6$	$d \geq 2$
initial value	$u_0 \in L^2(\Omega)$	$u_0 \in L^4(\Omega)$	$u_0 \in L^2(\Omega)$	$u_0 \in L^2(\Omega)$
boundary value	$u_B \in H^2$	$\|u_B\|^2 u_B \in H^2$	$u_B \in H^{3/2,3/4}$	$u_B \in W^{2,4}$
restriction on α, β	nothing	$\|\beta\| \leq \sqrt{3}\alpha$	nothing	nothing

The purpose of this talk is to establish the following theorem which improves all the three theorems [**2**, Theorems 2.2, 2.4 and 3.1].

Main Theorem. *Let Ω satisfy* (**A1**) *and let μ_j, ν_j satisfy* (**A2**), *i.e., $|\nu_j| \leq \sqrt{3}\mu_j$ $(j = 1,2)$. Assume that $d \geq 2$, $u_0 \in L^2(\Omega)$ and $u_B \in W^{2,4}(\partial\Omega \times (0,T))$. Let u_j be a solution to $(\mathbf{CGL})_{\mu_j,\nu_j}$ $(j = 1,2)$. Then the solution depends continuously on the modelling parameter $\mu + i\nu$, i.e.,*

$$\|u_1(t) - u_2(t)\|_{L^2(\Omega)}^2 \leq 4\sqrt{(\mu_1 - \mu_2)^2 + (\nu_1 - \nu_2)^2}\,(1 + \mu_1^{-1} + \mu_2^{-1})e^{C_1 t} C_2(t),$$

where $C_1 = C_1(\gamma, \alpha, |\Omega|)$ and $C_2(t) = C_2(t, \alpha, \beta, \gamma, \|u_B\|_{W^{2,4}(\partial\Omega\times(0,t))}, \|u_0\|_{L^2(\Omega)})$ are constants.

The following proposition plays an essential role in the proof of Main Theorem.

Proposition. *Assume* (**A1**). *Let $u_B \in W^{2,4}(\partial\Omega \times (0,T))$. Then there exists the auxiliary function ψ such that $\psi(t) \in W^{2,4}(\Omega)$ a.a. $t \in (0,T)$, $\psi \in L^4(\Omega \times (0,T)) \cap H^1(\Omega \times (0,T))$ and*

$$\begin{cases} \Delta_x \psi = 0 & \text{in } \Omega \times (0,T), \\ \psi = u_B & \text{on } \partial\Omega \times (0,T). \end{cases}$$

References

[1] N. Okazawa, T. Yokota, *Global existence and smoothing effect for the complex Ginzburg-Landau equation with p-Laplacian*, J. Differential Equations **182** (2002), 541–576.

[2] Y. Yang, H. Gao, *Continuous dependence on modelling for a complex Ginzburg-Landau equation with complex coefficients*, Math. Meth. Appl. Sci. **27** (2004), 1567–1578.