高次元における一般の感応性関数をもつ Keller–Segel系の大域可解性

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本研究は仙葉隆教授 (福岡大学 理学部) との共同研究に基づく.

Consider positive solutions of the fully parabolic system,

$$\begin{cases} \tau u_t = \Delta u - \nabla \cdot (u \nabla \chi(v)) & \text{in } \Omega \times (0, \infty), \\ v_t = \Delta v - v + u & \text{in } \Omega \times (0, \infty), \end{cases} \tag{1}$$

under the homogeneous Neumann boundary conditions in a smooth bounded convex domain $\Omega \subset \mathbf{R}^n$ $(n \geq 2)$ with nonnegative smooth initial data. Here τ is a positive parameter, χ is a smooth function on $(0, \infty)$ satisfying $\chi' > 0$.

Theorem Assume that

$$\begin{cases} \lim_{s \to \infty} \chi'(s) = 0 & \text{if } n = 2, \\ \limsup_{s \to \infty} s \cdot \chi'(s) < \dfrac{n}{n-2} & \text{if } n \geq 3. \end{cases}$$

If τ is sufficiently small, the solution of (1) exists globally and remains bounded uniformly in time.