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DISCUSSION

Rejoinder on: Light tail asymptotics in multidimensional reflecting processes for queueing networks

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First of all, I sincerely thank all the four discussants for taking time to carefully read the paper. Incorporating their comments and suggestions, the original manuscript has been revised. This includes some structural changes.

1. The old Sect. 4.3, which details the Markov additive approach, is now moved to the new Sect. 5, and Sect. 4 is largely rewritten. So, Sects. 5–8 are shifted to Sects. 6–9.
2. New results are added in the new Sect. 6.1.
3. During the review of this paper, some new papers came out. They are referred to in the new Sect. 9, Concluding remarks.

I also thank the discussants for sharing discussions. It is my pleasure to respond to their comments and questions. It is noted that the discussants argued on the original version while my answers refer to the present version.

1 Answer to Prof. Florin Avram

Thank you very much for the information on the Russian and even Japanese literature. As for the two dimensional SRBM, I would like to make clear the difference of the tail asymptotics of $\mathbb{P}(\boldsymbol{Z} \in x\boldsymbol{v} + \mathbb{R}^2_+)$ and $\mathbb{P}(\langle \boldsymbol{Z}, \boldsymbol{v}\rangle > x)$ as $x \to \infty$ for a nonnegative vector $\boldsymbol{v}$ and a random vector $\boldsymbol{Z}$ subject to the stationary distribution of the two dimensional SRBM under the stability condition. In Avram et al. (2001), the

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large deviations rate function is obtained, so both asymptotics can be computed in the sense of the rough asymptotic although extra assumptions, such as the $\mathcal{M}$-matrix condition, are needed. On the other hand, the second asymptotics are found in Dai and Miyazawa (2010), but they are exact asymptotics and do not require any extra assumption. I will further discuss on this issue in Sect. 2.

I absolutely agree that great contributions to this area have been made by the Russian school leading by Borovkov, Mougul'skii, Malyshev and some others. They are very strong in analysis. However, their results seem to have been not well diffused because of technical complexity. The network problems are so complicated, and therefore analytic results are getting hard to understand. To relax this situation, geometrical or graphical presentations may be helpful. I have not emphasized this aspect, but it is a spirit of this paper.

I also agree that there are so many related results on the asymptotic problems in stochastic processes. The area is too large for me, and thank you for helpful inputs.

2 Answer to Prof. Tomasz Rolski

Thank you very much for enthusiastic discussions. I agree that the Kella–Whitt martingale is a convenient tool for deriving the stationary equation in terms of moment generating functions for the SRBM. However, in finding the domain of the moment generating function of the stationary distribution, we cannot directly work on the stationary equation of moment generating functions because their existence is questioned. To get out this circular situation, Dai and Miyazawa (2010) used other types of test functions, namely, twice continuously differentiable functions with compact supports. The moment generating functions are obtained as their limits. This may not be necessary, but it works well.

Your question about the convergence domain is exactly what I am now studying. For the two dimensional SRBM, let I be the large deviations rate function, and let $\mathcal{D}$ be the domain of the moment generating function of its stationary distribution. It is proved in Dai and Miyazawa (2010) that

$$I(\boldsymbol{v}) \geq \sup\big\{\langle \boldsymbol{\theta}, \boldsymbol{v}\rangle; \boldsymbol{\theta} \in \mathcal{D}\big\}, \qquad \boldsymbol{v} \geq \mathbf{0}, \tag{2.1}$$

and the equality in this formula was conjectured. It should be noticed that (2.1) is not a direct consequence of Chernoff inequality as discussed in Lieshout and Mandjes (2008), Miyazawa and Rolski (2009).

As mentioned in Sect. 1 of this rejoinder, this rate function I is explicitly obtained in Avram et al. (2001), but its expression is much different from the right side of (2.1). So, it is not an easy task to verify this conjecture.

However, it is turned out by my recent study with Jim Dai that this conjecture is not always true except for $\boldsymbol{v} = (v, 0)$ or $(0, v)$ for $v > 0$. This suggests that the domain $\mathcal{D}$ itself is insufficient to determine the rate function, while it is sufficient to determine the tail asymptotics in the coordinate directions and those of $\mathbb{P}(\langle \boldsymbol{Z}, \boldsymbol{v}\rangle > x)$ because

$$-\lim_{x\to\infty} \frac{1}{x} \log \mathbb{P}\big(\langle \boldsymbol{Z}, \boldsymbol{v}\rangle > x\big) = \sup\{t \geq 0; t\boldsymbol{v} \in \mathcal{D}\}, \qquad \boldsymbol{v} \geq \mathbf{0}. \tag{2.2}$$

Thus, the domain is important but does not have full information even for rough asymptotics. This is something to be unexpected for us, and we are now writing a paper on this topic.

Thank you for deriving the stationary equation in terms of moment generating functions for the multidimensional reflecting Lévy process and the explicit results for the tandem queue case. I think they can be used to get the tail asymptotics of the stationary distribution. In particular, the tandem queue may be a good example for the multidimensional asymptotics for $d \geq 3$.

3 Answer to Prof. Peter Taylor

Thank you very much for interesting questions. It is indeed interesting to compare the large deviations technique with the analytic function approach. This is also closely related to Tomasz's question and my answer in Sect. 2. Since the rate function I can be obtained as the minimization problem of a certain cost function under the framework of sample path large deviations, two approaches may be considered to be dual in the sense of the minimization and maximization of nonnegative functions. However, the optimization in the large deviations approach is a variational problem on a space of functions, while that in the analytic function approach has variables in a finite dimensional vector space. Thus, the two problems are so different in formulation, and I have no idea of how to connect them.

Another difference in the two approaches is technical assumptions. For example, the large deviations approach for the SRBM needs a good property of a sample path, the so-called strong solution for generating a reflecting process. For this, a certain extra assumption is required for the reflection matrix R of the SRBM. This is not the case for the analytic function approach, but the large deviations approach can be used for a larger class of asymptotic problems. Thus, both approaches have their own merits and demerits.

As for the question on the program for deriving the convergence domain, the problem of finding the initial value $\boldsymbol{\theta}$ is converted to the problem on geometries produced by the moment generating functions of interior and boundary transitions, in the revised version. The latter geometric question can be answered by the stability condition as detailed for $d = 2$ in Sect. 6.3. In other words, the existence of a good initial point may be solvable by the stability condition. The stability problem for $d \geq 4$ remains unsolved, so the problem of finding a good initial point may be hard as well. However, I hope the geometrical interpretations may be helpful to challenge these problems.

4 Answer to Prof. Yiqiang Zhao

Thank you very much for explaining the details of the kernel method. In the revised version, I discussed more about the analytic function approach including the kernel method. However, I was mainly concerned with moment generating functions, so I appreciate your detailed discussions on the kernel method using generating functions.

I like to add one thing to the kernel method. This is the validity of the stationary equation (0.1), which is called the "fundamental form" in your report. We know its validity for $|x|, |y| \leq 1$, but we cannot simply make analytic extensions of the generating functions to the region $|x|, |y| > 1$ because the extension paths may across the curve $H(x, y) = 0$. This issue is the same as the geometric (or exponential) tightness of a stationary distribution, equivalently, the light tail in my terminology.

I do agree that generating functions are easier to use for deriving the asymptotics than those for moment generating functions because the state space is discrete. However, there are some drawbacks. First, it is not convenient to consider the tail asymptotics in an arbitrary direction because polynomials are not suitable for such asymptotics. Second, it lacks a geometric interpretation of the kernel curve $H(x, y) = 0$ in the x–y plane. For example, the interior of this curve is generally not convex while the corresponding set generated by the moment generating functions is convex. This convexity may not be needed for analysis, but it is certainly helpful to find the convergence domain, that is, the region for the fundamental form to be valid. I think both functions can help each other.

Thanks also for general remarks on the analytic function approach. I agree that the analytic method deserves special attention for the tail asymptotic problem. This reminds me of its applications to other areas such as combinatorics and partial differential equations. There may arise some other areas. I hope the techniques developed in different areas will contribute to each other.

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