Thermal Cloud Effects in the Dynamics of Bose Condensates in Optical Lattices

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Outline

- Introduction
- Thermal cloud effects in optical lattices
- Damping of collective modes
- Generalized GP equation in optical lattices
- Instability of the condensate
- Summary and outlook

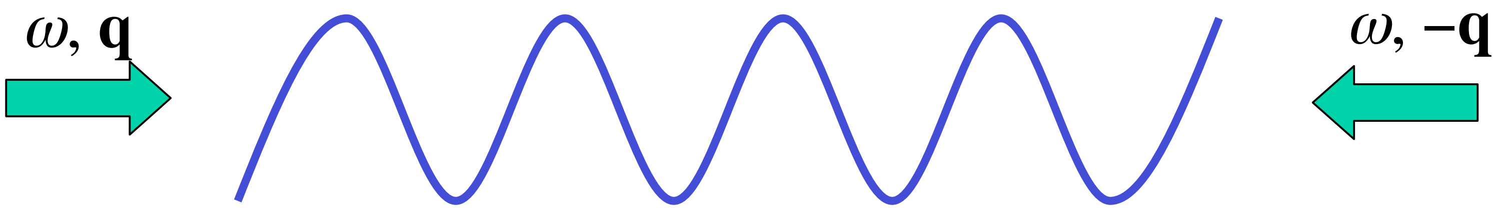
In this talk, I will mainly focus on giving an overview. Detailed discussion about the kinetic theory in optical lattices based on 2PI formalism will be given by Satoru Konabe's poster presentation.

Introduction

- Ultracold atoms in optical lattice potentials
- Review of *T*=0 condensate dynamics

Ultracold Atomic Gases in Periodic Potentials (Optical Lattices)

- Periodic potentials produced by two counter propagating laser beams.



$$V_{\rm opt}(z) = sE_R\cos^2 qz = sE_R\cos^2\left(\frac{\pi}{d}z\right)$$

d : lattice constant fixed by laser wavelength.

$E_R = \hbar^2 q^2/2m$: recoil energy.

s : dimensionless parameter describing strength of the lattice potential.

- Ideal crystal-like systems: **no impurities, possibilities of tuning the lattice parameters.**

Physics in the Presence of Optical Lattices

- No interaction (spin polarized Fermi gas, very dilute Bose gas)

  Interference in momentum distribution, Bloch oscillations

- **Weak interaction (shallow optical lattices)**

  Josephson oscillations, damping of condensate oscillations, Landau and dynamical instabilities

- Strong interaction (deep optical lattices)

  strongly-correlated quantum gases, superfluid-Mott insulator transition

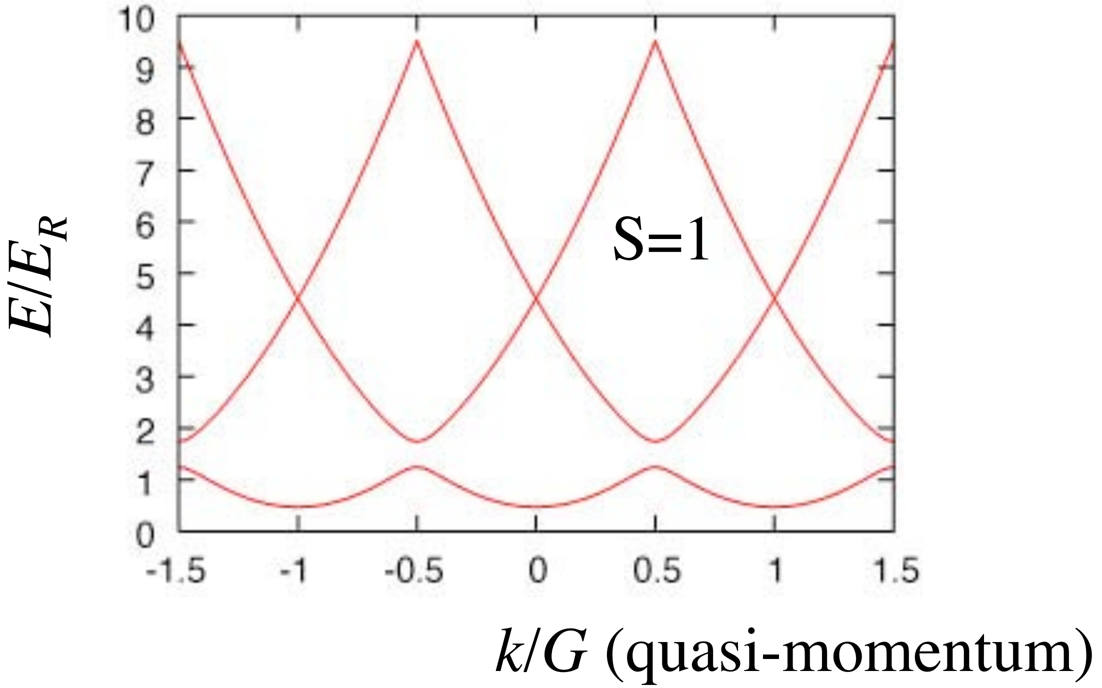
Ideal Bose Gas in 1D Optical Potential

- Bloch wave function

$$\frac{\hbar^2}{2m}\frac{d^2}{dz^2}u_k(z) + V_{\mathrm{opt}}(z)u_k(z) = E_k u_k(z)$$

$$u_k(z) = \sum_n u_{kn}\exp[i(k+nG)z], \quad G = 2q$$

- Bloch Energy Band



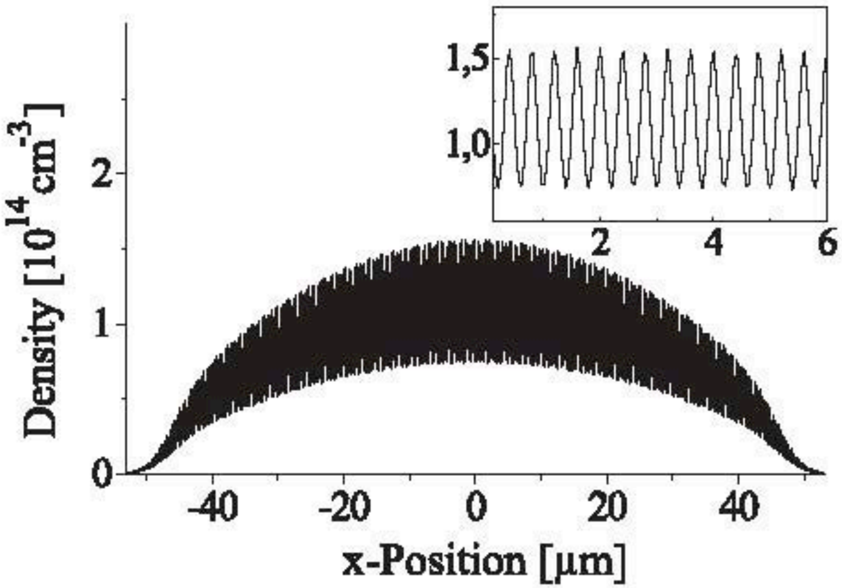
Condensate in Optical Lattices (T=0)

- Gross-Pitaevskii (GP) equation

$$i\hbar\frac{\partial\Phi}{\partial t} = \left(-\frac{\hbar^2\nabla}{2m} + V_{\mathrm{opt}} + V_{\mathrm{trap}} + g|\Phi|^2\right)\Phi$$

$$g = \frac{4\pi\hbar^2}{m}a$$ (a: s-wave scattering length)

- Equilibrium density profile



Condensate density profile averaged over the lattice constant d has the Thomas-Fermi profile:

$$n(\mathbf{r}) = \frac{\mu}{\tilde{g}}\left(1 - \frac{r_\perp^2}{R_\perp^2} - \frac{z^2}{Z^2}\right)$$

$\tilde{g}$ renormalized coupling constant

Condensate Bloch Band

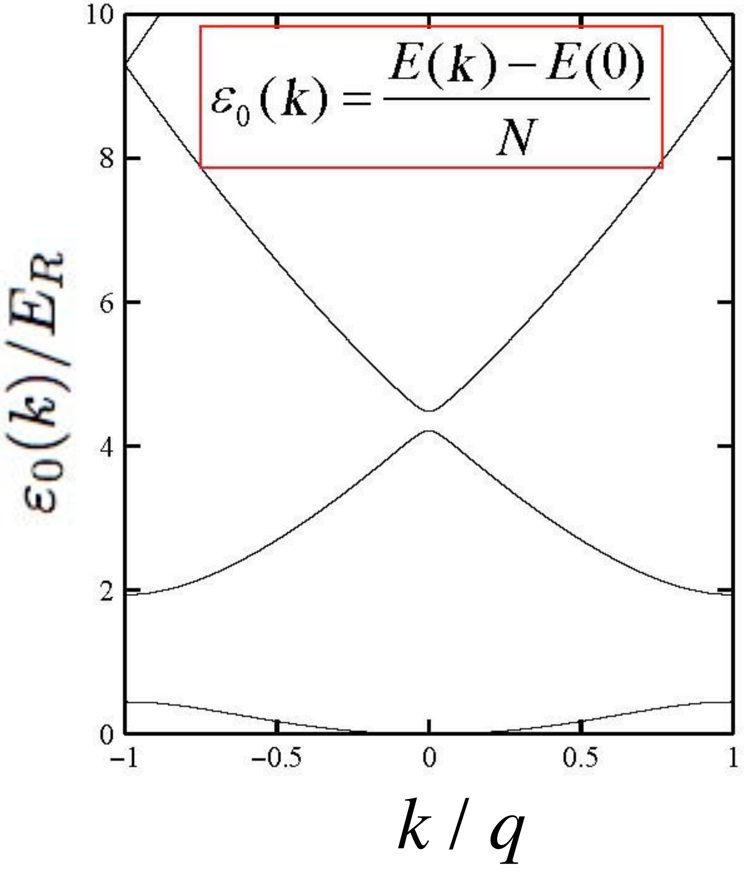
- Ground state solution can be generalized to solutions carrying quasi-momentum. Look for stationary solutions (Bloch solutions).

$$\Phi_{k_c}(z) = e^{ik_c z} u_{k_c}(z)$$

with u_{kc} periodic function

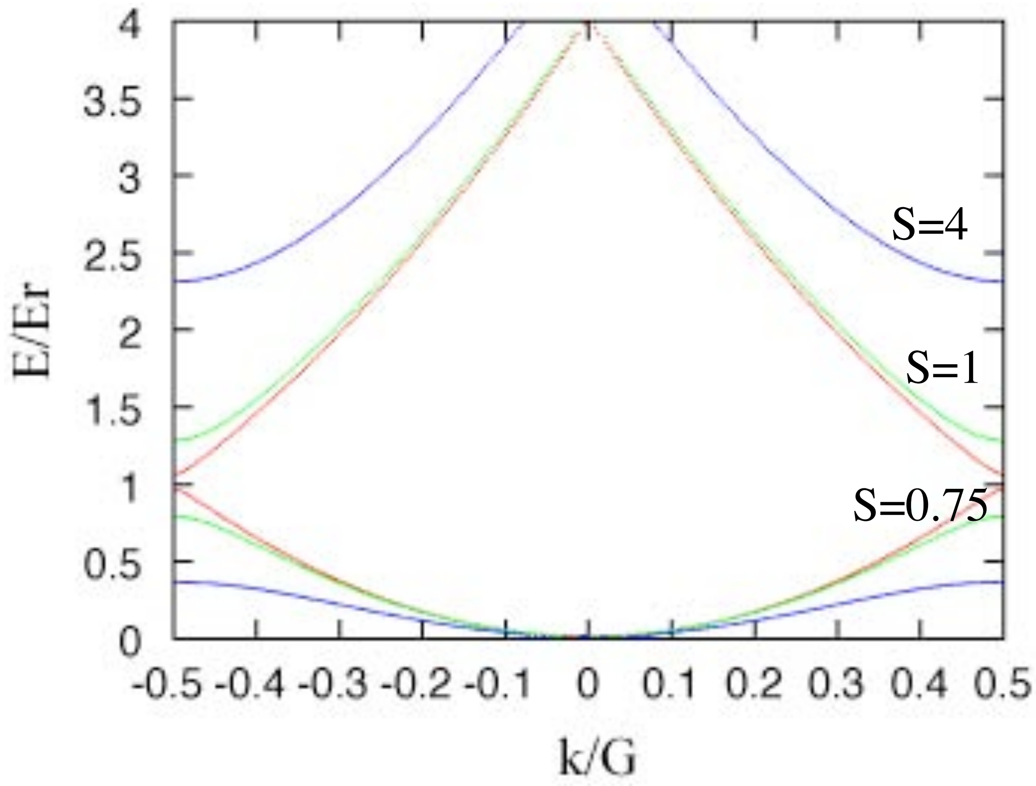
$$u_{k_c}(z+d) = u_{k_c}(z)$$

$$-\frac{\hbar^2}{2m}\left(\frac{d}{dz} - ik_c\right)^2 + [g|u_{k_c}(z)|^2 + V_{\mathrm{opt}}(z)]u_{k_c}(z) = \mu(k_c)u_{k_c}(z)$$



Excitation Spectrum (V_{trap}=0)

- Bogoliubov-Bloch excitations



- Phonon-like dispersion law in the long wavelength limit

$$E(k) \simeq \hbar c k \qquad c = \sqrt{\frac{n}{m^*}\frac{\partial \mu}{\partial n}}$$

(m^* : effective mass)

Coarse-Grained Hydrodynamic Equation

Kraemer et al. (2003)

- Introduce smoothed density averaged over lattice spacing, and write down the energy functional.

$$E = \int d\mathbf{r} \left\{ \frac{mn}{2} \left[v_x^2 + v_u^2 + \left(\frac{m}{m^*} \right) v_z^2 \right] + \frac{\tilde{g}n}{2} + nV_{\mathrm{trap}} \right\}$$

m^* : effective mass $\tilde{g}$: renormalized coupling constant

- One derives course-grained hydrodynamic equations.

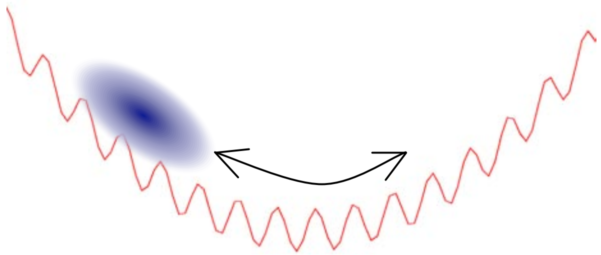
$$\frac{\partial n}{\partial t} + \partial_x (n v_x) + \partial_y (n v_y) + \partial_z \left(\frac{m}{m^*} n v_z \right) = 0$$

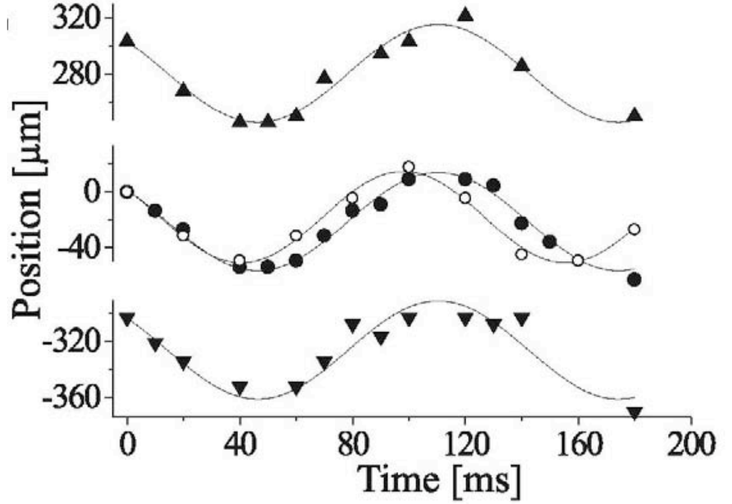
$$m \frac{\partial \mathbf{v}}{\partial t} + \nabla (\tilde{g} n + V_{\mathrm{trap}}) = 0$$

- We will later generalize this course-grained GP theory to finite temperatures, including the effect of thermal cloud atoms.

Dipole Oscillations

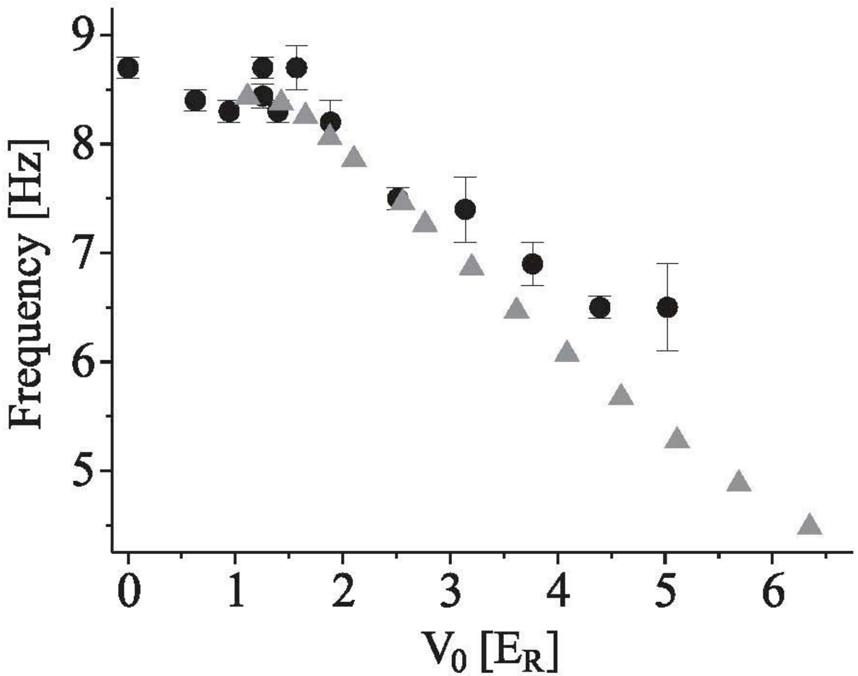
- 1D optical lattice + 3D harmonic trap potential





Cataliotti et al. (2001)

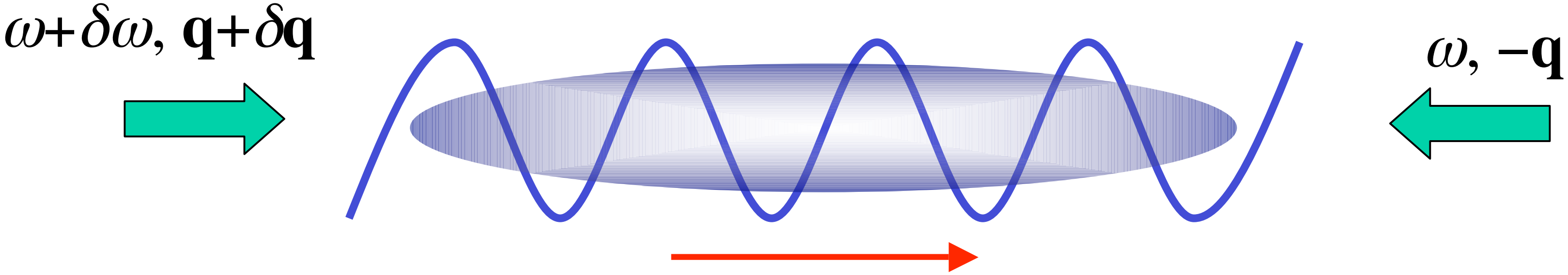
mode frequency v.s. lattice height



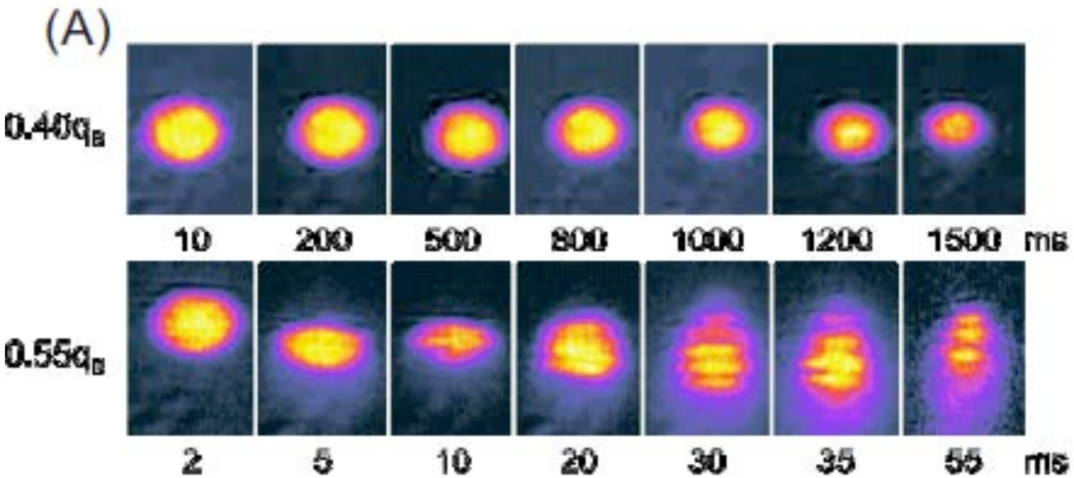
$$\frac{\omega}{\omega_z} = \sqrt{\frac{m}{m^*}}$$

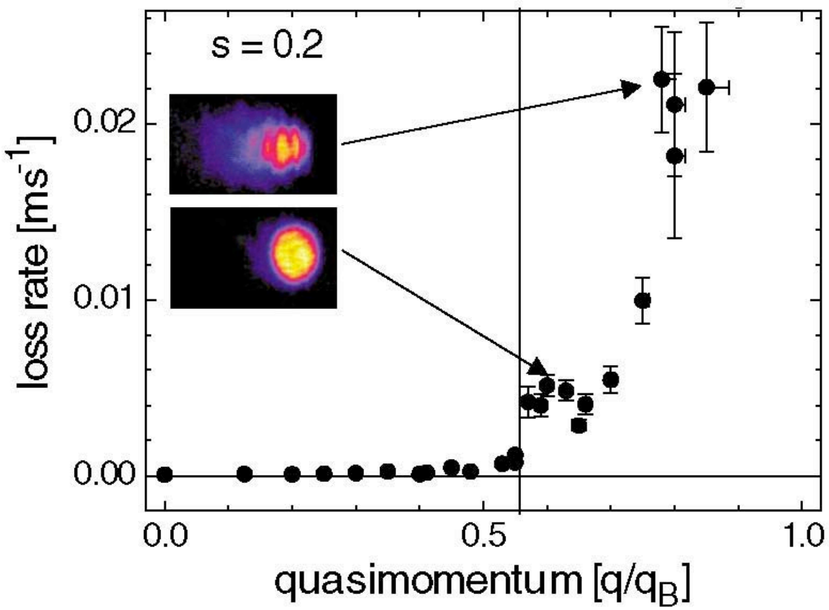
Condensates in Moving Lattices

- Moving optical lattices can produced by controlling the laser frequencies. Fallani et al., (2004)



- Dynamical instability above a critical velocity.





Dynamical Instability

- In the tight-binding limit, the dispersion relation in the presence of the lattice potential moving with the velocity *v*, is given by

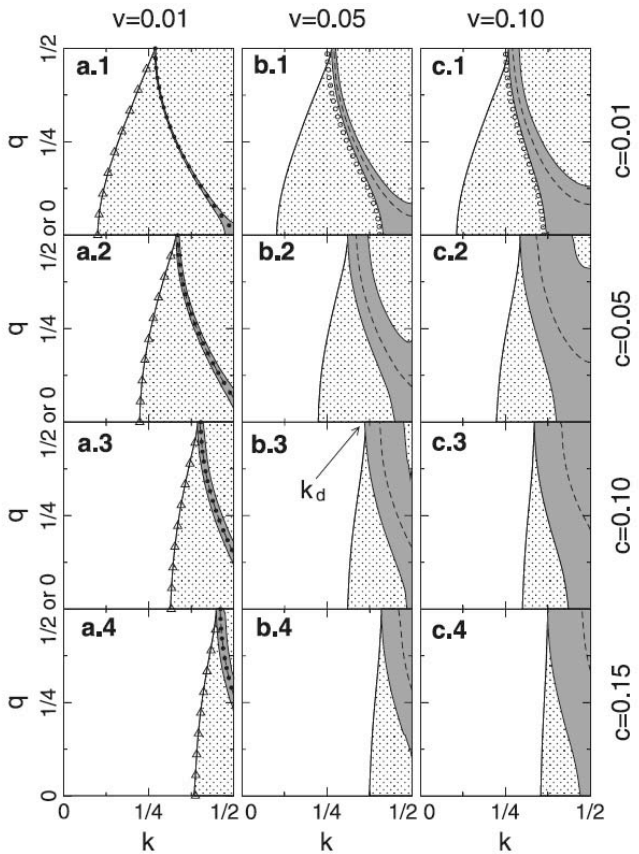
$$E(p) = \sqrt{\frac{\tilde{g}n}{m^*}\cos\frac{mv_0 d}{\hbar}p} \pm \frac{p}{m^* d}\sin\frac{mvd}{\hbar}$$

- Dynamical instability occurs if

$$\cos\frac{mvd}{\hbar} < 0 \quad \Rightarrow \quad \frac{mv}{\hbar} > \frac{\pi}{2d}$$

- Excitation energies possess the **imaginary part**, which leads to the exponential increase of the amplitude of collective oscillations.

- This kind of instability is well understood within the usual (T=0) GP equation !



Wu and Niu (PRA 2001)

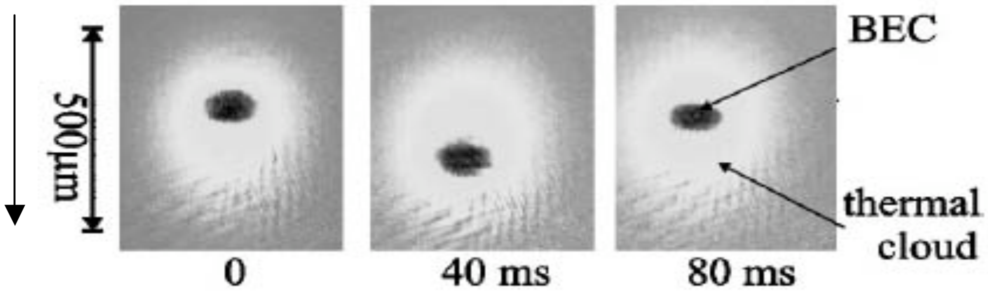
Motivation:
Thermal Cloud Effects in the Condensate Dynamics at Finite Temperatures

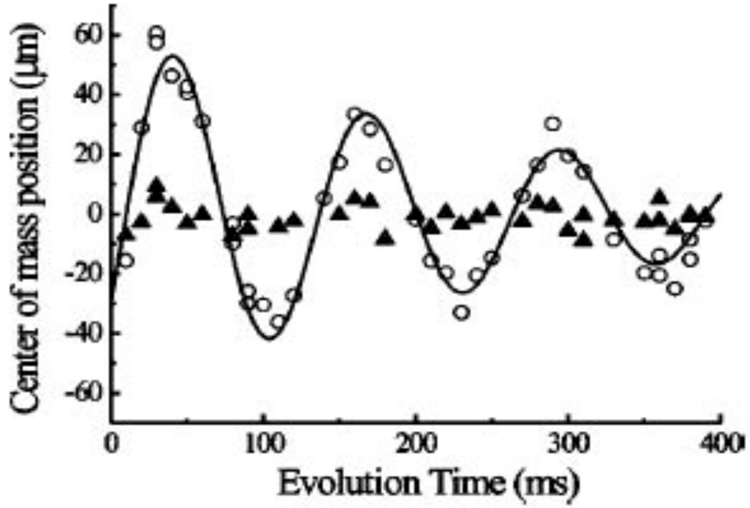
- Damping of the condensate oscillations.
- Instability of the condensate in the presence of the thermal cloud.

Damping of the Condensate Oscillation

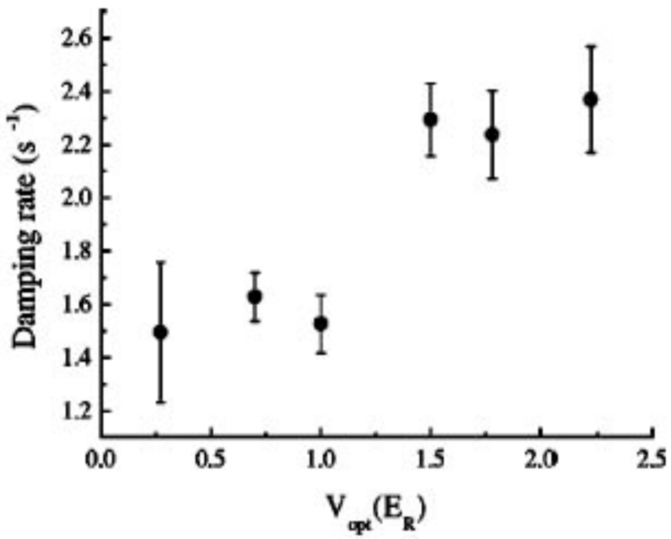
F. Ferlaino,et.al (2002)

The direction
of the lattice



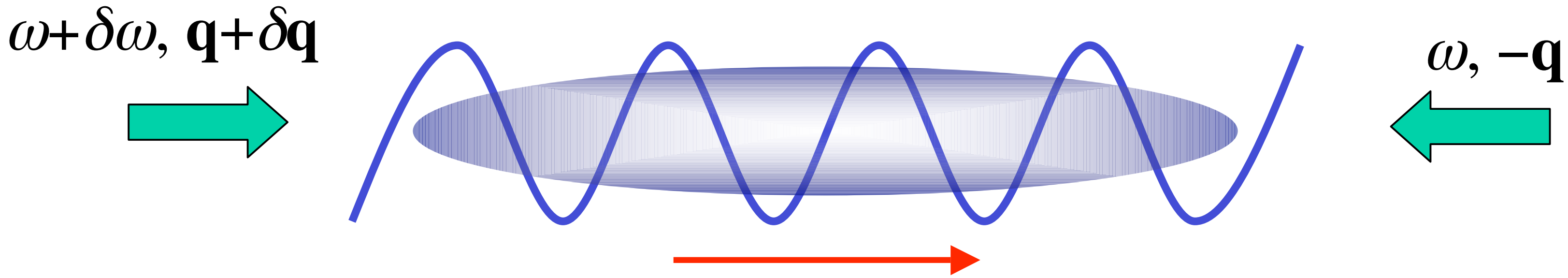


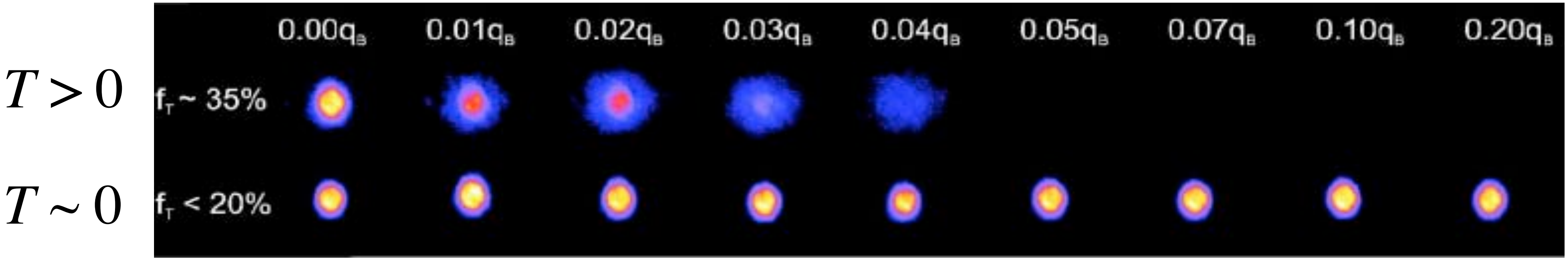
- Thermal cloud atoms cannot coherently move in the presence of optical lattice potentials.
- Dipole oscillations exhibit strong damping with increasing lattice depth.



Instability of the Condensate in the Presence of Thermal Cloud

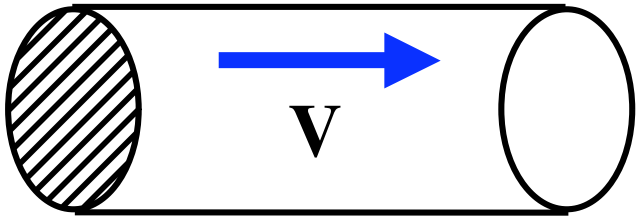
L.de.Sarlo et al. PRA **72** 013603 (2004)





- In the presence of the thermal cloud, one observes a strong reduction of the condensate atoms, even when the lattice velocity is well below the critical velocity for the dynamical instability.

Energetic Instability



- If the condensate is moving with the velocity $\mathbf{v}$, the excitation energy is given by $\epsilon(\mathbf{p}) \to \epsilon(\mathbf{p}) + \mathbf{p}\cdot\mathbf{v}$
- The excitation energy can become negative $\epsilon(\mathbf{p}) + \mathbf{p}\cdot\mathbf{v} < 0$

  when the velocity exceeds the critical velocity **(Landau criterion)**

$$v_c = \min \frac{\epsilon(\mathbf{p})}{p}$$

- The excitations are spontaneously produced, making the condensate unstable.

- What is the role of thermal cloud in optical lattices?
- What is the microscopic mechanism of the instability?

Finite-Temperature Theory of Bose Condensates in Optical Lattices

- In order to understand the role of thermal cloud in optical lattices, we want to develop finite-temperature theory.

- **Hartree-Fock-Popov formalism**

  Equilibrium calculation
  Excitation spectrum
  Landau damping

- **Nonequilibrium kinetic theory (Konabe's poster)**

  2PI effective action → Generalized GP Equation for the condensate; Kinetic equation for the noncondensate

Hartree-Fock-Popov Calculations

Arahata and Nikuni, JLTP (in press)

- GP equation

$\Phi(\mathbf{r}) = \langle\hat{\psi}(\mathbf{r})\rangle \quad n_c(\mathbf{r}) = |\Phi(\mathbf{r})|^2$ **condensate**

$\tilde{\psi}(\mathbf{r}) = \hat{\psi}(\mathbf{r}) - \Phi(\mathbf{r}) \quad \tilde{n}(\mathbf{r}) = \langle\tilde{\psi}^\dagger(\mathbf{r})\tilde{\psi}(\mathbf{r})\rangle$ **noncondensate**

$$\mu\Phi(\mathbf{r}) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{opt}}(z) + V_{\mathrm{trap}}(\mathbf{r}) + g|\Phi(\mathbf{r})|^2 + 2g\tilde{n}(\mathbf{r})\right]\Phi(\mathbf{r})$$

- Bogoluibov equations

$$\hat{\mathcal{L}}u_i(\mathbf{r}) - gn_c(\mathbf{r})v_i(\mathbf{r}) = E_i u_i(\mathbf{r})$$
$$\hat{\mathcal{L}}v_i(\mathbf{r}) - gn_c(\mathbf{r})u_i(\mathbf{r}) = -E_i v_i(\mathbf{r})$$

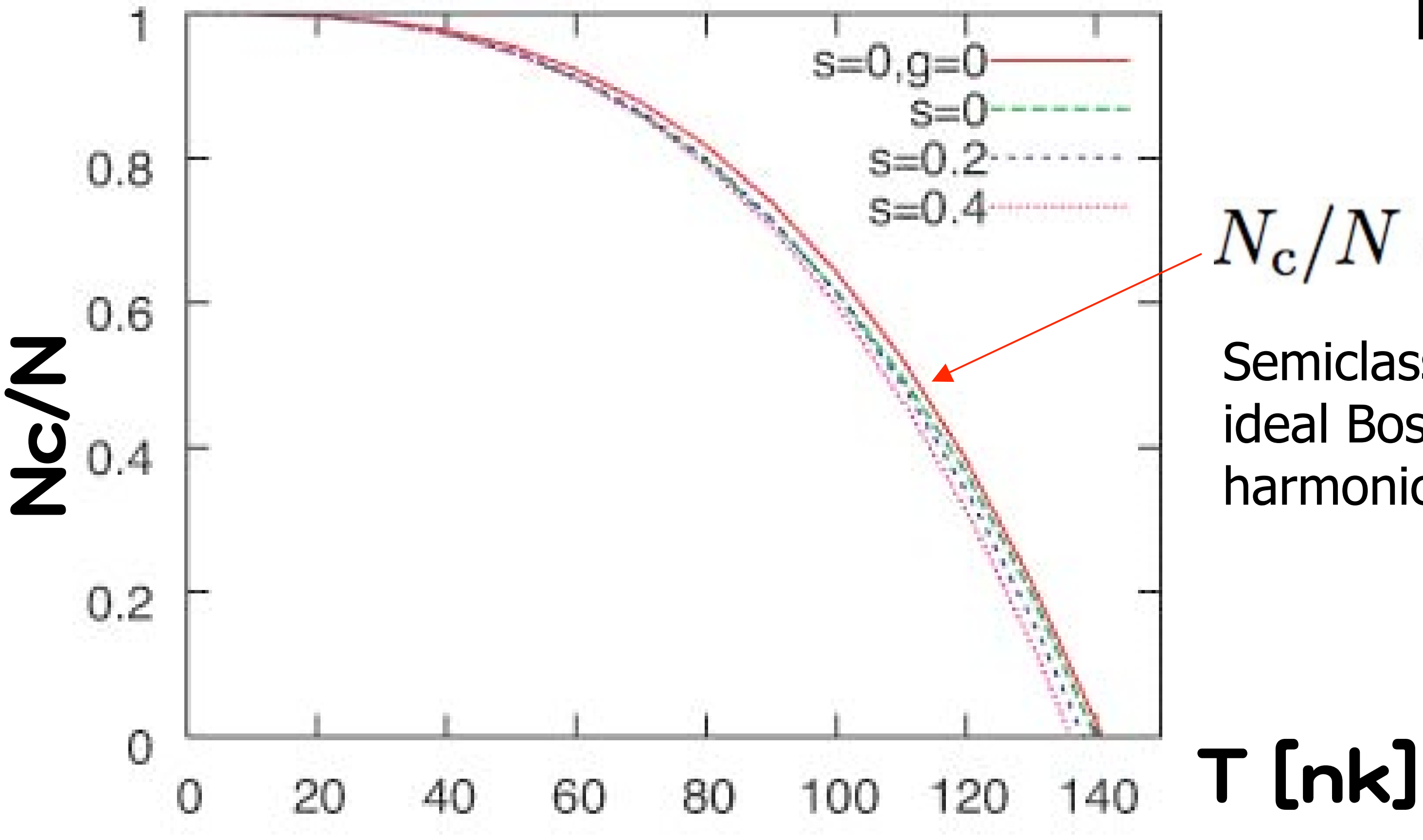
$$\hat{\mathcal{L}} = -\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{opt}}(z) + V_{\mathrm{trap}}(\mathbf{r}) + 2gn(\mathbf{r}) - \mu$$

$$\tilde{n}(\mathbf{r}) = \sum_i \left[|v_i(\mathbf{r})|^2 + \frac{|u_i(\mathbf{r})|^2 + |v_i(\mathbf{r})|^2}{e^{\beta E_i} - 1}\right]$$

Condensate Fraction

Calculations using the trap parameters for Florence experiment:

ω_z	9.0 Hz
$\omega_\perp$	92 Hz
λ	795 nm
N	400000



$$N_c/N = 1 - (T/T_c^0)^3$$

Semiclassical prediction of an ideal Bose gas in a 3D harmonic trap

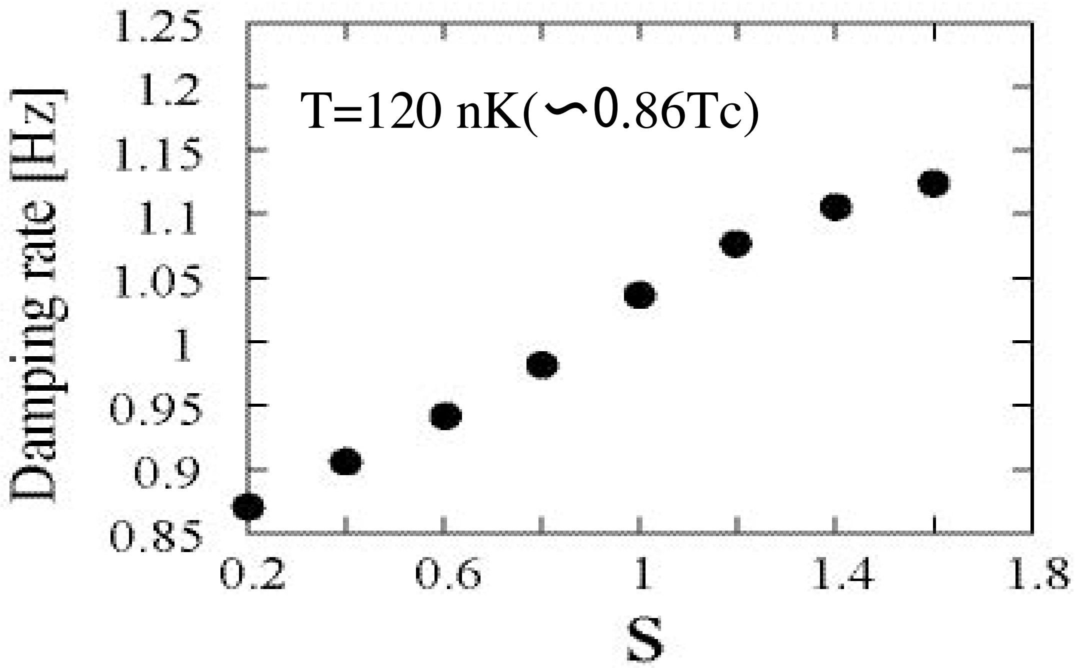
Damping of the Dipole Oscillation

- Landau damping rate

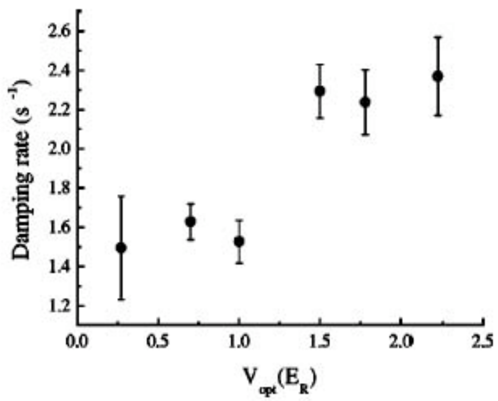
$$\Gamma = 4\pi g \sum |A_{ij}|^2 (f_i - f_j)\delta(\hbar\omega + E_i - E_j)$$

$$A_{ij} = \int d\mathbf{r}\Phi[u(u_i^* u_j + v_i^* v_j - v^* i u_j] - v(u_i^* u_j + v_i^* v_j - u^* v_j))]$$

《Lattice depth dependence of Landau damping 》



Experimented data

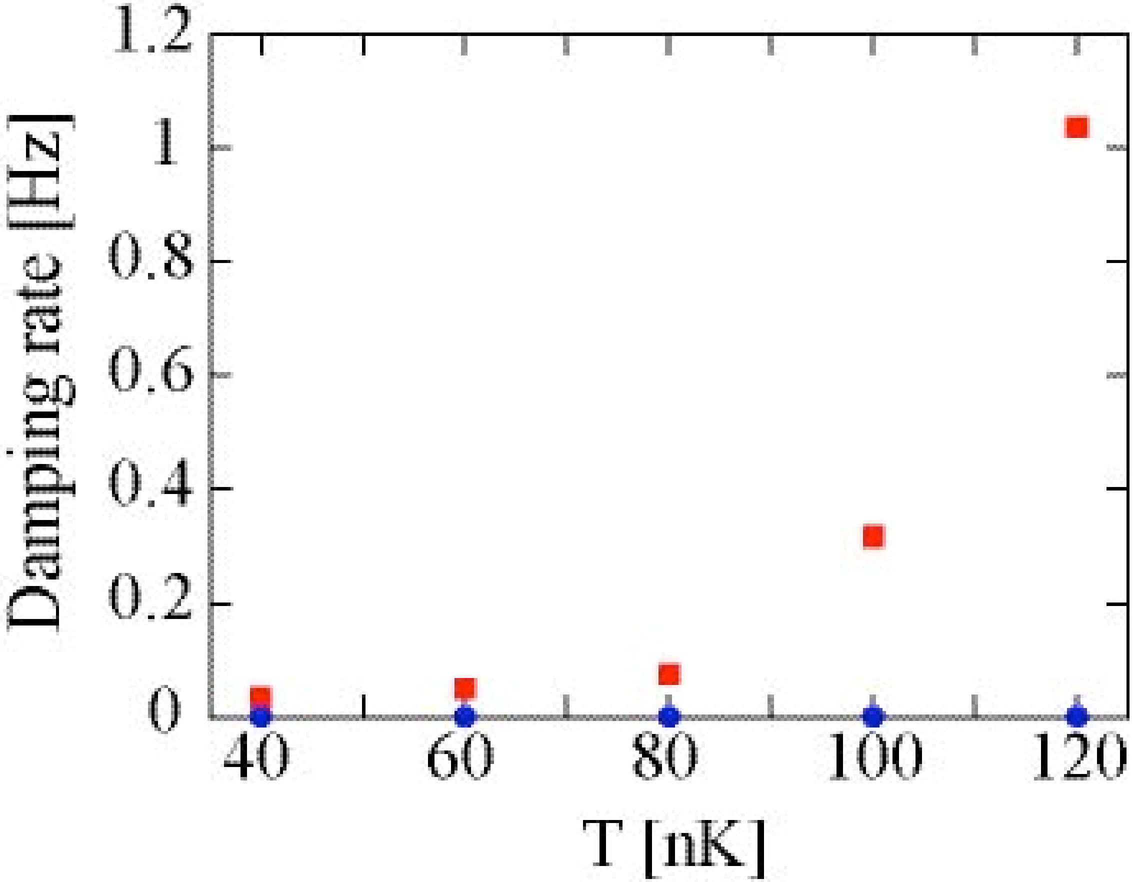


F. Ferlaino, et.al (2002)

Reasonable agreement with the experimental data.

Temperature Dependence of the Damping Rate

- Landau damping rate



Damping rate decreases significantly with decreasing temperature !!

Non-equilibrium Kinetic Theory for Bose Condensates in Optical Lattices

- In order to describe non-equilibrium behaviors, such as the instability of the condensate, one has to derive a kinetic theory including the effect of periodic potentials.
- We briefly review the ZNG formalism describing the coupled dynamics of the trapped condensate and thermal cloud .
- We derive a **coarse-grained generalized GP (GGP)** equation at finite temperatures, including the effect of lattice potentials.
- Our formalism is a natural extension of the coarse-grained condensate hydrodynamics derived by Kaemer et al..

Review of the ZNG formalism

- Generalized GP equation can be written as

$$i\hbar\frac{\partial\Phi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{trap}}(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2 + 2g\tilde{n}(\mathbf{r},t)\right]\Phi(\mathbf{r},t) - \int d\mathbf{r}'\int dt' F(\mathbf{r},t;\mathbf{r}'t).$$

$$\tilde{n}(\mathbf{r},t) = \langle\tilde{\psi}^\dagger(\mathbf{r},t)\tilde{\psi}(\mathbf{r},t)\rangle$$

- The function F describes the exchange of atoms between the condensate and thermal cloud. It involves noncondensate Green's functions, such as

$$G^<(\mathbf{r},t;\mathbf{r}',t') = -i\langle\tilde{\psi}^\dagger(\mathbf{r},t)\tilde{\psi}(\mathbf{r}',t')\rangle$$

- One also has the equation of motion for the noncondensate Green's function (which is often called Kadanoff-Baym equation).

Gradient Expansion

- To truncate the nonlocal term involving the function F, we assume that the macroscopic variables vary slowly in space and time. We thus make use of the approximation

$$\begin{aligned}\Phi(\mathbf{r}',t') &= \sqrt{n_c(\mathbf{r}',t')}e^{i\theta(\mathbf{r}',t')} \\ &\simeq \sqrt{n_c(\mathbf{r},t)}e^{i[\theta(\mathbf{r},t)+\partial_t\theta(\mathbf{r},t)(t'-t)+\nabla\theta(\mathbf{r},t)\cdot(\mathbf{r}'-\mathbf{r})]} \\ &\equiv \Phi(\mathbf{r},t)e^{i[\varepsilon_c(\mathbf{r},t)(t'-t)/\hbar+p_c(\mathbf{r},t)\cdot(\mathbf{r}'-\mathbf{r})/\hbar]}\end{aligned}$$

- For the noncondensate Green's function, we separate out the variables describing "slow" and "fast" processes.

$$G(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) = \int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}G(\mathbf{k},\omega;\mathbf{R},T)$$

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2,\quad t=t_1-t_2,\quad T=(t_1+t_2)/2$$

The slow dynamics described by the $(\mathbf{R},T)$ is treated semiclassically.

Coupled ZNG equations

- With these approximations, one obtains a closed set of the condensate and thermal cloud.
- Generalized GP equation for the condensate:

$$i\hbar \frac{\partial \Phi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\mathrm{trap}}(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2 + 2g\tilde{n}(\mathbf{r},t) - i\hbar R(\mathbf{r},t) \right] \Phi(\mathbf{r},t)$$

- Semiclassical kinetic equation for the noncondensate distribution:

collisional exchange of atoms

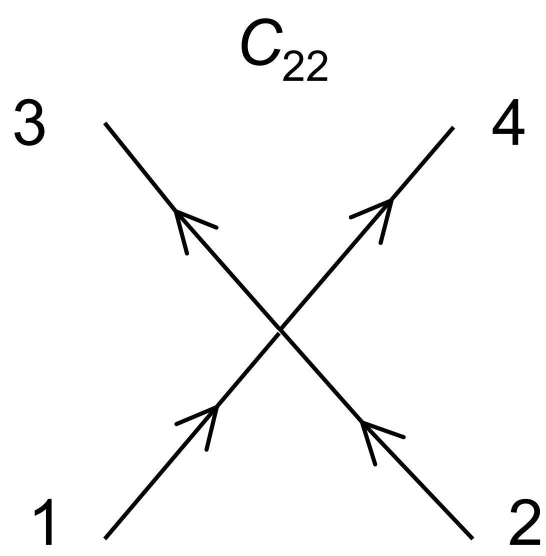
$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} - \nabla U \cdot \nabla_{\mathbf{p}} \right] f(\mathbf{p},\mathbf{r},t) = C_{12}[f,\Phi] + C_{22}[f]$$

collisions between thermal cloud atoms

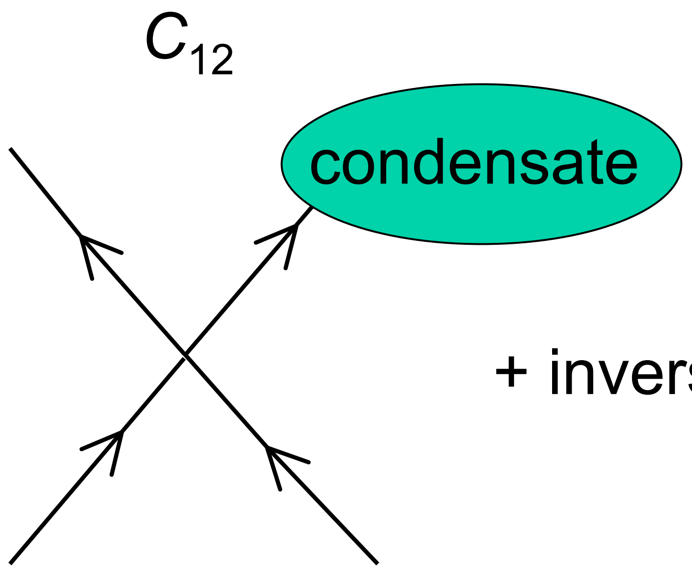
$$U(\mathbf{r},t) = U_{\mathrm{ext}}(\mathbf{r}) + 2g\tilde{n}(\mathbf{r},t) + 2gn_c(\mathbf{r},t)$$

Collision Terms

- Collision term in the kinetic equation.



collisions between thermal cloud atoms



+ inverse

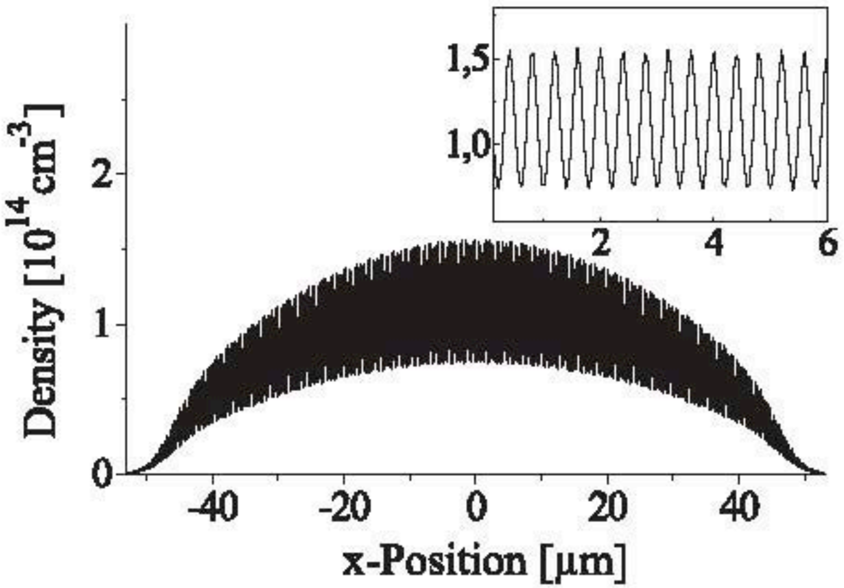
collisions between atoms in the condensate and thermal cloud (colllisional exchange)

- Dissipative term in the GP equation.

$$R(\mathbf{r},t) = \frac{1}{2n_c}\int \frac{d\mathbf{p}}{(2\pi\hbar)^3} C_{12}$$

Finite-Temperature Dynamics in Optical Lattices

- The ZNG coupled equations have been successfully used to describe the dynamics of trapped Bose gases at finite temperatures.
- In order to extend the kinetic theory to include optical lattices, one has to deal with the rapid spatial variation associated with the underlying lattice potential.



condensate density profile in a lattice potential + harmonic confinement

- For this purpose, it is convenient to work with the effective action, which involves integrations over position.

2PI Formalism for Bose gases in Optical Lattices

Konabe and Nikuni, JLTP (in press)

- We derive the 2PI effective action for the condensate wavefunction and the noncondensate Green's functions.

$$\Gamma[\Phi, G] = \Gamma_\Phi[\Phi, G] + \Gamma_G[G] = S_{\mathrm{GGP}}[\Phi, G] + \Gamma_G[G]$$

$$\begin{aligned} S_{\mathrm{GGP}} = -\int d\mathbf{r} \int dt \Phi^*(\mathbf{r}, t)\Big[-i\hbar\frac{\partial}{\partial t} - \frac{\hbar^2\nabla}{2m} + V_{\mathrm{opt}}(z) + V_{\mathrm{trap}}(\mathbf{r}) \\ + \frac{g}{2}|\Phi(\mathbf{r}, t)|^2 + 2g\tilde{n}(\mathbf{r}, t)\Big]\Phi(\mathbf{r}, t) \\ - \int d\mathbf{r} \int d\mathbf{r}' \int dt \int dt' \Phi^*(\mathbf{r}, t) F(\mathbf{r}, t; \mathbf{r}'t)\Phi(\mathbf{r}, t) \end{aligned}$$

- The generalized GP equation is obtained from

$$\frac{\delta S_{\mathrm{GGP}}[\Phi, G]}{\delta \Phi^*} = 0$$

- One obtains

$$i\hbar \frac{\partial \Phi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\text{trap}}(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2 + 2g\tilde{n}(\mathbf{r},t) \right] \Phi(\mathbf{r},t) - \int d\mathbf{r}' \int dt' F(\mathbf{r},t;\mathbf{r}'t).$$

- In order to eliminate rapid variations associated with lattice potentials, we must go back to the effective action, and introduce the Bloch functions.

$$\Phi_{k_c}(z) = e^{ik_c z} u_{k_c}(z) \qquad \tilde{\phi}_k(z) = e^{ikz} \tilde{u}_k(z)$$

rapid oscillation associated with lattice potentials

- Introduce coarse-grained quantities (average over lattice spacing)

$$\Phi(\mathbf{r},t) = \sqrt{n_c} e^{i\theta} \rightarrow \bar{\Phi}(\mathbf{r}_\perp, z, t) = \sqrt{\bar{n}_c} e^{i\bar{\theta}}$$

$$n_c(\mathbf{r},t) \rightarrow \bar{n}_c(\mathbf{r}_\perp, z, t), \qquad \theta(\mathbf{r},t) \rightarrow \bar{\theta}(\mathbf{r}_\perp, z, t)$$

Coarse-Grained GP Equation

- We derive a coarse-grained effective action for the smoothed variables.

$$S_{\rm GGP}[\Phi] \to \bar{S}_{\rm CG}[\bar{\Phi}]$$

- From the functional derivative, and making the gradient expansion, we finally obtain the finite-*T* coarse-grained GP equation

$$i\hbar \frac{\partial \bar{\Phi}(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2 \nabla_\perp^2}{2m} + \varepsilon_{\rm opt}(k_c, \mathbf{r}, t) + V_{\rm trap}(\mathbf{r}) + i\hbar \bar{R}(\mathbf{r},t) \right] \bar{\Phi}(\mathbf{r},t)$$

$\varepsilon_{\rm opt}(k_c, \mathbf{r}, t)$ local energy of the condensate in an optical lattice with the finite momentum $p_c = \hbar k_c$.

= modified dispersion along the lattice direction

$\bar{R}(\mathbf{r},t)$ dissipative term describing the exchange of atoms between the condensate and thermal cloud.

Generalized GP Hydrodynamic Equations

- We consider the low condensate velocity limit ($k_c << q$).

$$\frac{\partial \bar{n}_c(\mathbf{r},t)}{\partial t} + \nabla_\perp \cdot [\bar{n}_c(\mathbf{r},t)\bar{\mathbf{v}}_{c\perp}(\mathbf{r},t)] + \frac{\partial}{\partial z}\left[\left(\frac{m}{m^*}\right)\bar{n}_c(\mathbf{r},t)\bar{v}_{cz}(\mathbf{r},t)\right] = -\bar{\Gamma}(\mathbf{r},t)$$

$$m\frac{\partial \bar{\mathbf{v}}_c(\mathbf{r})}{\partial t} + \nabla\left[\bar{\mu}_c(\mathbf{r},t) + \frac{1}{2}\left(\frac{m}{m_\mu^*}\right)m\bar{v}_{cz}^2 + \frac{m}{2}\bar{\mathbf{v}}_{c\perp}^2\right] = 0$$

$$\bar{\mu}_c(\mathbf{r},t) = -\frac{\hbar^2}{2m}\frac{\nabla_\perp^2\sqrt{\bar{n}_c(\mathbf{r},t)}}{\sqrt{\bar{n}_c(\mathbf{r},t)}} + \mu_{\rm opt}(\bar{n}_c,\bar{\tilde{n}}) + V_{\rm trap}(\mathbf{r})$$

condensate chemical potential in an optical lattice

$$\frac{1}{m^*} = \left.\frac{\partial^2 \varepsilon_{\rm opt}(k_c)}{\hbar^2 \partial k_c^2}\right|_{k_c=0} \qquad \frac{1}{m_\mu^*} = \left.\frac{\partial^2 \mu_{\rm opt}(k_c)}{\hbar^2 \partial k_c^2}\right|_{k_c=0}$$

effective masses

Summary of the Kinetic Theory in Optical Lattices

- Condensate dynamics is described by the coarse grained generalized GP equation.

- Noncondensate Green's function can also be expressed in terms of the Bloch wave function.

$$G(\mathbf{r},t;\mathbf{r}',t) = \sum_{k,k'} \tilde{u}_k(z)\tilde{u'_k}^*(z')G_{kk'}(\mathbf{r},t;\mathbf{r}'t)$$

Instability of the Condensate

- Collisional damping
- Landau damping
- Critical velocity

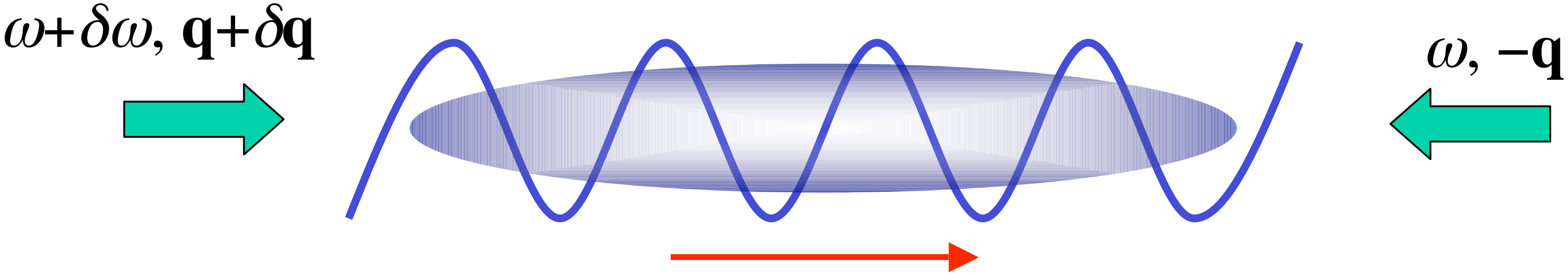
Konabe and Nikuni, J. Phys. **B** 39, S101 (2006)

Iigaya, Konabe, Danshita, and Nikuni, PRA **74**, 053611 (2006)

Konabe and Nikuni, JLTP (in press)

Instability in a Moving Lattice

- We use the coarse-grained GGP equation to discuss the instability of a condensate in a moving lattice.



- We calculate damping of condensate collective modes. We will show that the damping rate can become **negative** in the presence of the moving optical lattice.

- This means that the fluctuation of the condensate exponentially grows, leading to the instalibity.

$$\delta n_c \propto e^{\Gamma t}$$

Collisional Damping Rate

- In calculating the dissipative term in the GGP equation, we assume that is the thermal cloud is in static equilibrium distribution (Bose distribution function). ⇒ Static thermal cloud approximation
  [Williams and Griffin (2001)]

- The frequency and damping rate for the phonon mode propagating along the lattice direction are given by

$$\omega = \Omega - i\Gamma_c$$

$$\Omega = c^* k_z \pm v_c^{\mu} k_z$$

$$\Gamma_c = \frac{1}{2\tau}\left(1 \pm \frac{v_c^{\mu}}{c^*}\right)$$

- Condensate velocity and sound velocities are defined as

$$v_c^{\mu} = \frac{1}{\hbar}\frac{\partial \mu_{\mathrm{opt}}}{\partial k_c} \qquad c^* = \sqrt{\frac{n_c}{m^*}\frac{\partial \mu_{\mathrm{opt}}}{\partial n_c}}$$

Landau Damping Rate

- The noncondensate density fluctuation is treated within the linear response theory:

$$\delta\bar{\tilde{n}} = \chi^0_{\tilde{n}\tilde{n}}(\mathbf{k},\omega)2\tilde{g}\delta\bar{n}_c$$

- For simplicity, we calculate the density response function in the Hartree-Fock approximation

$$\Gamma_L = \frac{\tilde{g}mc^* k_z}{\pi\hbar^3}\frac{k_{\mathrm{B}}T}{(c^*\pm v_c^\mu)^2+2c^{*2}}(c^*\pm v_c^\mu)$$

- Iigaya et al., PRA (2006) used Hartree-Fock-Popov approximation for the tight-binding model to calculate the Landau damping rate.

Instability of the Condensate

- The expressions for the collsional and Landau damping show that the **damping rate can be negative** when

$$v_c^\mu > c^*$$

- This indicates the growth instability

$$\delta n_c \propto e^{\Gamma t}$$

- This condition turns out to be **precisely the same** as the usual Landau criterion for the **negative excitation energy**.

- This result clearly shows the crucial role of the thermal cloud in the optical lattice, and give an insight into the microscopic mechanism of the Landau instability.

Summary

- We discussed the effects of thermal cloud atoms in Bose-condensed gases in the presence of optical lattices.
- We derived the coarse-grained GGP equation at finite temperatures.
- The effects of a lattice potential is incorporated into the equation of motion for the condensate by a modified dispersion (effective mass) and a renormalized coupling constant.
- We used the coarse-grained GGP equation to discuss the Landau instability.

Outlook

- Coupled non-equilibrium dynamics of the condensate and thermal cloud. This will involve a kinetic equation for the noncondensate distribution function in the presence of an optical lattice.
- Two-fluid hydrodynamics in the presence of an optical lattice.
- Dynamics of a two-component Fermi gas in the presence of an optical lattice.