Damping of condensate oscillations of a trapped Bose gas in a 1D optical lattice at finite temperatures

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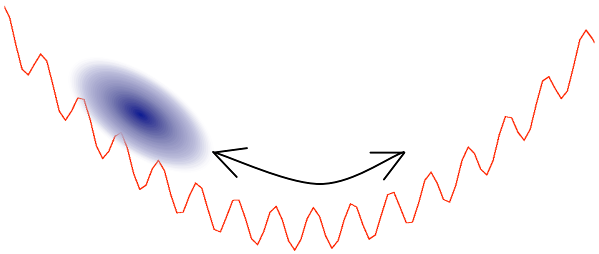
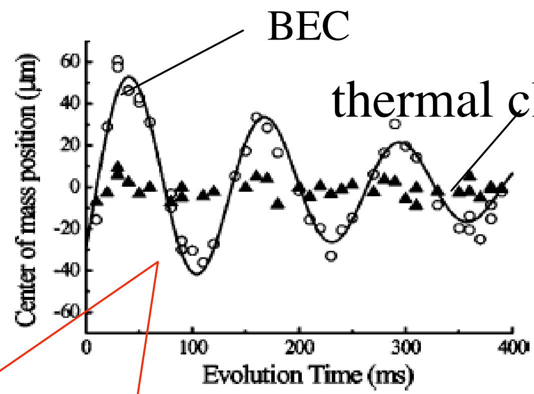
Introduction: Importance of thermal components

●Dipole Oscillation

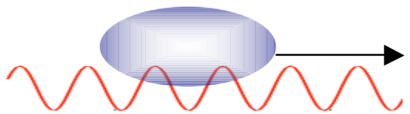


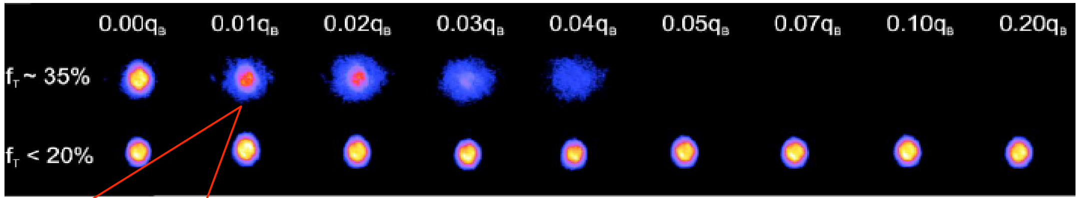


F. Ferlaino,et.al PRA **66,** 011604

damping of the collective mode

●Current decay





L.de.Sarlo et al. PRA **72** 013603

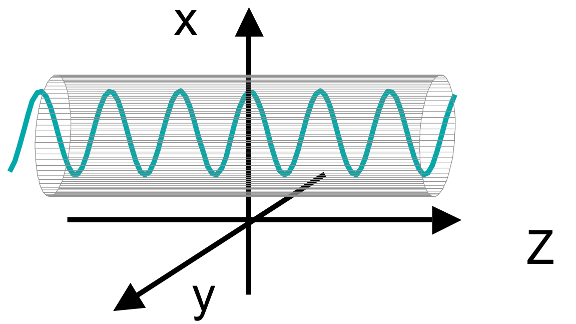
Breakdown of the superfluidity

Introduction: Importance of radial excitations

Florence experiments

- 1D optical lattice + a highly anisotropic harmonic confinement

$$V_{\rm ext} = V_{\rm trap} + V_{\rm op},$$
$$V_{\rm trap} = \frac{m}{2}\left[\omega_\perp^2(x^2+y^2)+\omega_z^2 z^2\right],$$
$$V_{\rm op} = sE_{\rm R}\cos^2(kz),$$



- T=0 condensate dynamics is often discussed by using 1D GP equations, ignoring the radial degree of freedom

- parameters

ω_z	9.0 Hz
$\omega_\perp$	92 Hz
λ	795 nm
N	400000

Temperature →

$\hbar\omega_z/k_B$	0.43nK
$\hbar\omega_\perp/k_B$	4.41nK
E_R/k_B	175 nK

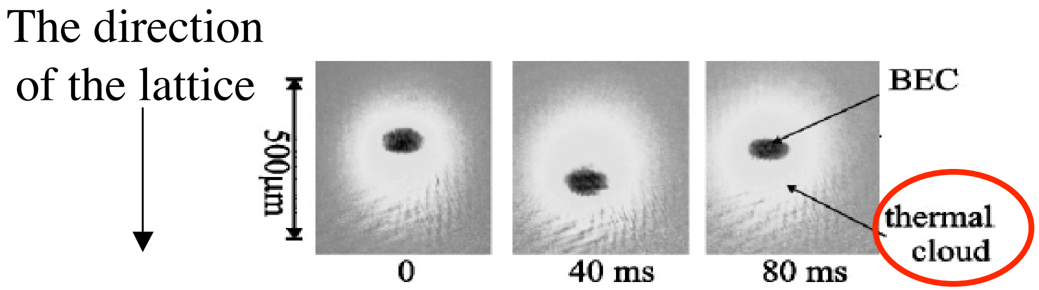
<120 nk (Florence)

Cannot ignore radial excitations!!

Introduction: our study

- We study
  - **equilibrium properties** of Bose-condensed gases
  - **Landau damping** of the collective mode

- We emphasize
  the roles of **radial thermal excitations**



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Quasi 1D Modeling of a Trapped Bose Gas

- **Expand the field operator in terms of the radial wave function.**

$$\hat{\psi}(\mathbf{r}) = \sum_{\alpha} \hat{\psi}_{\alpha}(z)\phi_{\alpha}(x,y)$$

α ; The index of the single-particle state with the eigenvalue ϵ_{α}

$$\left[-\frac{\hbar^2}{2m}\nabla_{\rho}^2 + \frac{m}{2}\omega_{\rho}^2\rho^2\right]\phi_{\alpha}(\rho) = \epsilon_{\alpha}\phi_{\alpha}(\rho) \qquad \rho = (x,y)$$

- **Quasi 1D Hamiltonian**

$$\hat{H} = \sum_{\alpha}\int dz \hat{\psi}_{\alpha}^{\dagger}(z)\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{m}{2}\omega_z^2 z^2 + V_{\mathrm{op}}(z) + \epsilon_{\alpha}\right]\hat{\psi}_{\alpha}(z)$$
$$+ \sum_{\alpha\alpha'\beta\beta'}\frac{g_{\alpha\alpha'\beta\beta'}}{2}\int dz \hat{\psi}_{\alpha}^{\dagger}\hat{\psi}_{\beta}^{\dagger}\hat{\psi}_{\beta'}\hat{\psi}_{\alpha'},$$

The renormalized coupling constant $g_{\alpha\alpha'\beta\beta'} \equiv g\int dxdy\phi_{\alpha}^{*}\phi_{\beta}^{*}\phi_{\beta'}\phi_{\alpha'}$

- **We use the Hartree Fock Bogoliubov popov(HFB-Popov) approximation.**

- **From the numerical solutions of the GP equation, we find**

$$|\Phi_1/\Phi_0| < 10^{-6}$$

first excited radial mode (Φ_1) — lowest radial mode (Φ_0)

$$\langle\hat{\psi}_{\alpha}\rangle = \Phi\delta_{\alpha 0}$$

Quasi 1D Modeling of a Trapped Bose Gas

- **GP equation**

$$\mu\Phi = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{m}{2}\omega_z^2 z^2 + V_{op} + \epsilon_0 + g_{0000}|\Phi|^2 + 2\sum_{\alpha'\beta} g_{0\alpha'\beta 0}\left\langle \tilde{\psi}_\beta^\dagger \tilde{\psi}_{\alpha'} \right\rangle \right] \Phi$$

- **The coupled Bogoliubov equations**

$$L_\alpha u_{j\alpha} + \sum_{\alpha'} \left\{ \left(2g_{\alpha'}^{\alpha} n_0 + g_{\alpha'\beta\beta'}^{\alpha} n_{\beta\beta'} \right) u_{j\alpha'} - g_{\alpha'}^{\alpha} n_0 v_{j\alpha'} \right\} = E_j u_{j\alpha}$$

$$L_\alpha v_{j\alpha} + \sum_{\alpha'} \left\{ \left(2g_{\alpha'}^{\alpha} n_0 + g_{\alpha'\beta\beta'}^{\alpha} n_{\beta\beta'} \right) v_{j\alpha'} - g_{\alpha'}^{\alpha} n_0 u_{j\alpha'} \right\} = -E_j u_{j\alpha}$$

where

$$\begin{aligned} L_\alpha &\equiv -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{m}{2}\omega_z^2 z^2 + V_{\rm op} + \epsilon_\alpha - \mu \\ n_0(z) &= |\Phi|^2(z) \\ g_{\alpha'}^{\alpha} &= g_{\alpha\alpha'00} \\ g_{\alpha'\beta\beta'}^{\alpha} n_{\beta\beta'} &= \sum_{\beta\beta'} g_{\alpha\alpha'\beta\beta'}\left\langle \tilde{\psi}_\beta^\dagger \tilde{\psi}_{\beta'} \right\rangle \end{aligned}$$

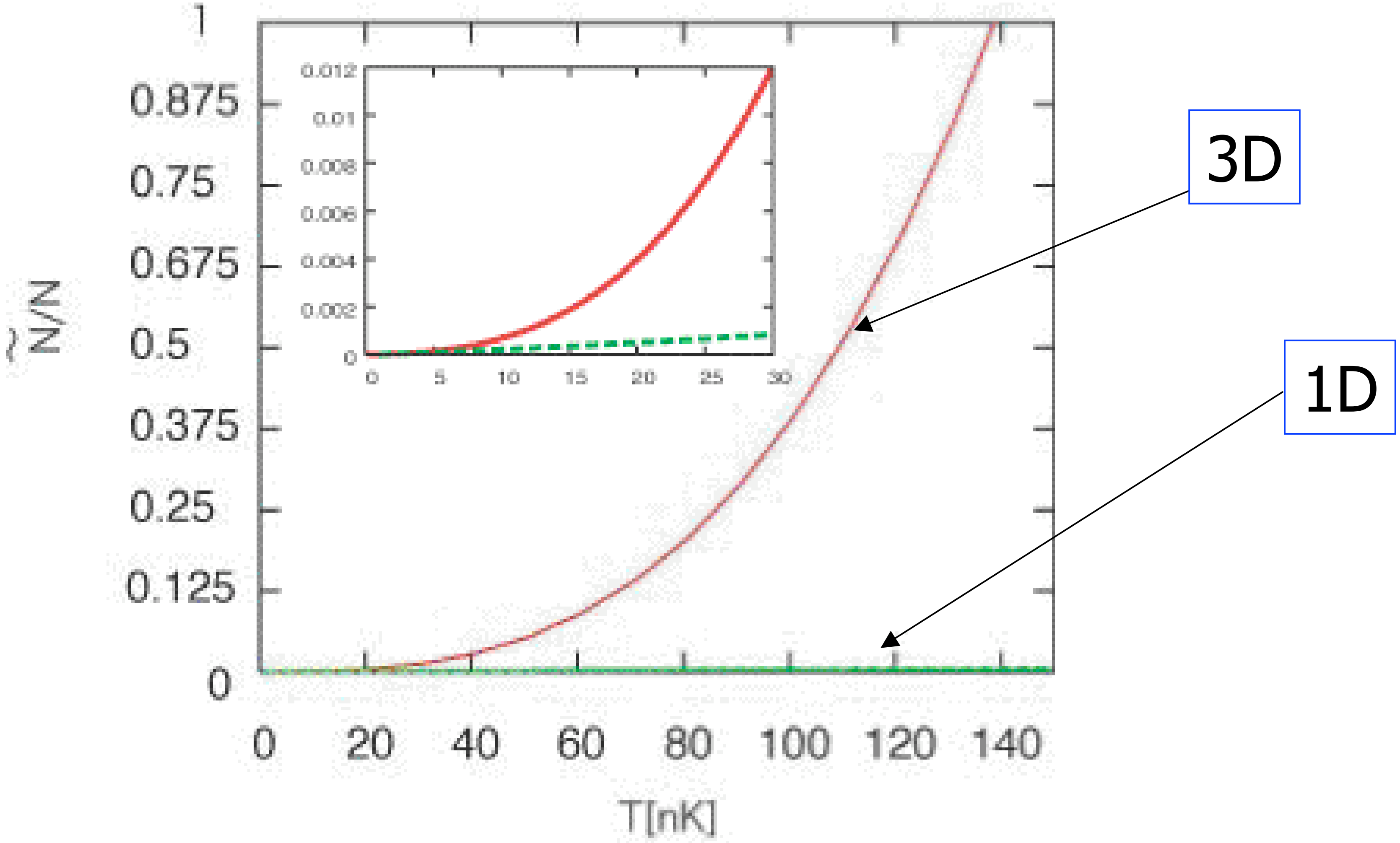
- **The noncondensate density**

$$\tilde{n} = \sum_{j\alpha\beta}\left[\left(u_{j\alpha}u_{j\beta} + v_{j\alpha}v_{j\beta} \right) N(E_j) + v_{j\alpha}v_{j\beta} \right]$$

where $N(E_j) = \dfrac{1}{[\exp(\beta E_j) - 1]}$

Results

<<Thermal fraction>>



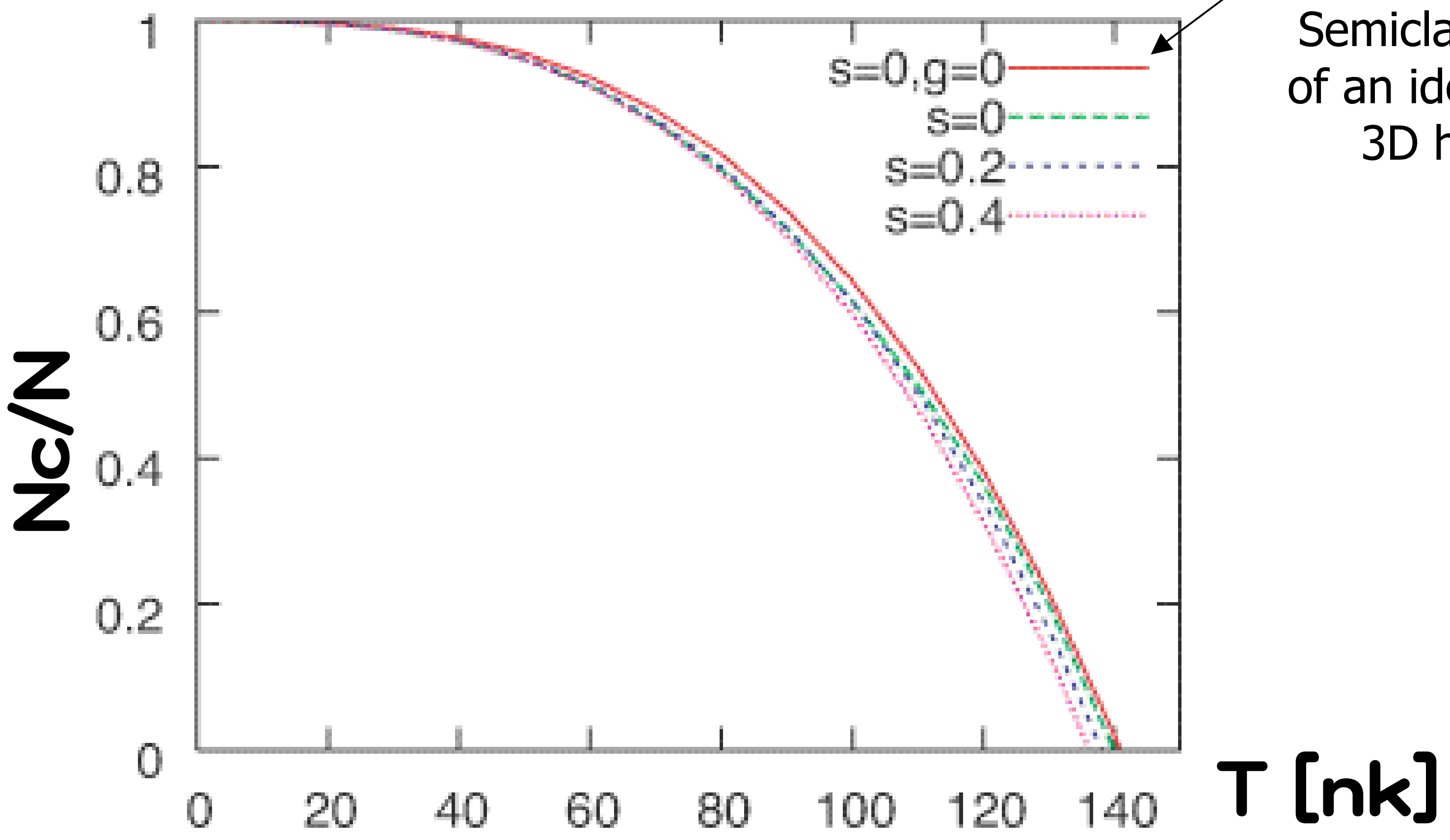
Radial excitations cannot be ignored !!

Results

<<Condensate fraction>>

$$N_c/N = 1 - (T/T_c^0)^3$$

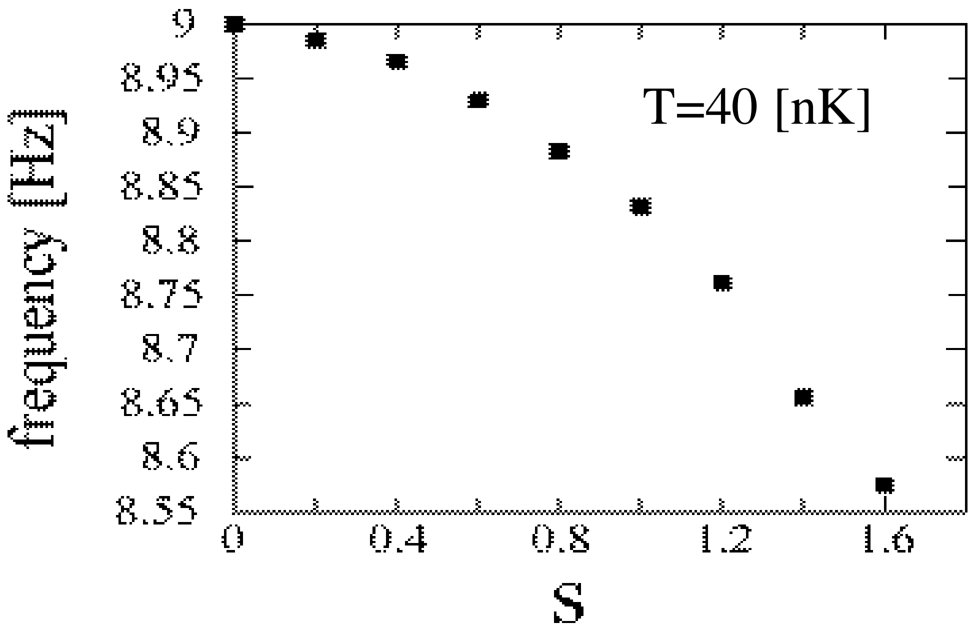
Semiclassical prediction of an ideal Bose gas in a 3D harmonic trap



Tc can be determined !!

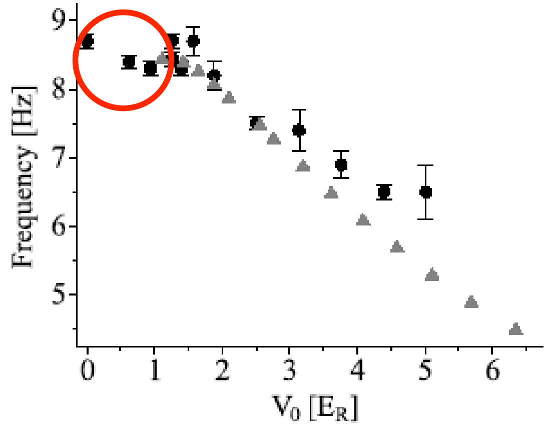
Results

<<Dipole mode frequency>>



(lattice depth)

Experimented data



Consistent with Florence experiments !!

Results

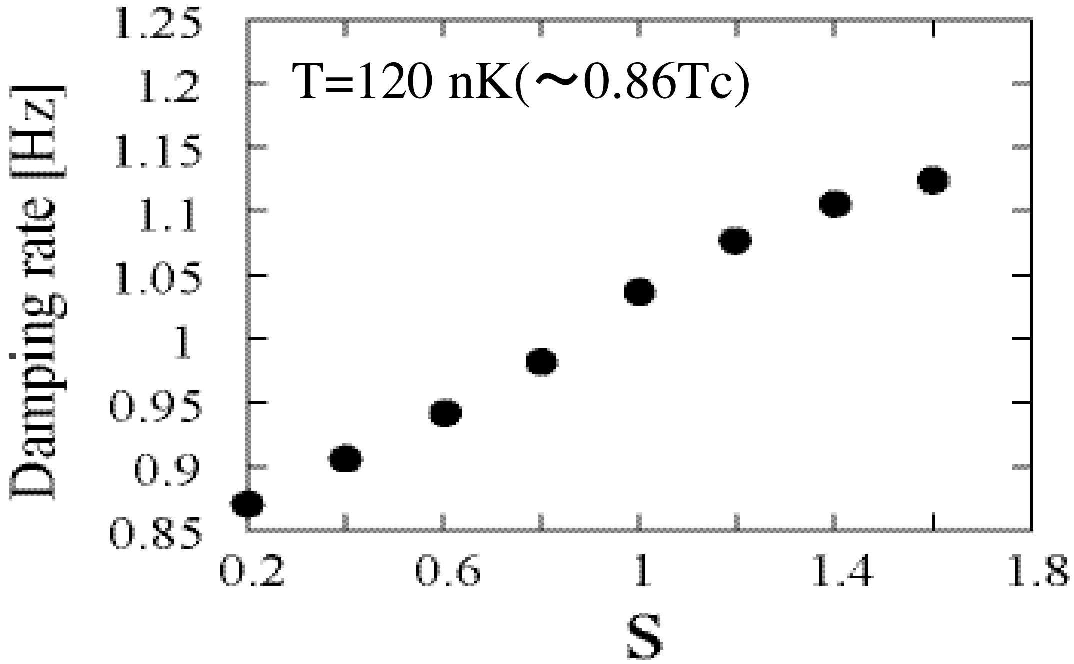
$$\gamma_L = 4\pi \sum_{\alpha,\alpha'} g_{\alpha\alpha' 00} \sum_{i \neq j} |A_{ij}^{\alpha\alpha'}|^2 \left(f_i^0 - f_j^0\right) \delta\left(\hbar\omega + \epsilon_i - \epsilon_j\right)$$

$$A_{ij}^{\alpha\alpha'} \equiv \int dz \Phi \{ u_{10} \left[u_{i\alpha}^* u_{j\alpha'} + v_{i\alpha}^* v_{j\alpha'} - v_{i\alpha}^* u_{j\alpha'} \right] - v_{10} \left[u_{i\alpha}^* u_{j\alpha'} + v_{i\alpha}^* v_{j\alpha'} - u_{i\alpha}^* v_{j\alpha'} \right] \}$$

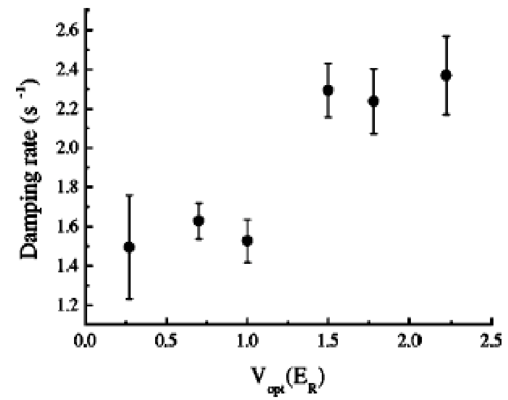
S.Giorgini, PRA**57**, 2949

Pitaevskii,L.P.and Stringari,S.
Bose-Einstein Condensation
(CLARENDON PRESS OXFORD 2003)

≪Lattice depth dependence of Landau damping ≫



Experimented data



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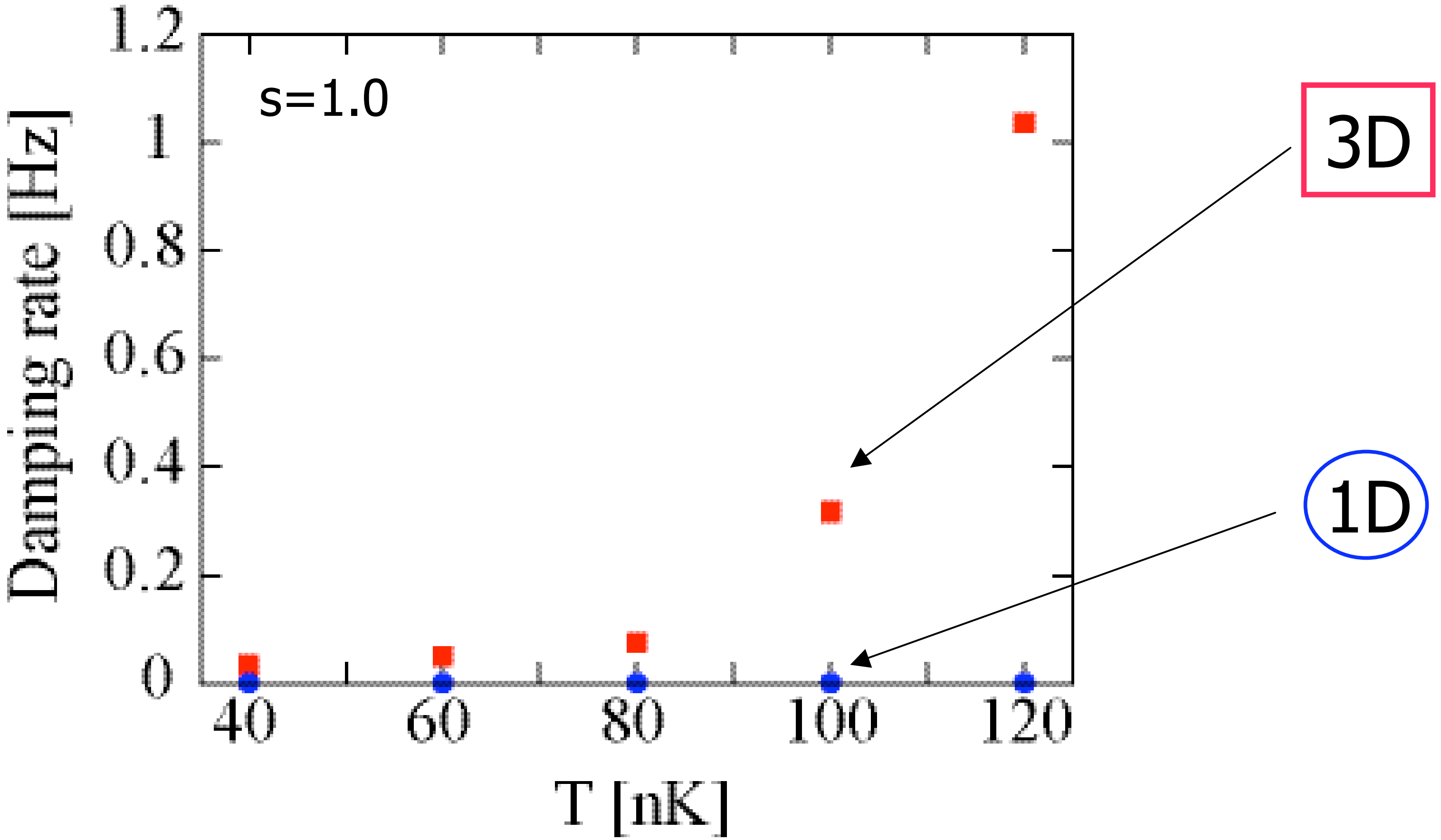
Qualitatively OK !

Results

<<temperature dependence of Landau damping >>



Damping rate decreases prominently with decreasing temperature !!

Summary

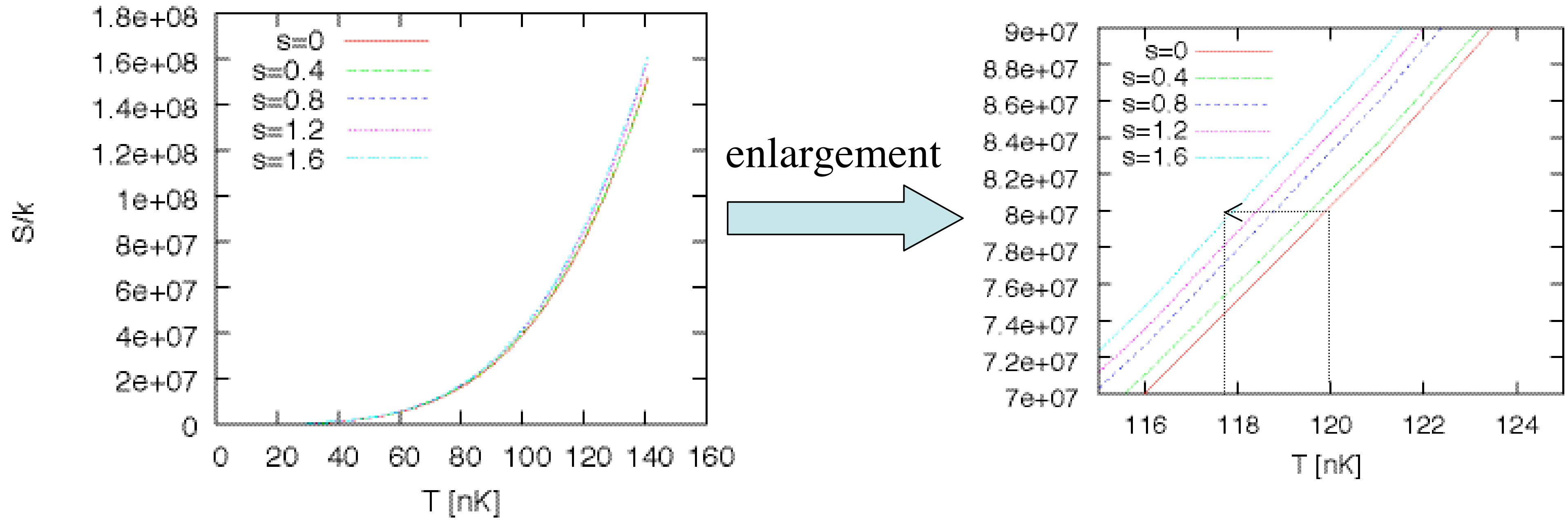
- We have calculated the transition temperature and the condensate fraction of a trapped Bose gas in a 1D optical lattice.
- While we treated the condensate wavefunction only with the lowest radial mode, we took into account the radial excitations for thermal cloud.
- We have calculated damping rate of collective modes with varying lattice depth and temperature.
- Result for the damping rate is consistent with experimented data.
- **The experimentally observed damping can be understood as Landau damping**.
- **We shows that the radial thermal excitations are important in both equilibrium condensate fractions and Landau damping rate**.

Future study

- Landau damping and Landau instability in current carrying condensate.

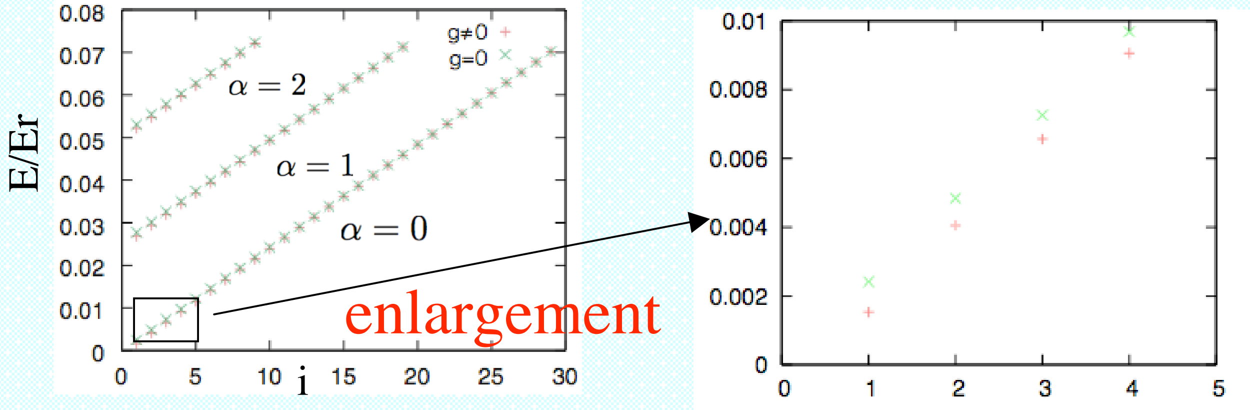
Entropy of Bose gases

We calculate entropy for ideal bose gas

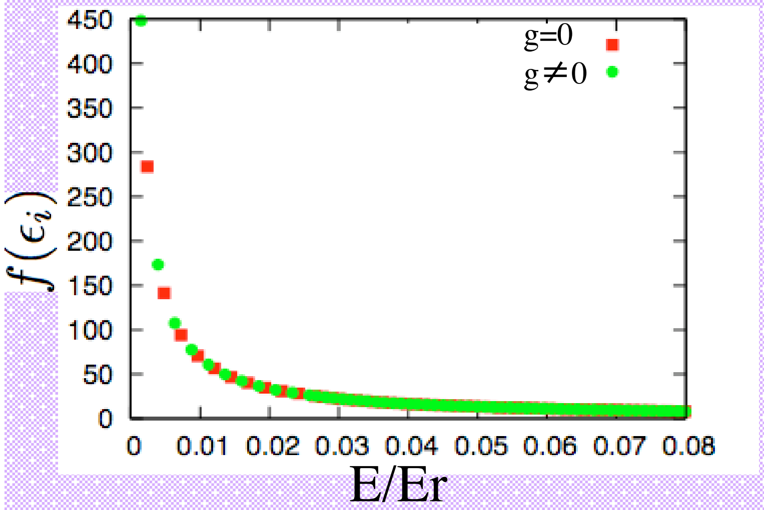


T = 120 nK in no lattice → T=118 nK in lattice depth s=1.6

Energy spectra(s=1.2)



$$f_i^0 = \frac{1}{exp(\epsilon_i) - 1}$$



Motivation

Previous studies on a 1D optical lattice at finite temperatures

◆Damping of Bogoliubov excitations in optical lattices

S.Tsuchiya and A.Grifin PRA **70** 023611

calculation of Landau damping of Bogoliubov phonon

◆Finite temperature treatment of ultracold atoms in a one dimension optical lattice

B.G.Wild, P.B.Blakie, and D.A.W.Hutchinson PRA **73** 023604

calculation of equilibrium properties

Problems

●Tight binding ・・ consider only 1st Bloch band

valid if $k_{\mathrm{B}}T \ll E_{\mathrm{R}}$

●Pure 1D ・・ ignore the radial excitations

valid if $k_{\mathrm{B}}T \ll \hbar\omega_{\perp}$

E_{R}; recoil energy

$\omega_{\perp}$; trap frequency in the radial direction

In the present work,we take into account the effect of radial excitations.

We study

❑equilibrium properties of Bose-condensed gases

❑Landau damping rate