# Conversion Efficiencies of Heteronuclear Feshbach Molecules

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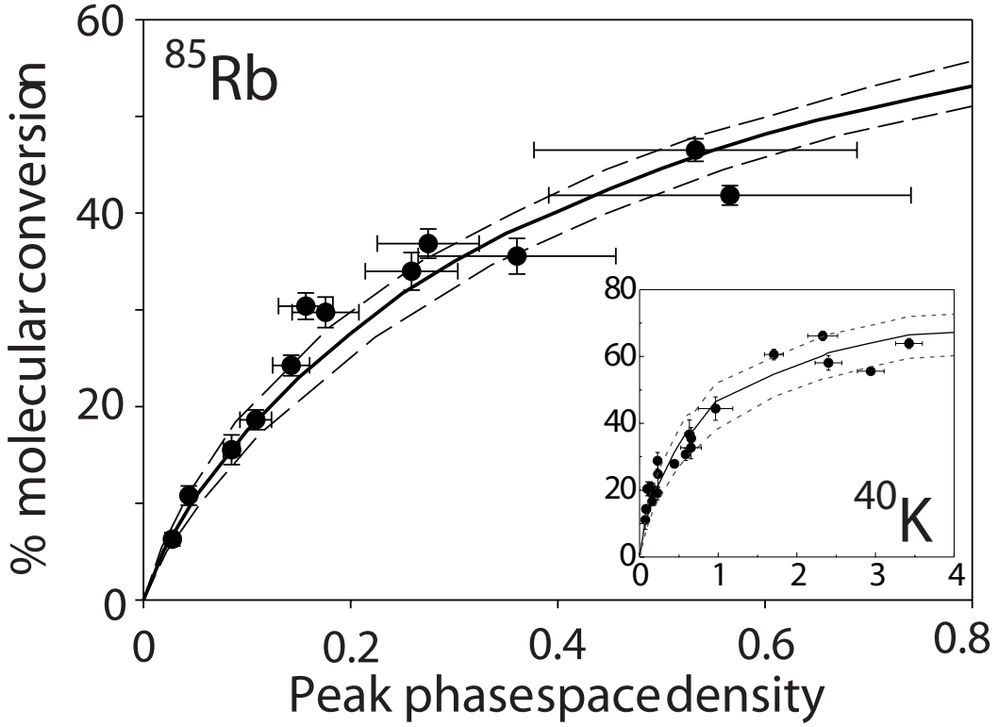
# Molecular Conversion Efficiencies and SPSS model

## Hodby et al. PRL, 94, 120402 (2005)

Trapped Atomic Gas : $^{85}$Rb (bosonic)

Trapped Atomic Gas : $^{40}$K (fermionic)

**Conversion efficiencies v.s. peak phase space density**




Dots : experimental data

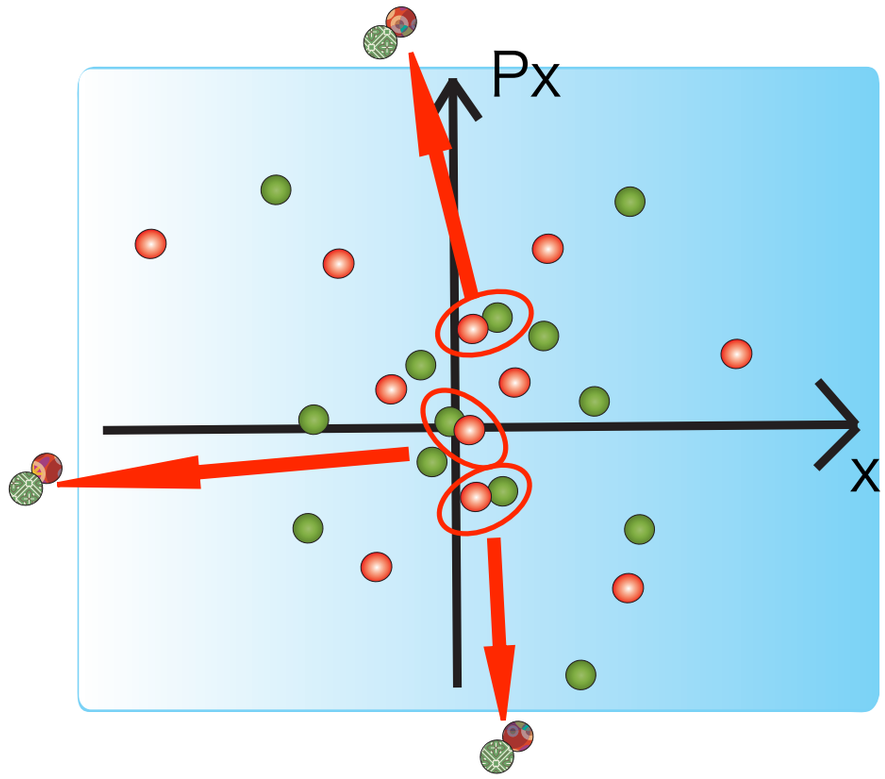
Solid line : SPSS model

Dashed line : uncertainty of the SPSS model

**Stochastic Phase Space Sampling (SPSS) model**

A pair of atoms close enough in phase space forms a molecule.

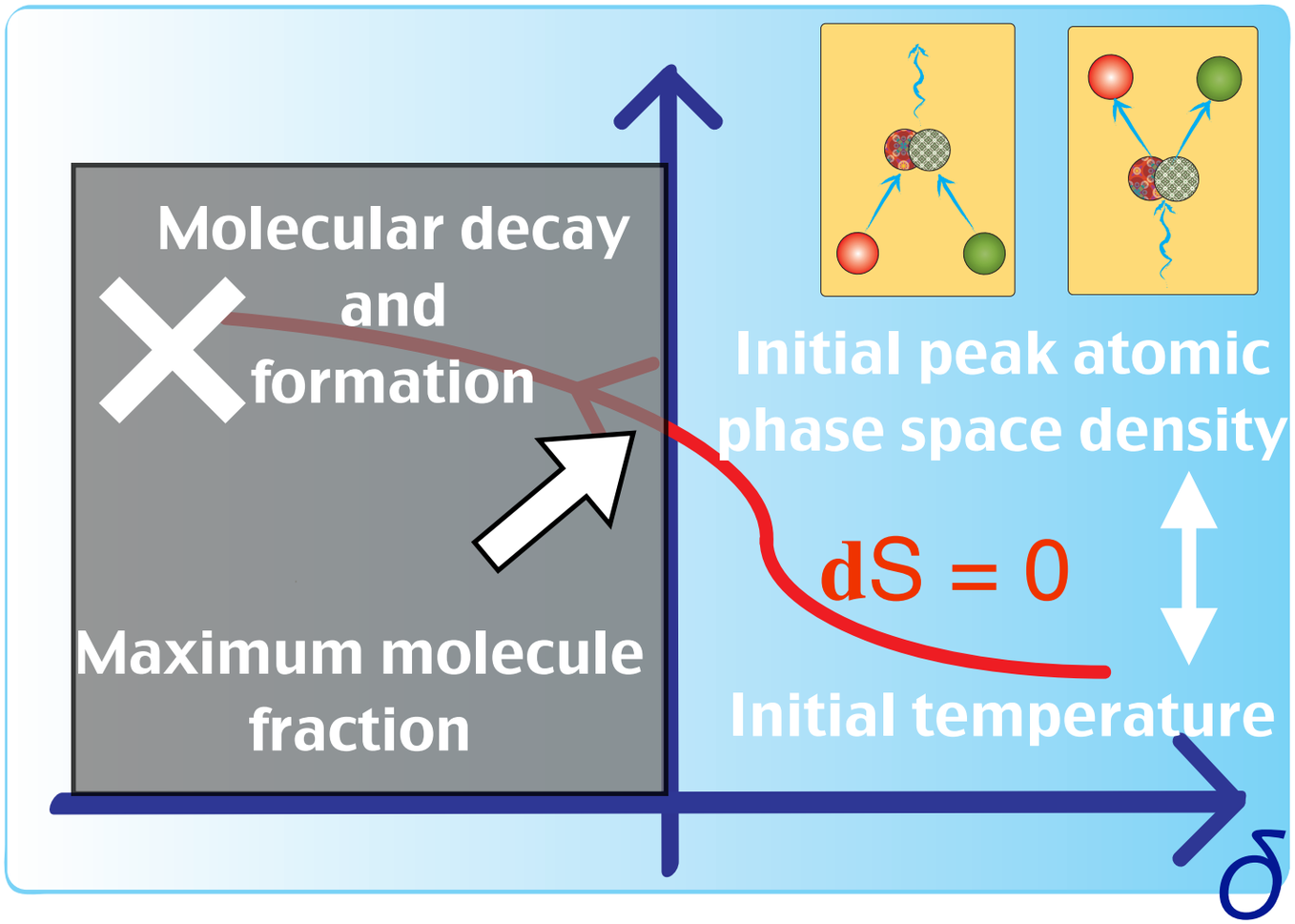
Single fitting parameter is needed.



# Saturation Mechanism of Molecular Formation

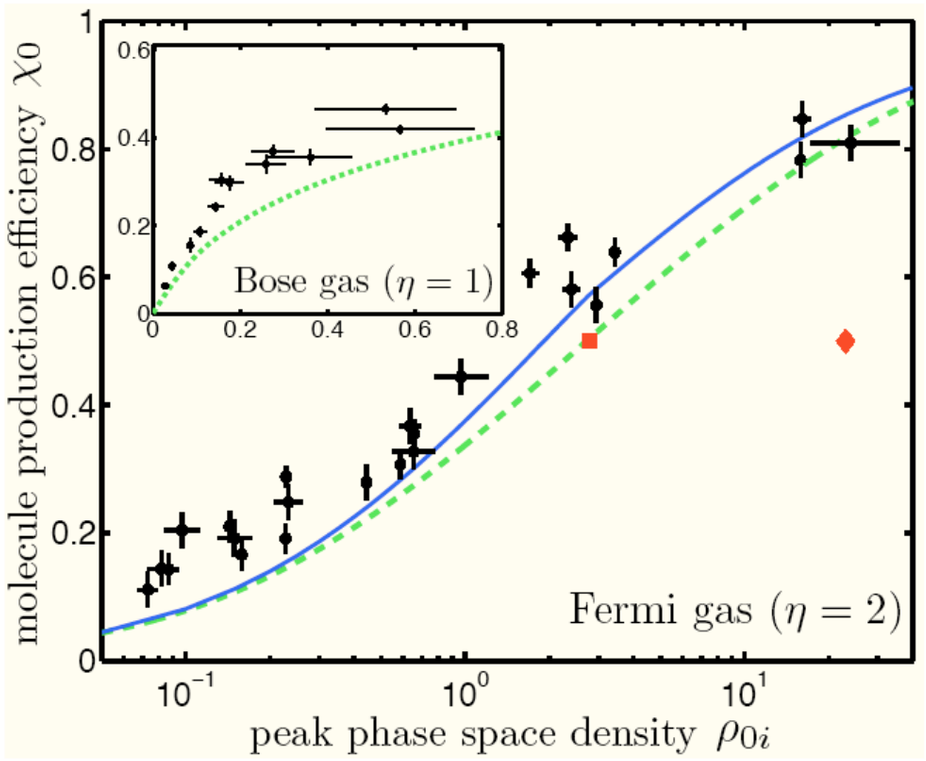
## Williams et al., New J. Phys.,8, 150 (2006)

Conservation lows of momentum and energy
via the atom-molecule conversion
cannot be satisfied
for negative detunings.




Conversion efficiency is determined by
molecular population at $\delta = 0$

**Conversion efficiencies V.S.**
**initial peak phase space density**




Dots : experimental data (Hodby et al.)

Green : classical gas

$$2\chi_0 + \ln\left(\frac{\eta}{2}\frac{\chi_0}{1-\chi_0}\right) = \ln\rho_{0i}$$

Blue : quantum statistics

**No fitting parameters!**

# Experimental Study on Conversion Efficiencies of Heteronuclear Feshbach Molecules

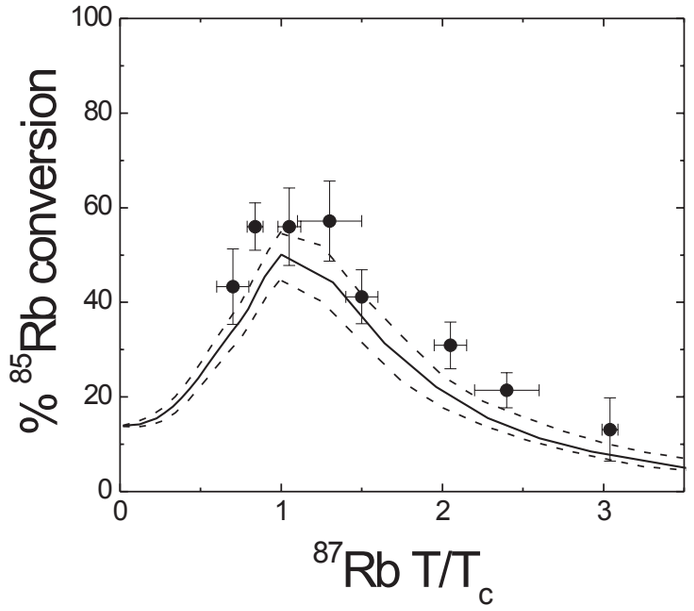
## Papp and Wieman, PRL, 97, 180404 (2006)

$^{87}\mathrm{Rb}$ : majority

$^{85}\mathrm{Rb}$ : minority

Feshbach resonance is observed between $^{87}\mathrm{Rb}$ and $^{85}\mathrm{Rb}$ .

Adiabatic conversion efficiency




Dots : experimental data

Solid line : SPSS model **( $^{85}$Rb : classical)**

Dashed line : uncertainty of the SPSS model

We apply the theory by Williams et al.
to heteronuclear Feshbach molecules,
and compare with the experimental results.

# Conversion Efficiencies of Heteronuclear Feshbach Molecules

Ideal gas mixture

**2 constraints**

① Total number of atoms

$$N_{\mathrm{tot}} = N_{87} + N_{85} + 2N_m$$

② Initial number ratio

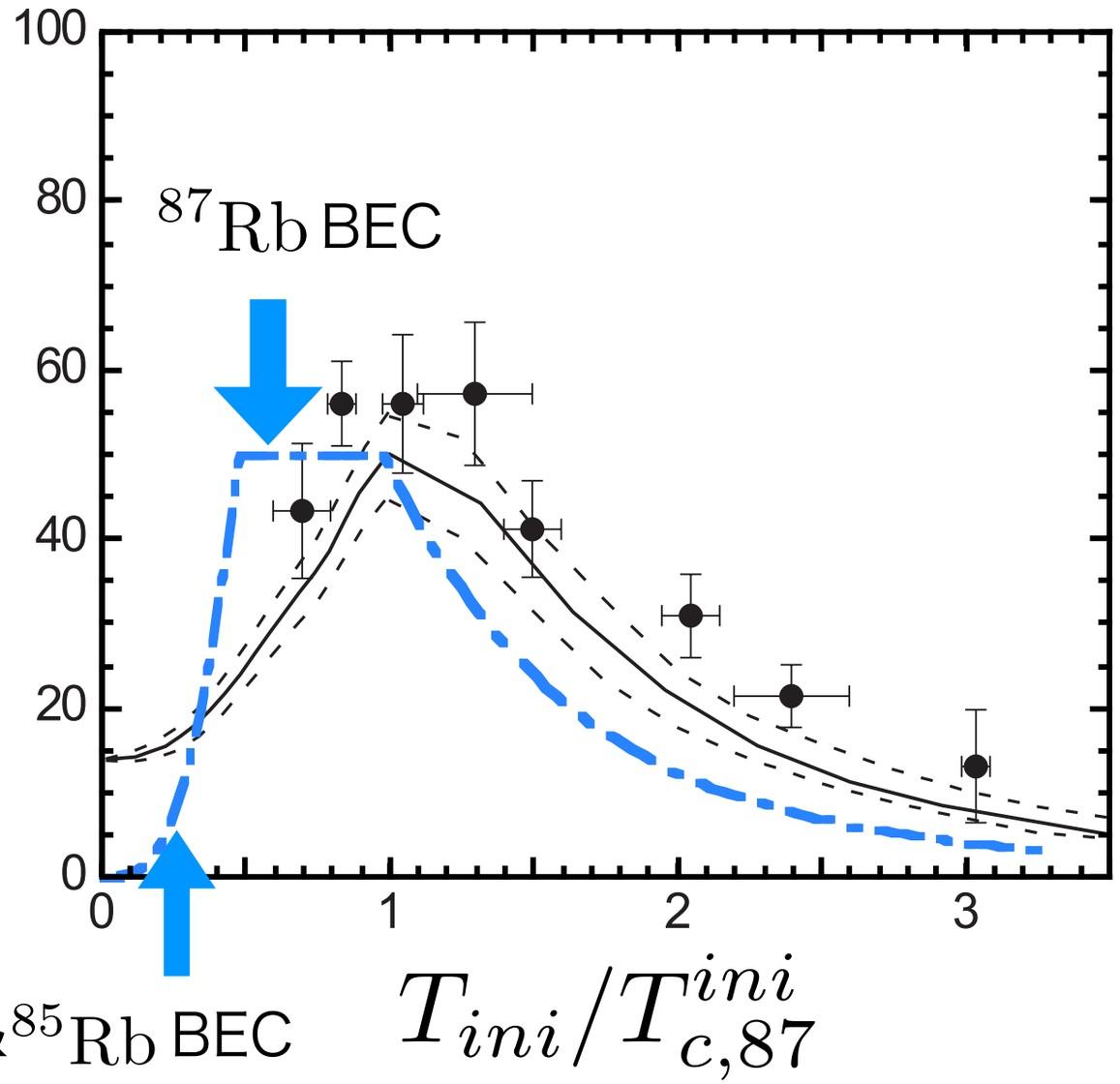
$$\frac{N_{85} + N_m}{N_{87} + N_m} = \frac{N_{85,ini}}{N_{87,ini}} \equiv \alpha$$

We set numbers.

$$N_{87,ini} = 300,000$$
$$N_{85,ini} = 40,000$$

Conversion efficiency is defined by

$$\chi_0 \equiv \frac{N_m}{N_{85,ini}} \quad .$$



# Conversion Efficiencies of Heteronuclear Feshbach Molecules

| | Majority | | Minority | | Molecule |
|---|---|---|---|---|---|
| ✓ | boson | + | fermion | <-> | boson |
| | boson | + | fermion | <-> | fermion |
| | fermion | + | boson | <-> | fermion |
| | fermion | + | fermion | <-> | boson |

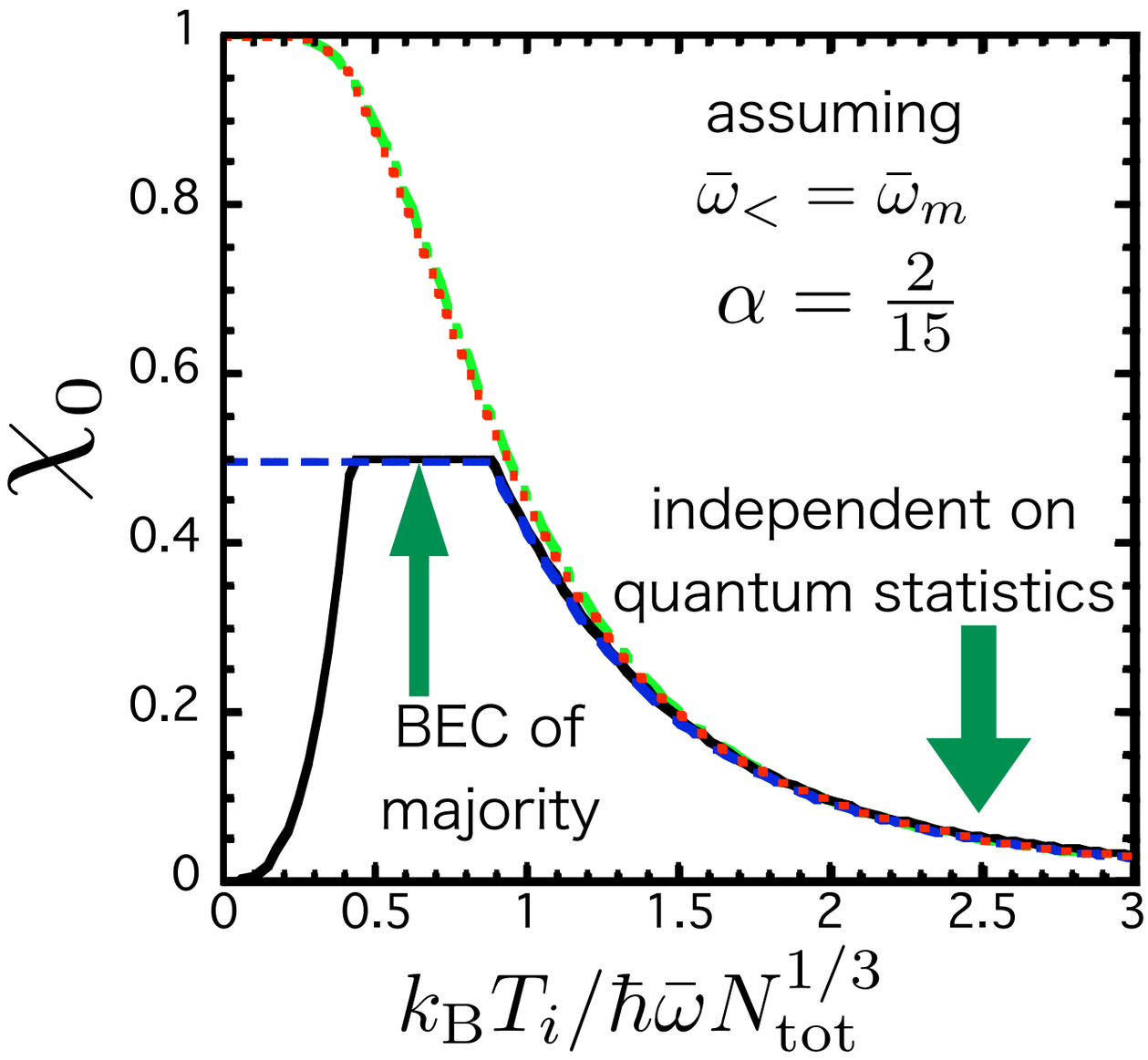
**Observation of heteronuclear Feshbach resonance**

(except bosonic $^{87}\mathrm{Rb}$ & bosonic $^{85}\mathrm{Rb}$)

bosonic $^{23}\mathrm{Na}$ & fermionic $^{6}\mathrm{Li}$ Stan et al., P.R.L., (2004)

bosonic $^{87}\mathrm{Rb}$ & fermionic $^{40}\mathrm{K}$ Inouye et al., P.R.L., (2004)

# Molecular Conversion Efficiency as a Function of Initial Temperature




**Majority : boson**

black : b(>)+b(<) <-> b(m),

blue : b(>)+f(<) <-> f(m),

$$z_> z_< = z_m$$

↓ BEC of majority

1

This makes a plateau.

The maximum conversion efficiency is

$$\chi_{0,max} = \frac{\bar{\omega}_<^3}{\bar{\omega}_<^3 + \bar{\omega}_m^3}$$

**Majority : fermion**

red : f(>)+b(<) <-> f(m),

green : f(>)+f(<) <-> b(m),

The maximum conversion efficiency is

$$\chi_{0,max} = 100\%$$

# Molecular Conversion Efficiency as a Function of Initial Temperature

## fermion(majority)+boson(minority)<->fermion(molecule)

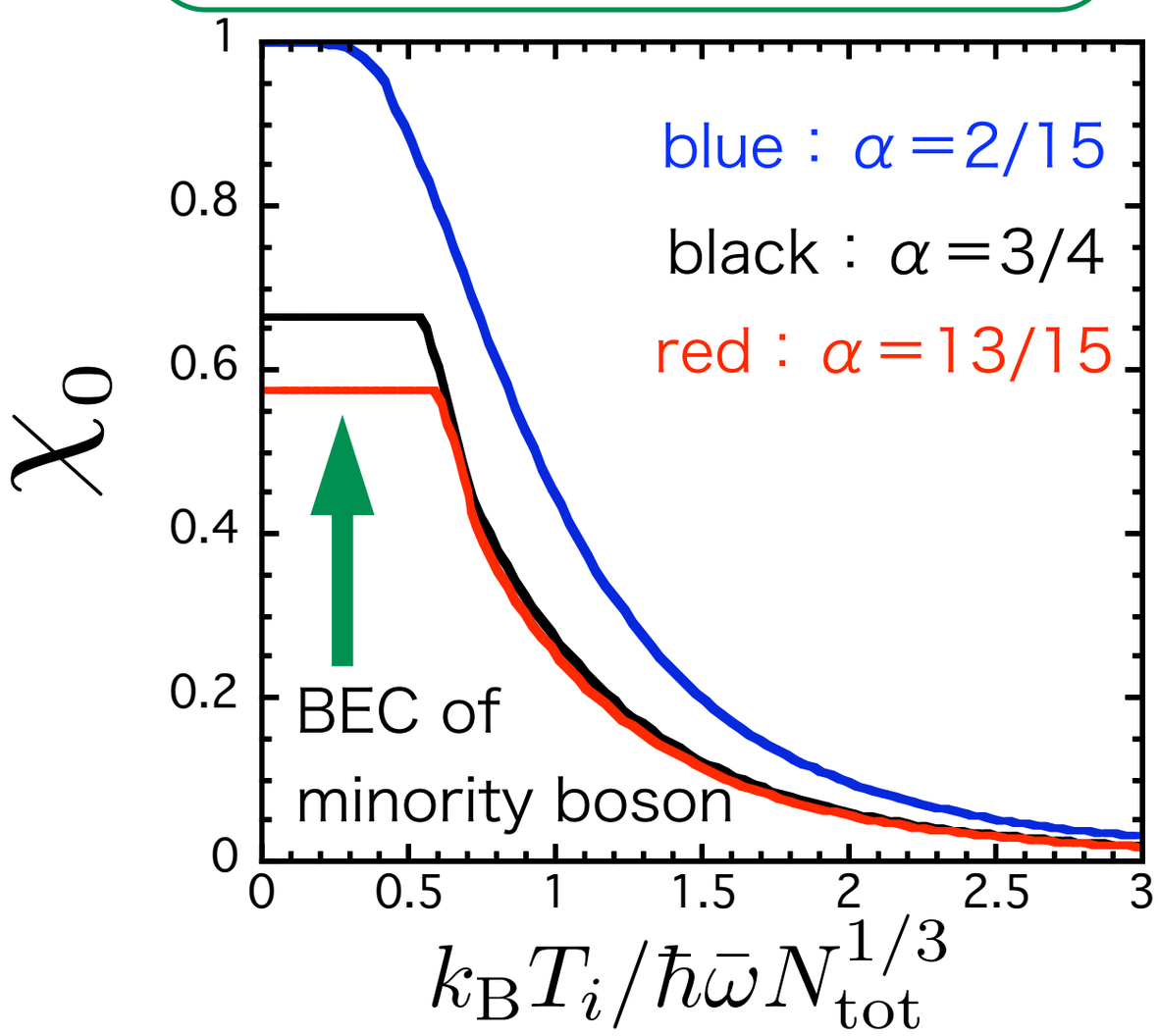
Transition temperature at $\delta = 0$ is

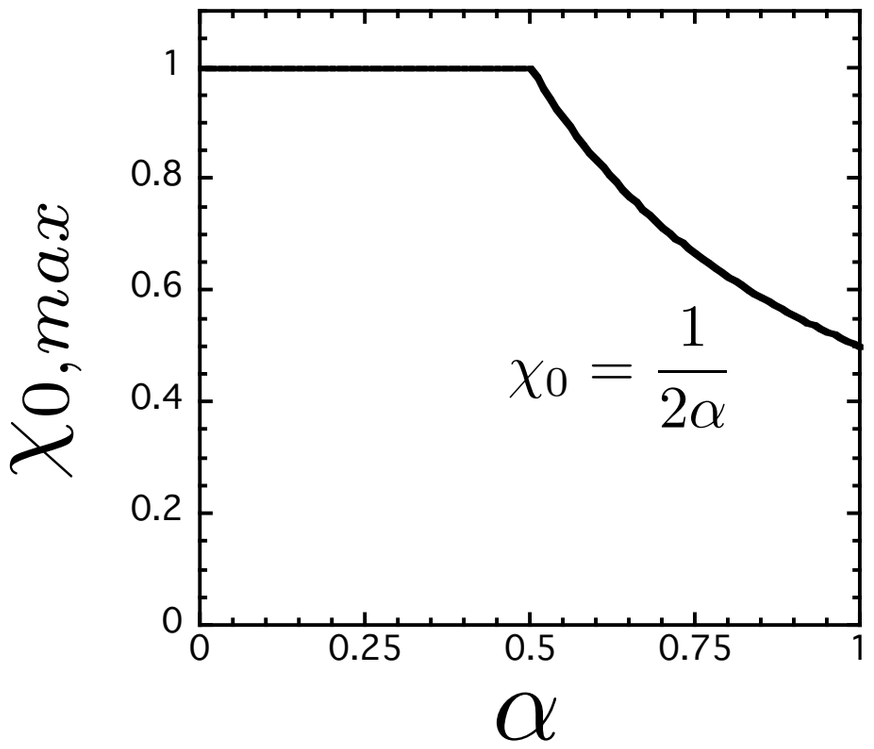
$$k_B T_c = \hbar\bar{\omega}_< \left[ \left( \alpha - \frac{\bar{\omega}_>^3}{\bar{\omega}_m^3 + \bar{\omega}_>^3} \right) \frac{N_{\rm tot}}{(1+\alpha)\zeta(3)} \right]^{1/3}$$

Condition for BEC

$$\alpha - \frac{\bar{\omega}_>^3}{\bar{\omega}_m + \bar{\omega}_>} > 0$$

assuming $\bar{\omega}_> = \bar{\omega}_< = \bar{\omega}_m$




maximum conversion efficiency

50%～100%

# Molecular Conversion Efficiency as a Function of Initial Peak Phase Space Density

Connecting the entropy, we have the explicit formula for conversion efficiency,

$$S(\delta = 0) = S_i$$

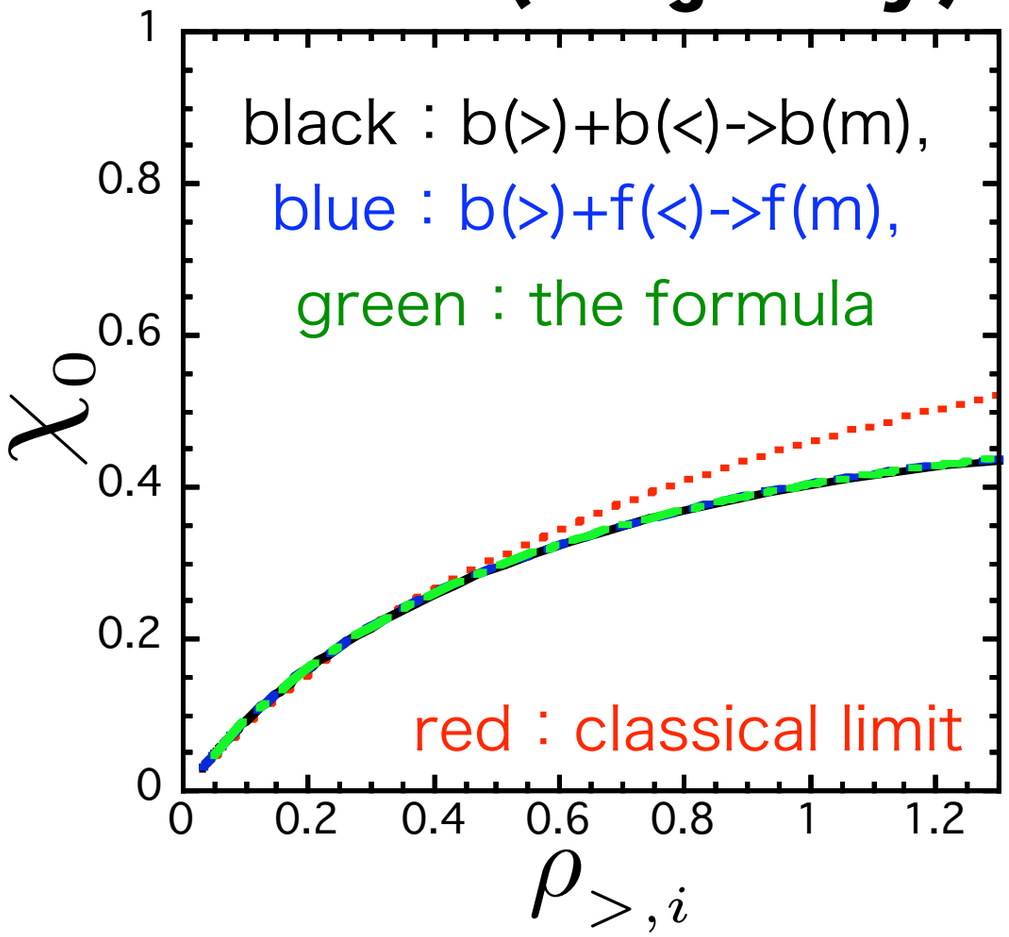
assuming

majority : Bose(Fermi) distribution

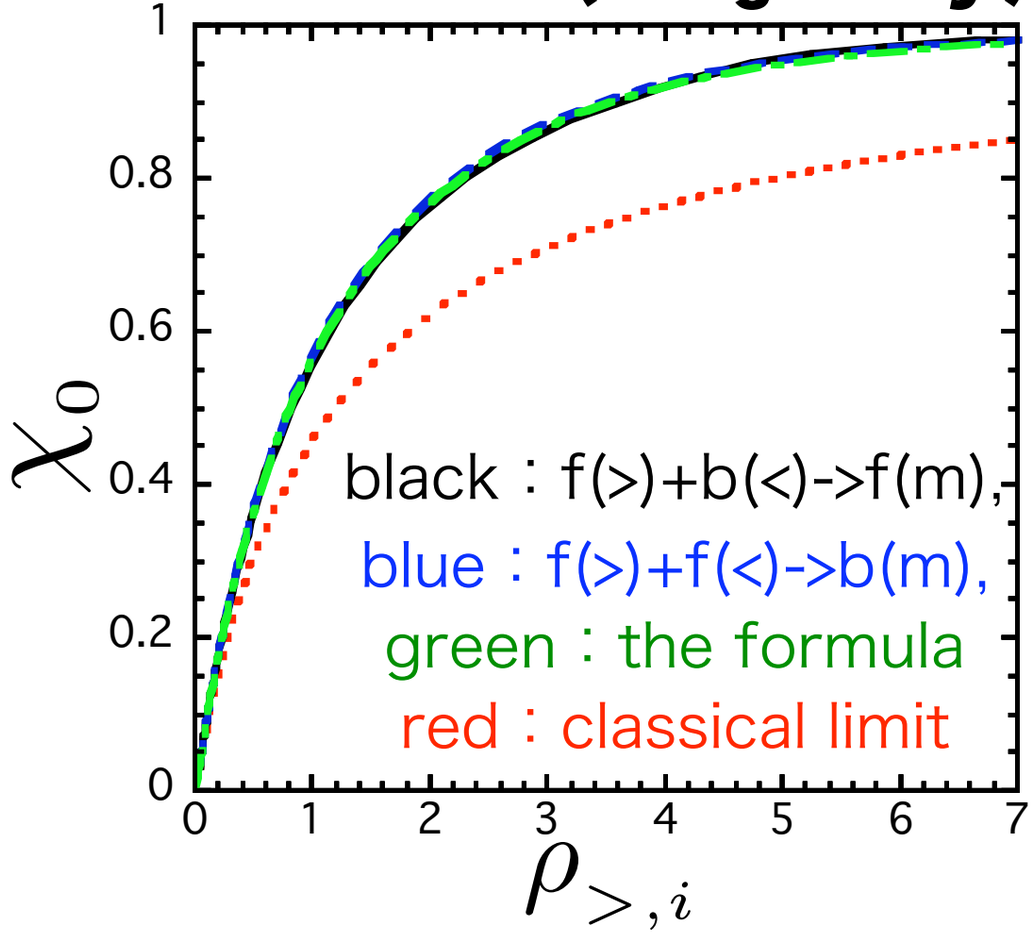
minority and molecule : Boltzmann distribution

$$\chi_0 = \chi_0(\rho_{>,i}, \alpha, \bar{\omega}_m/\bar{\omega}_<)$$

## boson(majority)




## fermion(majority)




assuming

$$\alpha = \frac{2}{15}$$

&

$$\bar{\omega}_m = \bar{\omega}_<$$

# Molecular Conversion Efficiency as a Function of Initial Peak Phase Space Density

Again,
fermion(majority)+boson(minority)<->fermion(molecule)

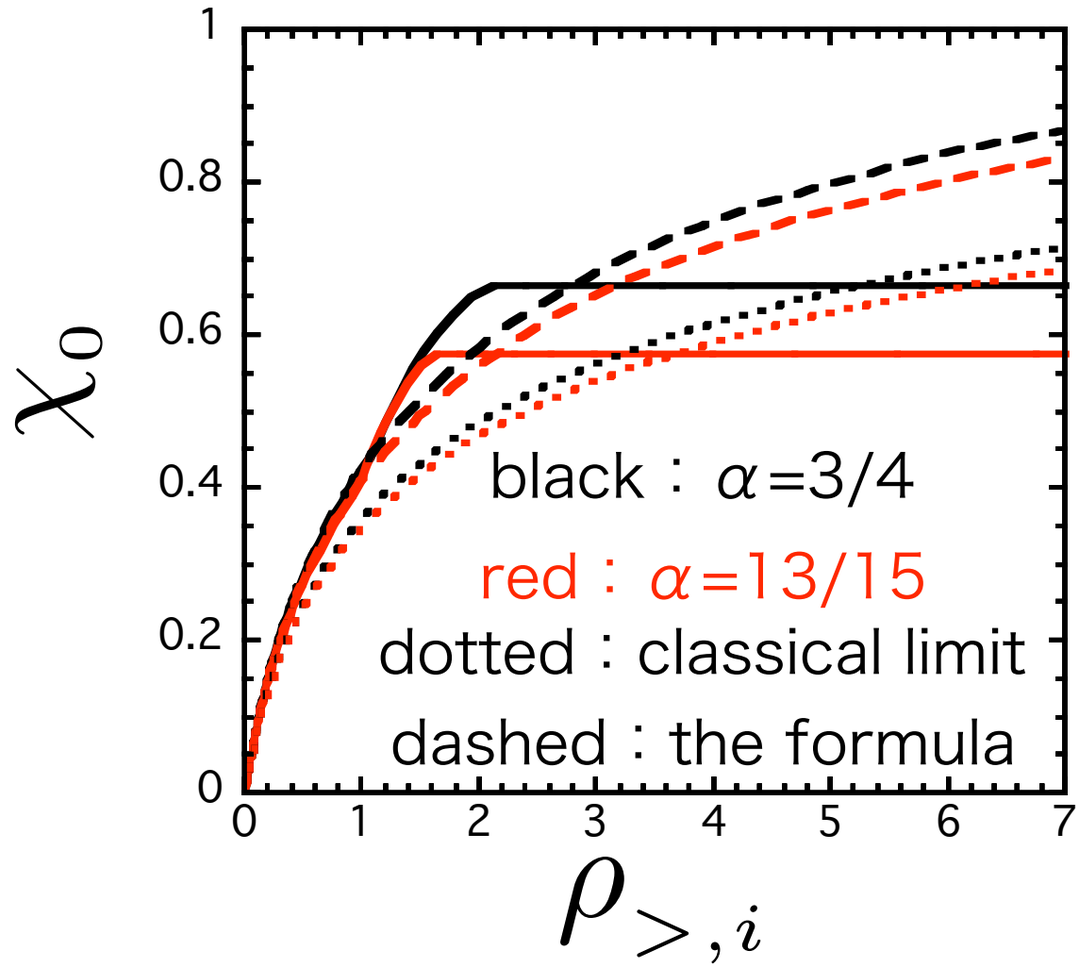
If the number ratio satisfies the condition

$$\alpha > \frac{\bar{\omega}_{>}^3}{\bar{\omega}_m^3 + \bar{\omega}_{>}^3} \quad ,$$

we have the BEC of minority bosons.

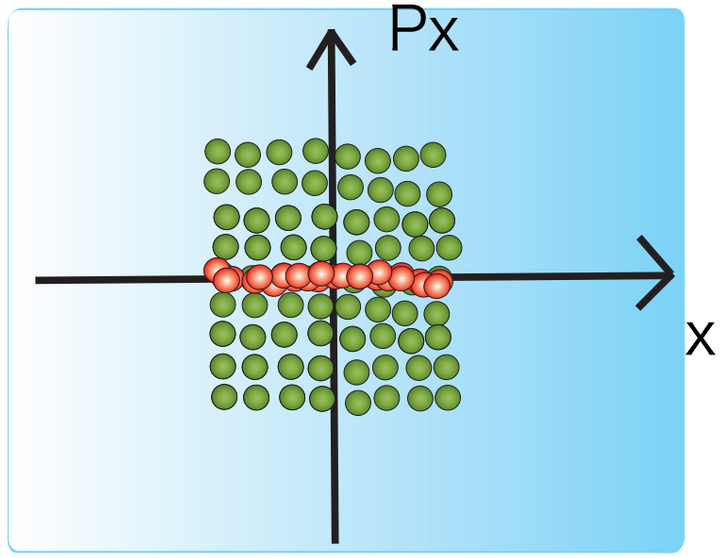
Our assumption breaks down
in this case.

The Bose statistics plays a crucial role
where the results deviate
from the approximation formula.



# Difference between SPSS model and Theory by Williams et. al.

**boson + fermion ⇔ fermion**




**SPSS model**

As temperature **decreases**,
conversion efficiency **decreases**.

**Theory by Williams et al.**

As temperature **decreases**,
conversion efficiency has a **plateau** or **increases**.

**boson + boson ⇔ boson**

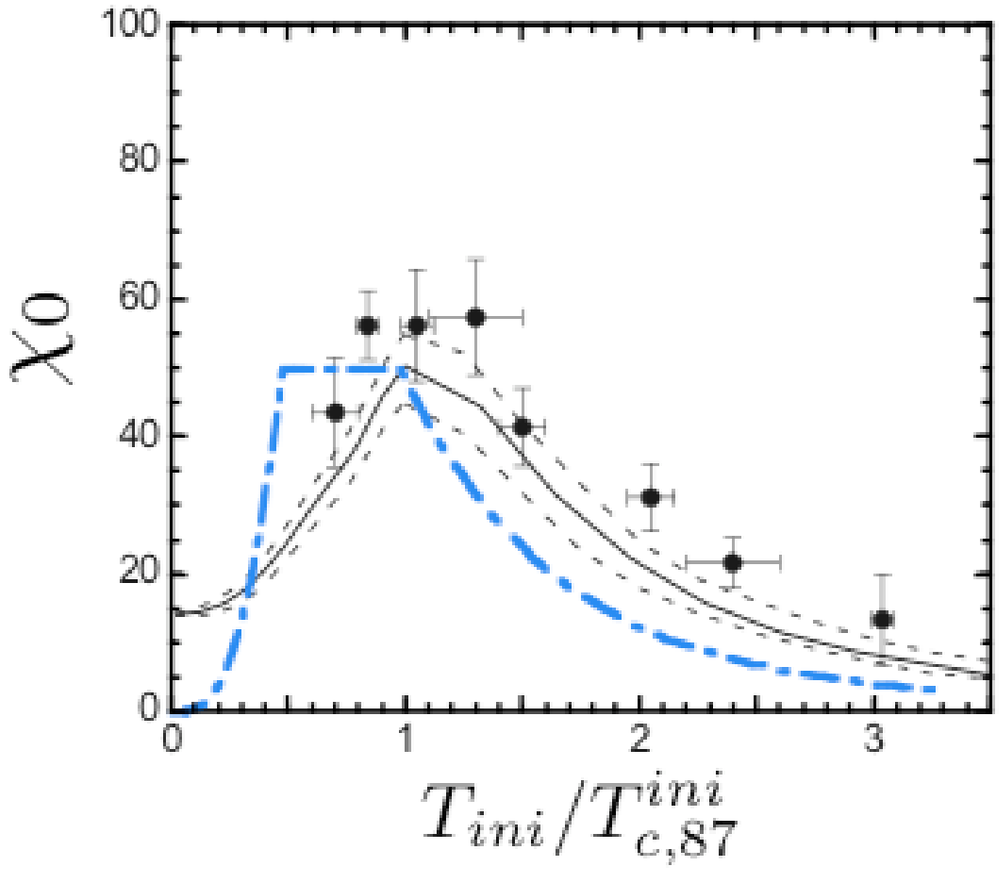
at low-temperature region

**SPSS model**

If minor component were BEC,
conversion efficiency quickly reaches **100%**.

**Theory by Williams et al.**

Conversion efficiency drops to **0%**.



# Summary

- Theory by Williams et al. reproduces
  the peak behavior of conversion efficiency
  of heteronuclear molecule of $^{87}$Rb and $^{85}$Rb.
- Quantum statistics plays a crucial role
  in conversion efficiency at low-temperature region.
- Conversion efficiency does not reach 100%,
  but has a plateau, if atoms condense at $\delta = 0$.
- SPSS model and theory by Williams et al.
  lead to qualitatively different results,
  when applying to heteronuclear Feshbach molecules.