CM 周期の代数的整数論への応用の紹介

加塩朋和 *

2021 年 3 月 10 日～12 日

プロジェクト研究集会 2020

* E-mail: kashio_tomokazu@ma.noda.tus.ac.jp

歴史

- (CM) 周期記号:
  G. Shimura, Automorphic forms and periods of abelian varieties, J. Math. Soc. Japan 31 (1979), 561–592.
- 絶対 CM 周期記号:
  H. Yoshida, On absolute CM-periods, Proc. Symposia Pure Math. 66, Part 1 (1999), 221–278.
- p 進絶対 CM 周期記号:
  K-, H. Yoshida, On p-adic absolute CM-periods. I, Amer. J. Math. 130 (2008), no. 6, 1629–1685.
- [CM 周期: p 進周期] (比):
  K-, Fermat curves and a refinement of the reciprocity law on cyclotomic units. J. Reine Angew. Math. 741 (2018), 255-273.
  K-, On a common refinement of Stark units and Gross-Stark units, preprint (arXiv:1706.03198).

CM 周期 (CM = complex multiplication = 虚数乗法)

- 円周率 $\pi = 2\int_0^1 \frac{1}{\sqrt{1-x^2}}dx = 3.1415\ldots$
  $\int \frac{1}{\sqrt{1-x^2}}dx = \int \frac{dx}{y}$ on $y^2 = 1-x^2 \Leftrightarrow x^2+y^2 = 1.$
- レムニスケート周率 $\varpi = 2\int_0^1 \frac{1}{\sqrt{1-x^4}}dx = 2.6220\ldots$
  $\int \frac{1}{\sqrt{1-x^4}}dx = \int \frac{dx}{y}$ on $y^2 = 1-x^4$ $\overset{(X,Y)=(\frac{2y+2}{x^2},\frac{4y+4}{x^3})}{\dashrightarrow}$ $E\colon Y^2 = X^3+4X.$
- $End(E) \ni [\phi\colon (X,Y) \mapsto (-X, \sqrt{-1}Y)]$,
  $\phi^2 = [-1\colon (X,Y) \mapsto (X,-Y)]$, $\phi^4 = id \Rightarrow End(E) \cong \mathbb{Z}[i]$.

- 虚数乗法をもつ E: 楕円曲線 $/\overline{\mathbb{Q}}$, i.e., $End(E)\otimes_{\mathbb{Z}}\mathbb{Q} = K$: 虚二次体
  $\Rightarrow \pi^{-1}\int_\gamma \frac{dx}{y} =: p_K(id, id)$: 志村五郎氏の周期記号(の特別な場合),
  Well-defined only up to $\overline{\mathbb{Q}}^\times$ ($\because$ モデル $E/\overline{\mathbb{Q}}$, 閉路 γ の取り方).

CM 周期と整数論

Theorem 1

$f(z)$: 保型形式, 重さ k, フーリエ係数 $\in \overline{\mathbb{Q}}$, $\tau \in K$: 虚二次体, $Im(\tau) > 0$.

$$\Rightarrow f(\tau) \in \overline{\mathbb{Q}} \cdot p_K(id, id)^k.$$

一般化

K: CM 体 (総実代数体 K^+ 上の虚二次拡大体) に対し

- "K の虚数乗法をもつアーベル多様体の周期積分" で,
- "K^+ 上の Hilbert 保型形式の CM 点 $\in K$ での値" や
- "K の代数的 Hekce 指標の L 関数の臨界値" の超越数部分が表せる.

CM 周期（一般の場合）

- $A/\overline{\mathbb{Q}}$: n 次元アーベル多様体 s.t. $End(A) \otimes_{\mathbb{Z}} \mathbb{Q} \cong K$: $2n$ 次 CM 体.

$\Rightarrow$ $H^0(A, \Omega^1_A)$ (正則 1 形式全体) $\curvearrowleft End(A) \otimes_{\mathbb{Z}} \mathbb{Q} = K$
$\cong \bigoplus_{\sigma\in\Xi} \sigma$: n 個の 1 次元表現 $\sigma \in Hom_{\mathbb{Q}}(K, \mathbb{C})$ の直和,
$\Xi \subset Hom_{\mathbb{Q}}(K, \mathbb{C})$: A の CM 型, $Hom(K, \mathbb{C}) = \{\sigma, \overline{\sigma} \mid \sigma \in \Xi\}$.
$A(\mathbb{C}) := \mathbb{C}^n/L \hookleftarrow {}^{\forall}$ CM 型 Ξ, $L := \{(\xi(z))_{\xi\in\Xi} \mid z \in \mathcal{O}_K\}$.

- $p_K(\sigma, \Xi) := \pi^{-1} \int_{\gamma} \omega_{\sigma}$ (${}^{\forall}$ CM 型 Ξ, ${}^{\forall}\sigma \in \Xi$),

$K \overset{\sigma}{\curvearrowright} \omega_{\sigma} \in H^0(A, \Omega^1_A)$, γ: $A(\mathbb{C})$ の閉路.

- $p_K(\sigma, \Xi) =: \prod_{\xi\in\Xi} p_K(\sigma, \xi)$ s.t. $p_K(\sigma, \xi)p_K(\sigma, \overline{\xi}) = 1$ と "分解" できる:

$\sigma, \tau \in Hom_{\mathbb{Q}}(K, \mathbb{C}) \Rightarrow {}^{\exists}\Xi_i \ni \sigma$ s.t. $\sum_i n_i \Xi_i := \sum_i \sum_{\xi\in\Xi_i} n_i \xi = \tau - \overline{\tau}$

$\Rightarrow p_K(\sigma, \tau) := \prod_i p_K(\sigma, \Xi_i)^{\frac{n_i}{2}}$ ($\because$ 志村の単項関係式).

- Well-defined up to $\overline{\mathbb{Q}}^{\times}$.

※ CM 周期 $\Leftarrow H^1_{dR}(A) \times H^B_1(A(\mathbb{C})) \to \mathbb{C}$, $(\omega, \gamma) \mapsto \int_{\gamma} \omega$.

Theorem 2 (Chowla-Selberg 公式)

- モジュラー判別式: $\Delta(z) = q\prod_{n=1}^{\infty}(1-q^n)^{24}$, $q = e^{2\pi iz}$, 重さ 12.
- K: 虚二次体, $\chi = \left(\frac{-d}{*}\right)$, $L(s,\chi) = \sum_n \chi(n)n^{-s}$.

$$\text{(CSF)} \quad \exp\left(\frac{12hL'(0,\chi)}{L(0,\chi)}\right) = (2\pi)^{12h}\prod_{\bar{\mathfrak{a}}\in Cl_K}|\Delta(\bar{\mathfrak{a}})|^2.$$

$\Delta(\bar{\mathfrak{a}}) := N(\mathfrak{a})^{12}\Delta(\frac{\omega_1}{\omega_2})\omega_2^{-12}$ if $\mathfrak{a} = \mathbb{Z}\omega_1 \oplus \mathbb{Z}\omega_2$, $Im(\omega_1/\omega_2) > 0$.

- 解析的類数公式 $L(0,\chi) = \frac{2h}{w}$.
- Lerch の公式 $L'(0,\chi) = \sum_{a=1}^{d}\chi(a)\log(\Gamma(\frac{a}{d})) - L(0,\chi)\log d$.

Corollary 3

$$\pi p_K(id,id)^2 \equiv \prod_{a=1}^{d}\Gamma(\tfrac{a}{d})^{\frac{w\chi(a)}{2h}} \mod \overline{\mathbb{Q}}^{\times}.$$

Corollary 3

$$\pi p_K(id,id)^2 \equiv \prod_{a=1}^{d} \Gamma(\tfrac{a}{d})^{\frac{w\chi(a)}{2h}} \mod \overline{\mathbb{Q}}^\times.$$

Example 4 ($K=\mathbb{Q}(i)$, $h=1$, $d=4$, $\chi\colon (\mathbb{Z}/4\mathbb{Z})^\times \to \{\pm 1\}$)

$$\pi^{\frac{1}{2}} p_{\mathbb{Q}(i)}(id,id) \equiv \prod_{a=1}^{4} \Gamma(\tfrac{a}{4})^{\chi(a)} = \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}.$$

c.f. $p_K(id,id) \overset{\text{虚二次体}}{\equiv} \pi^{-1}\int_\gamma \frac{dx}{y} \overset{\substack{K=\mathbb{Q}(i)\\ y^2=1-x^4}}{\equiv} \pi^{-1}\cdot 2\int_0^1 \frac{1}{\sqrt{1-x^4}}dx$

$= \pi^{-1}\cdot\varpi = \dfrac{\pi^{\frac{-1}{2}}\Gamma(\frac{1}{4})}{2\Gamma(\frac{3}{4})}.$

虚二次体 $\Rightarrow$ CM 体, Γ 関数 $\Rightarrow$??? $\rightsquigarrow$ 吉田敬之氏の絶対周期記号(予想)

吉田予想

Definition 5 (Barnes の多重 Γ 関数)

$$\Gamma(x,(\omega_1,\dots,\omega_r)) \qquad (x,\omega_i>0)$$
$$:= \exp\left(\frac{d}{ds}\sum_{m_1,\dots,m_r\geq 0}(x+m_1\omega_1+\cdots+m_r\omega_r)^{-s}\Big|_{s=0}\right).$$

Example 6

$\Gamma(x,(1)) = \exp\left(\frac{d}{ds}\sum_{m\geq 0}(x+m)^{-s}\Big|_{s=0}\right) \overset{\text{Lerch}}{=} \frac{\Gamma(x)}{\sqrt{2\pi}}.$

Conjecture 7 (絶対 CM 周期記号)

$${}^{\forall}K,\sigma,\tau,\ p_K(\sigma,\tau) \equiv \prod \Gamma(x,\boldsymbol{\omega})^a \times \prod b^c \bmod \overline{\mathbb{Q}}^{\times}\ \text{の形.}$$

※ $x,\omega_i,b,c\in\widetilde{K}^+$, $a\in\mathbb{Q}$ は明示的, “新谷基本領域” の取り方による.

Example 8 (円分体の場合 (アーベル体 $\Rightarrow$ 多重じゃない Γ))

$F_n \colon x^n + y^n = 1 \rightsquigarrow J(F_n)$ の成分は $\mathbb{Q}(\zeta_m)$ $(m \mid n)$ の虚数乗法を持つ.

$$\int_{\exists\gamma} x^{r-1} y^{s-n} dx = B(\tfrac{r}{n}, \tfrac{s}{n}) = \frac{\Gamma(\frac{r}{n})\Gamma(\frac{s}{n})}{\Gamma(\frac{r+s}{n})} \quad (0 < r, s, r+s < n).$$

Example 9

$C \colon y^2 = \frac{7+\sqrt{41}}{2}x^6 + (-10-2\sqrt{41})x^5 + 10x^4 + \frac{41+\sqrt{41}}{2}x^3 + (3-2\sqrt{41})x^2 + \frac{7-\sqrt{41}}{2}x + 1,$
$C' \colon y^2 = \frac{7-\sqrt{41}}{2}x^6 + (-10+2\sqrt{41})x^5 + 10x^4 + \frac{41-\sqrt{41}}{2}x^3 + (3+2\sqrt{41})x^2 + \frac{7+\sqrt{41}}{2}x + 1.$
$\rightsquigarrow J(C), J(C')$ は $K = \mathbb{Q}(\sqrt{2\sqrt{5}-26})$ の虚数乗法を持つ. (※ $\mathbb{Q}$ 上 non-abelian)
$\omega_{\mathrm{id}} := \frac{2dx}{y} + \frac{(\sqrt{5}-1)x dx}{y}$ (C 上), $\omega'_{\mathrm{id}} := \frac{2dx}{y} + \frac{(\sqrt{5}-1)x dx}{y}$ (C' 上).

$$\pi^{-1}\int_\gamma \omega_{\mathrm{id}} \int_{\gamma'} \omega'_{\mathrm{id}} \fallingdotseq \prod_{20\text{ 個の }(x_1,x_2)} \Gamma_2\left(x_1 + \frac{3-\sqrt{5}}{2}x_2, \left(1, \frac{3-\sqrt{5}}{2}\right)\right)$$
$$\times \left(\frac{\sqrt{5}-1}{2}\right)^{\frac{19\sqrt{5}+42}{123}} \frac{(\sqrt{5}+13)\sqrt{-8\sqrt{5}+20+(\sqrt{5}+15)\sqrt{2\sqrt{5}-26}}}{6560}.$$

$(\frac{1}{41}, \frac{5}{41}), (\frac{2}{41}, \frac{10}{41}), (\frac{4}{41}, \frac{20}{41}), (\frac{5}{41}, \frac{25}{41}), (\frac{8}{41}, \frac{40}{41}), (\frac{9}{41}, \frac{4}{41}), (\frac{10}{41}, \frac{9}{41}), (\frac{16}{41}, \frac{39}{41}), (\frac{18}{41}, \frac{8}{41}), (\frac{20}{41}, \frac{18}{41}),$
$(\frac{21}{41}, \frac{23}{41}), (\frac{23}{41}, \frac{33}{41}), (\frac{25}{41}, \frac{2}{41}), (\frac{31}{41}, \frac{32}{41}), (\frac{32}{41}, \frac{37}{41}), (\frac{33}{41}, \frac{1}{41}), (\frac{36}{41}, \frac{16}{41}), (\frac{37}{41}, \frac{21}{41}), (\frac{39}{41}, \frac{31}{41}), (\frac{40}{41}, \frac{36}{41}).$

応用と問題

数値例 & 精密化

Bouyer-Streng, Examples of CM curves of genus two defined over the reflex field, LMS J. Comput. Math., 18 (2015), no. 1, 507–538.

- “代数的数部分”=?, “相互法則 (明示的な Galois 群の作用)”?.

Stark 予想

K-, On the algebraicity of some products of special values of Barnes' multiple gamma function. Amer. J. Math. 140 (2018), no. 3, 617–651.

- 吉田予想 ⇒ Stark 単数 $\exp(\zeta'(0,\tau)) \in \overline{\mathbb{Q}}$: e.g., Ex. 8 ⇒ $\cos(\frac{a}{n}) \in \overline{\mathbb{Q}}$.
- 精密化: 単数性?, ∈???, Rubin's Integral refinement version?

Kronecker 極限公式の拡張

Yoshida, Absolute CM-periods, Math. Surv. Monogr. 106(2003), Chap.V.

- $E(z,s) := \sum_{(m,n)\neq(0,0)} \frac{y^s}{|mz+n|^{2s}} \overset{\text{(KLF)}}{=} \frac{\pi}{s-1} + 2\pi(\gamma - \log(2\sqrt{y}|\Delta(z)|^{\frac{1}{12}})) + O(s-1)$.

$$\boxed{\log|\Delta(z)|} \overset{\text{KLF}}{\Leftrightarrow} E(z,s) \Rightarrow \zeta_K(s), L(s,\chi), \Gamma(\tfrac{a}{d}) \rightsquigarrow \text{CSF}.$$

$K = \mathbf{Q}(\sqrt{-1})$. Then (E, θ) is of CM-type $(K, \{\mathrm{id}\})$. Since dx/y is a $\mathbf{Q}$-rational holomorphic differential 1-form, we get

$$\pi p_K(\mathrm{id}, \mathrm{id}) \sim \int_0^1 \frac{dx}{\sqrt{1-x^4}}.$$

Recall the well known formula for the beta function $B(p,q)$ (cf. [WW], p. 253):

$$B(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q),$$

$$\text{(1.3)} \qquad \int_0^1 x^{p-1}(1-x)^{q-1}dx = B(p,q), \qquad \Re(p) > 0, \quad \Re(q) > 0.$$

We get

$$\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{1}{4}B(\frac{1}{4}, \frac{1}{2}) = \frac{\sqrt{\pi}}{4}\frac{\Gamma(1/4)}{\Gamma(3/4)}.$$

Hence we obtain

$$\sqrt{\pi} p_{\mathbf{Q}(\sqrt{-1})}(\mathrm{id}, \mathrm{id}) \sim \frac{\Gamma(1/4)}{\Gamma(3/4)}.$$

More generally, let K be an imaginary quadratic field of discriminant $-d$ and let χ be the Dirichlet character which corresponds to the quadratic extension $K/\mathbf{Q}$. Then the Chowla-Selberg formula ([SC], §12) states

$$\text{(1.4)} \qquad \pi p_K(\mathrm{id}, \mathrm{id})^2 \sim \prod_{a=1}^{d-1} \Gamma(\frac{a}{d})^{w\chi(a)/2h},$$

where w is the number of roots of unity contained in K and h is the class number of K. Except for the cases $K = \mathbf{Q}(\sqrt{-1})$ and $K = \mathbf{Q}(\sqrt{-3})$, there is no known way to derive (1.4) from the direct evaluation of an elliptic integral. We will give a geometric proof of (1.4) in the next section.

where w is the number of roots of unity contained in K and h is the class number of K. Except for the cases $K = \mathbf{Q}(\sqrt{-1})$ and $K = \mathbf{Q}(\sqrt{-3})$, there is no known way to derive (1.4) from the direct evaluation of an elliptic integral. We will give a geometric proof of (1.4) in the next section.

Yoshida, Absolute CM-periods $\cdots$, p63–

- “geometric proof”: $F_n: x^n + y^n = 1$ (Ex. 8)$\Rightarrow p_{\mathbb{Q}(\zeta_{d_K})} \Rightarrow p_{\mathbb{Q}(\sqrt{-d_K})}$.
- “direct(?)”: $\mathbb{Q}(\zeta_{d_K}) \supset \mathbb{Q}(\sqrt{-d}) \rightsquigarrow F_{d_K} \Leftrightarrow E$ with CM by $\mathbb{Q}(\sqrt{-d_K})$ $\rightsquigarrow \int_\gamma \frac{dx}{y}$ の “変数変換”.

e.g.,

- $E: y^2 = x^3 - 595x - 5586$, $End(E) \cong \mathbb{Z}[\sqrt{-7}]$
  (CSF) $\pi p_{\mathbb{Q}(\sqrt{-7})}(id, id)^2 \equiv \prod_{a=1}^{7} \Gamma(\frac{a}{7})^{\frac{\chi(a)}{2}} = \frac{\Gamma(\frac{1}{7})\Gamma(\frac{2}{7})\Gamma(\frac{4}{7})}{\Gamma(\frac{3}{7})\Gamma(\frac{5}{7})\Gamma(\frac{6}{7})}$
  $\Rightarrow \int_\gamma \frac{dx}{\sqrt{x^3-595x-5586}} \doteqdot \frac{\Gamma(\frac{1}{7})\Gamma(\frac{2}{7})\Gamma(\frac{4}{7})}{\pi} = B(\frac{1}{7}, \frac{2}{7}) = \int_0^1 t^{\frac{-6}{7}}(1-t)^{\frac{-5}{7}}$.
- $\mathbb{Q}(\sqrt{-5})$, $h_{\mathbb{Q}(\sqrt{-5})} = 2 \Rightarrow \int_\gamma \frac{dx}{\sqrt{x^3+\sqrt[4]{5}x^2-(5+3\sqrt{5})x+\sqrt[4]{5}(5+\sqrt{5})}} \doteqdot$
  $\sqrt{\int_0^1 t^{\frac{-19}{20}}(1-t)^{\frac{-11}{20}} \int_0^1 t^{\frac{-17}{20}}(1-t)^{\frac{-14}{20}}}$.

p 進類似 $\cdots$ de Rham の同型 $\Rightarrow$ p 進 Hodge の比較同型

- CM 周期: $H^1_{dR}(A) \times H^B_1(A) \to \mathbb{C}$, $(\omega_\sigma, \gamma) \mapsto \pi \cdot p_K(\sigma, \Xi)$.

$\Leftarrow$ $H^1_{dR}(A) \otimes_{\overline{\mathbb{Q}}} \mathbb{C} \cong H^1_B(A) \otimes \mathbb{C}$ & $H^B_1(A) \times H^1_B(A) \to \mathbb{Q}$.

- $H^1_{dR}(A) \otimes_{\overline{\mathbb{Q}}} B_{dR} \cong H^1_{p,et}(A) \otimes_{\mathbb{Q}_p} B_{dR}$, $H^1_B(A) \otimes \mathbb{Q}_p \cong H^1_{p,et}(A)$.

$\Rightarrow$ p 進周期: $H^1_{dR}(A) \times H^B_1(A) \to B_{dR}$, $(\omega_\sigma, \gamma) \mapsto \pi_p \cdot p_{K,p}(\sigma, \Xi)$.

Theorem 10 (Coleman の公式 on abs.Frob. $\curvearrowright F_n/\mathbb{F}_p$ $(p \nmid 2n)$)

$$G(\tfrac{a}{n}) = \frac{\Gamma \text{ 関数} \cdot p \text{ 進周期}}{\text{CM 周期}} := \frac{\frac{\Gamma(\frac{a}{n})}{\sqrt{2\pi}} \pi_p^{\frac{1}{2}-\langle \frac{a}{n} \rangle} \prod_{(b,n)=1} p_{\mathbb{Q}(\zeta_n),p}(id, \sigma_b)^{\frac{1}{2}-\langle \frac{ab}{n} \rangle}}{\pi^{\frac{1}{2}-\langle \frac{a}{n} \rangle} \prod_{(b,n)=1} p_{\mathbb{Q}(\zeta_n)}(id, \sigma_b)^{\frac{1}{2}-\langle \frac{ab}{n} \rangle}}.$$

$$[\text{abs.Frob.} \curvearrowright H^1_{cris}(F_n/\mathbb{F}_p)] \doteqdot p^{\frac{1}{2}-\langle \frac{a}{n} \rangle} \frac{G(\langle \frac{pa}{n} \rangle)}{\Phi_{\mathrm{cris}} G(\frac{a}{n})} \equiv \Gamma_p(\langle \tfrac{pa}{n} \rangle) \mod \mu_\infty.$$

円分体 ⇒ 一般の CM 体

Conjecture 11

K-, On a common refinement of Stark units and Gross-Stark units (arXiv:1706.03198)

Theorem 12

Cnj. 11 ⇒ rank 1 abel Gross-Stark 予想 (解決), Stark 単数の相互法則

Example 13

- Thm 10: $G(\frac{a}{n}) := \dfrac{\frac{\Gamma(\frac{a}{n})}{\sqrt{2\pi}} \pi_p^{\frac{1}{2}-\langle\frac{a}{n}\rangle} \prod_{(b,n)=1} p_{\mathbb{Q}(\zeta_n),p}(id,\sigma_b)^{\frac{1}{2}-\langle\frac{ab}{n}\rangle}}{\pi^{\frac{1}{2}-\langle\frac{a}{n}\rangle} \prod_{(b,n)=1} p_{\mathbb{Q}(\zeta_n)}(id,\sigma_b)^{\frac{1}{2}-\langle\frac{ab}{n}\rangle}}$,

$$\Rightarrow p^{\frac{1}{2}-\langle\frac{a}{n}\rangle} \frac{G(\langle\frac{pa}{n}\rangle)}{\Phi_{\mathrm{cris}} G(\frac{a}{n})} \equiv \Gamma_p(\langle\frac{pa}{n}\rangle) \mod \mu_\infty.$$

- $G(\frac{a}{n})G(\frac{n-a}{n}) = \frac{\Gamma(\frac{a}{n})}{\sqrt{2\pi}} \frac{\Gamma(\frac{n-a}{n})}{\sqrt{2\pi}} = \frac{1}{2\sin(\frac{a}{n}\pi)} \in \overline{\mathbb{Q}} \overset{\text{Thm.10}}{\curvearrowleft} \Phi_{cris}|_{\overline{\mathbb{Q}}_p} \fallingdotseq$ Frob. at p

応用・問題

数値例 ⇒ Bouyer-Streng, Examples of CM curves ...

- $\omega_i = \sum a_n^{(i)} t^n \frac{dt}{t} \in H^1_{cris}$, $\Phi_{cris}(\omega_0) = \alpha\omega_1 \Rightarrow \alpha = \lim_{\frac{n_k}{a^{(0)}_{n_k}} \to 0} \frac{p\sigma_p(a^{(0)}_{n_k})}{a^{(1)}_{pn_k}}$.

e.g., $E\colon y^2 = 1 - x^4$, $\omega = \frac{dx}{y} = \sum (-1)^{\frac{n-1}{4}} \binom{-\frac{1}{2}}{\frac{n-1}{4}}$

$$p \equiv 1 \bmod 4 \Rightarrow \alpha = \lim_{k\to\infty} \frac{p(-1)^{\frac{p^k-1}{4}} \binom{-\frac{1}{2}}{\frac{p^k-1}{4}}}{(-1)^{\frac{p^{k+1}-1}{4}} \binom{-\frac{1}{2}}{\frac{p^{k+1}-1}{4}}} = p \cdot \frac{\Gamma_p(\frac{3}{4})}{\Gamma_p(\frac{1}{4})\Gamma_p(\frac{1}{4})}.$$

精密化

- Coleman の公式の “1 の冪根部分” の復元.
- p 進周期そのもの? c.f. 吉田予想 $\cdots$ CM 周期 v.s. 多重ガンマ関数.