RIMS Workshop
Algebraic Number Theory and Related Topics

Organizers:
Tomokazu Kashio (Tokyo University of Science)
Masataka Chida (Tokyo Denki University)

Date: December 12 (Mon)– December 16 (Fri), 2022
Place: Room 420, Research Institute for Mathematical Sciences (RIMS), Kyoto University, Kyoto 606-8502, JAPAN +Zoom (hybrid)

概要 / Abstract

December 12 (Mon)

10:00 – 10:50 **王 沛鐸 / Peiduo Wang** (The University of Tokyo)
On generalized Fuchs theorem over p-adic polyannuli

In this talk, we study finite projective differential modules on p-adic polyannuli satisfying the Robba condition. Christol and Mebkhout proved the decomposition theorem (the p-adic Fuchs theorem) of such differential modules on one dimensional p-adic annuli under certain non-Liouvilleness assumption and Gachet generalized it to higher dimensional cases. On the other hand, Kedlaya proved a generalization of the p-adic Fuchs theorem in one dimensional case. We prove Kedlaya's generalized version of p-adic Fuchs theorem in higher dimensional cases.

11:10 – 12:00 **Abhinandan** (The University of Tokyo)
Syntomic complex and finite height crystalline representations

In the proof of p-adic crystalline comparison theorem, one of the most important steps in the approach of Fontaine and Messing is to establish a comparison between syntomic cohomology and p-adic étale cohomology via (Fontaine-Messing) period map. This approach was successfully generalized to the semistable case by Kato and a complete proof of crystalline and semistable comparison theorems for schemes was given by Tsuji. Few years ago, Colmez and Nizioł gave a new interpretation of the (local) Fontaine-Messing period map in terms of complexes of (φ, Γ)-modules and used it to prove semistable comparison theorem for p-adic formal schemes. In this talk, we will first look at a local generalisation (of crystalline version of this interpretation by Colmez and Nizioł) to crystalline coefficients (on syntomic side) and relative Wach modules (on (φ, Γ)-module side) introduced by the speaker. Then as a global application we will establish a comparison between syntomic complex with coefficients in a locally free relative Fontaine-Laffaille module and p-adic nearby cycles of the associated p-adic étale local system.

13:30 – 14:20 **吉崎 彪雅 / Hyuga Yoshizaki** (Tokyo University of Science)
The p-adic limits of class numbers in $\mathbb{Z}_p$-towers

In a previous study on Weber's class number problem, the speaker pointed out that the class numbers in the $\mathbb{Z}_2$-extension of the rational number field converge in the ring $\mathbb{Z}_2$ of 2-adic integers. In this talk, p being a prime number, we prove a similar result for a general $\mathbb{Z}_p$-extension over a global field. In addition, we prove an analogous assertion for a $\mathbb{Z}_p$-cover of a compact 3-manifold and discuss variants of Weber's class number problem in the spirit of arithmetic topology. Furthermore, we pursue numerical studies to establish an explicit formula for the p-adic limit of cyclic resultants, using p-prime-th roots of unity, the p-adic logarithm, and the Iwasawa invariants. Concrete examples we will exhibit are $\mathbb{Z}_p$-covers of torus knots and twist knots in S^3, and constant $\mathbb{Z}_p$-extensions of the function fields of elliptic curves. (This talk is based on a joint work with Jun Ueki at Ochanomizu University.)

14:40 – 15:30 **宮﨑 弘安 / Hiroyasu Miyazaki** (NTT Institute for Fundamental Mathematics)

モジュラス付きモチーフのホッジ実現について

On Hodge realization of motives with modulus

さまざまなコホモロジーを束ねることがモチーフ理論の究極目標の一つである。Voevodsky の混合モチーフ理論は $\mathbb{A}^1$-ホモトピー不変層のコホモロジーを束ねる理論として大きな成功を収めたが、裏を返せば、非-$\mathbb{A}^1$-ホモトピー不変なコホモロジーを捉えられないという問題も抱えている。この弱点を克服することを目的として、講演者はKahn-齋藤-山崎と共同で、Voevodsky の理論をモジュラス付きモチーフ理論に一般化した。本講演では、非-$\mathbb{A}^1$-ホモトピー不変なコホモロジーの基本的な例であるホッジコホモロジーを、少なくとも標数 0 の体上では、モジュラス付きモチーフの枠組みで表現できることを紹介する。本講演は Shane Kelly 氏との共同研究に基づく。

The theory of motives aims to unify various cohomology theories. Voevodsky's motive theory unifies the cohomology of $\mathbb{A}^1$-invariant sheaves, while it does not capture non-$\mathbb{A}^1$-invariant ones. In order to overcome this defect, I introduced motives with modulus in joint work with Kahn-Saito-Yamazaki. In this talk, I would like to explain that we can represent the Hodge cohomology in the category of motives with modulus, at least over a field of characteristic 0. This talk is based on joint work with Shane Kelly.

15:50 – 16:50 **坂内 健一 / Kenichi Bannai** * (Keio University / RIKEN)

On the Equivariant Polylogarithm Class for the Algebraic Torus associated with a Totally Real Field

The polylogarithm class is a motivic class constructed in the motivic cohomology of an algebraic group minus the identity. It has played an important role in the proof of the Beilinson conjecture and the Bloch-Beilinson-Kato conjecture in the case of the multiplicative group as well as that of elliptic curves. In this talk, we will discuss the Hodge realization of the equivariant polylogarithm for the case of the algebraic torus associated to a totally real field, and give its relation to the Shintani generating class. We will explain how equivariant action with respect to the unit group as well as existence of plectic structures conjecturally allow for obtaining important information from the polylogarithm. This is a joint work with Hohto Bekki, Kei Hagihara, Tatsuya Ohshita, Kazuki Yamada and Shuji Yamamoto.

December 13 (Tue)

10:00 – 10:50 **中山 裕大 / Yuta Nakayama** (The University of Tokyo)

Topics related to RSZ Shimura varieties

Rapoport, Smithling and Zhang invented Hodge type Shimura varieties and their integral models to pose a variant of Arithmetic Gan–Gross–Prasad conjecture. In this talk, we mainly show that their integral models are isomorphic to the integral models that Kisin and Pappas uniformly defined for Hodge type Shimura varieties with parahoric levels. Our method relies on canonicity, the universal properties introduced by Pappas that characterize the latter integral models. If we have time, we also describe a result on the variant of Arithmetic Gan–Gross–Prasad conjecture for Shimura curves.

11:10 – 12:00 **Joseph Muller** (Université Sorbonne Paris Nord / The University of Tokyo)

On the cohomology of the unramified PEL unitary Rapoport-Zink space of signature $(1, n-1)$

Rapoport-Zink (RZ) spaces are moduli spaces which classify the deformations of a p-divisible group with additional structures. It is equipped with compatible actions of p-adic and Galois groups, and their cohomology is believed to play a role in the local Langlands program. So far, the cohomology of RZ spaces is entirely known only in the cases of the Lubin–Tate tower and of the Drinfeld space; in particular both of them are RZ spaces of EL type. In this talk, we consider the unramified PEL unitary RZ space with signature $(1, n-1)$. In 2011, Vollaard and Wedhorn proved that it is stratified by generalized Deligne–Lusztig varieties, whose incidence relations mimic the combinatorics of the Bruhat–Tits building of a unitary group. We compute the cohomology of these strata and we draw some consequences on the cohomology of the RZ space at hyperspecial level. In particular, we prove that it is not admissible in general. When $n = 3, 4$ we deduce an automorphic description of the cohomology of the basic stratum in the corresponding Shimura variety via p-adic uniformization.

13:30 – 14:20 **松田 光智 / Koji Matsuda** (The University of Tokyo)

Modular Jacobian varieties over cyclotomic fields with the Mordell-Weil rank 0

The Mordell–Weil group of an elliptic curve over an algebraic number field is a finitely generated abelian group, hence its torsion subgroup is finite. It is easy to compute the torsion subgroup of the Mordell–Weil group of individual given elliptic curve. Conversely for given finite abelian group, it is a hard problem to determine if it is the torsion subgroup of the Mordell–Weil group of an elliptic curve or not. In 1977 Mazur solved this problem over $\mathbb{Q}$ by studying arithmetic properties of modular curves. In this talk, we determine all modular curves whose Jacobian varieties have Mordell–Weil rank zero among those defined over cyclotomic number fields, not only over the rational number field, and using it we show a certain classification result of possible torsion subgroups of elliptic curves over cyclotomic fields.

14:40 – 15:30 **奥山 裕介 / Yûsuke Okuyama** (Kyoto Institute of Technology)

非アルキメデス的体上の多項式力学系の潜在的半安定還元の停留性

Stationarity of potential semistable reductions for non-archimedean polynomial dynamics

A rational function of degree $d(>1)$ in one variable defined over an algebraically closed field K that is complete with respect to a non-trivial and non-archimedean absolute value can be reduced to that defined over the residual field k of K but of degree $\leq d$. Rumely determined algorithmically when for some projective coordinate change $h \in \mathrm{PGL}(2, K)$

of $\mathbb{P}^1_K$, the conjugation $h \circ f \circ h^{-1}$ of a given rational function $f \in K(z)$ of degree d has a "good reduction" (i.e., the reduction of $h \circ f \circ h^{-1}$ is still of degree d) and, more generally, determined all $h \in \mathrm{PGL}(2, K)$ for each of which the conjugation $h \circ f \circ h^{-1}$ has a GIT-(semi)stable reduction (in the ambient space $\mathbb{P}^{2d+1}_k$ of the space of rational functions defined over k and of degree d) in the Hilbert-Mumford sense. The GIT semistability has been successfully applied to the moduli of hypersurfaces, arrangements of points, algebraic curves, etc, which however cannot be iterated, and few things seem to have been ever known about how the above "potential semistable reduction" property behaves under iteration of rational functions.
In this talk, for polynomials, we would give an answer to this problem, which states that this "potential semistable reduction" property is stationary for the number of times of iteration of a given polynomial. We would also talk about a related harmonic analysis on the Berkovich projective line/hyperbolic space. This talks is based on our joint work with Professor Hongming Nie (Stony Brook University).

15:50 – 16:50 山木 壱彦 / **Kazuhiko Yamaki** * (University of Tsukuba)
Survey on the geometric Bogomolov conjecture

In Diophantine geometry, it is interesting to ask how the points of small height are distributed on a projective variety over a number field or a function field. The geometric Bogomolov conjecture, which is the main topic of this talk, is one of such questions over function fields. This conjecture was formulated in 2013 by the speaker. Then several significant partial results had been established by several authors, and in 2022, it was affirmatively solved in full generality by Xie and Yuan with the crucial aid of previous partial results. In this talk, we will begin by recalling the geometric Bogomolov conjecture with its background and then give a very brief outline of the proof.

December 14 (Wed)

09:30 – 10:20 高 智強 / **Chih-Chiang Kao** (Tokyo Institute of Technology)
Iterated Galois groups of $X^2 + c$ over quadratic number field with odd class number

Let K be a quadratic number field with odd class number. Consider the polynomial $f(X) = X^2 + c$ where c is an algebraic integer in K. Denote the n-th iteration of $f(X)$ by $f^n(X)$, then the Galois group $\mathrm{Gal}(f^n(X)/K)$ can be embedding into $[C_2]^n$ which is the n-th fold wreath product of cyclic group with 2 elements. We will give some criteria on the constant terms of $f^n(X)$ to determine that when the embedding is surjective for all $n \in \mathbb{N}$.
In the second part, we will focus on $K = \mathbb{Q}(\sqrt{d})$, where

(i) $d = 2, q$ — q is a prime congruent to 1 modulo 4;

(ii) $d = l, 2l, l_1 l_2$ — l, l_1, l_2 are primes congruent to 3 modulo 4;

(iii) $d = -1, -2, -l$ — l is a prime congruent to 3 modulo 4.

All the cases have odd class numbers. We will use the quadratic residue properties for the fundamental unit to give some sufficient conditions on c such that $\mathrm{Gal}(f^n(X)/K) \cong [C_2]^n$.

10:35 – 11:25 中原 徹 / **Toru Nakahara** (University of Malakand)

あるアーベル及び非アーベル拡大体の Monogenity
Monogenity of certain abelian and non-abelian extension fields

あるアーベル及び非アーベル拡大体の Monogenity について二つの定理を紹介する。
定理 A. 有理数体上拡大次数 $2(p-1)$ 次のアーベル体は二種類の例外を除いて non-monognic である。
定理 B. 有理数体上 4 次の二面体拡大体の整数基と monogenity とを決定する。

Our claims of recent research jointed with PhD/Postdoc scholars in Pakistan are as follows.
Theorem A *The non-cyclic abelian fields* $K = \boldsymbol{Q}(\zeta_{|p^*|}, \sqrt{\ell^*})$ *are non-monogenic except for the two classes of the fields with the conductors* $3p^* = |-3| \cdot p$ *for* $\ell^* = -3$ *or* $4p^* = |4 \cdot (-1)| \cdot p$ *for* $\ell^* = -4$ *under the conditions* $(p^*, \ell^*) = 1$, *with a prime number* p *and a squarefree odd number* $|\ell| > 3$ *or even* $|\ell^*| \geqq 2^3$ *of the conductor* $p^*\ell^*$ *for the conductor* $p^* = \pm p \equiv 1 \pmod 4$ *of a prime cyclotomic field* $k_{|p^*|}$ *and the conductor* ℓ^* *of a quadratic subfield* $k = \boldsymbol{Q}(\sqrt{l^*}) \subset k_{|\ell^*|}$ *with the odd field discriminat* $d_k = \ell^* = \pm\ell$ *or the even* $d_k = \ell^* = \pm 4\ell$.

This result applying an idea of [N. Khan et al, JNT **198**(2019), 43-51] is a generalization of [M. Sultan, T. N. Monatsh. für Math. **176**_ 1(2015) 153-162].

Theorem B *Let* K *be a Dihedral quartic field* $\boldsymbol{Q}(\sqrt{a+b\omega})$, *where* $a^2 + ab + b^2\frac{1-m}{4}$ *is a squrefree integer and the quadratic subfield* $k = \boldsymbol{Q}(\omega)$ *of* K *has the odd conductor* m *with* $\omega = \frac{1+\sqrt{m}}{2}$. *Then all the integral bases of* K *are given into the twelve families* ${}_m\mathrm{C}^a_{b\_b'}$ *with* $m \equiv 1, 5 \pmod 8$, $a \equiv 1, 3 \pmod 4), b\_b' \equiv 1\_3, 2\_4 \pmod 4$ $a \equiv 2, 4 \pmod 4), b\_b' \equiv 1\_3 \pmod 4$. *Here the twenty four = 32 -eight empty families can be summarized into twelve types and e.g. the family* ${}_1C^1_{1\_3}$ *denotes* $[1_8, 1_4, 1\_3_4] = [a \equiv 1_8, b \equiv 1_4, m \equiv 1\_3_4]$.

Integral bases of Dihedral quartic fields with the even or odd field discriminant of the quadratic subfields have been determined by K. S. Williams, et al [JNT **51**(1995), 87-201]. However their characterization is complicated using many parameters. Also one monogenic Dihedral quartic field containg a real quadratic subfield of conductor 5 has been shown by A. C. Kable [JNT **76**-1 (1999) 120-129]. Refer A. Yukie, *Number Theory* **2** - *Foundations of Algebraic Number Theory* -, 2014, page 131. Then this work involves a simple method to constract integral bases of such fields and classify the monogenity of all the Dihedral quartic fields together with the density [M. Ahmad and T. N., *Integral bases of Dihedral quartic fields and its Application,* in preparation].

11:40 – 12:30 **金村 佳範 / Yoshinori Kanamura** (Keio University)
$\sqrt{m}$ の整数係数連分数展開表示とその Pell 方程式への応用について
Some periodic integer continued fraction expansions of $\sqrt{m}$ and application to the Pell equations

整数係数連分数とは、正則連分数において自然数列を考えていた箇所で、整数列も許して得られる連分数のことを指す。本講演では、m を平方数ではない自然数として、$\sqrt{m}$ の整数係数連分数展開について考える。古くから、$\sqrt{m}$ の正則連分数展開表示は一意に得られることが知られている。一方、整数係数連分数展開表示を考えると一意性は成り立たず、一般には複数の表示が得られる。そこで、$\sqrt{m}$ の整数係数連分数展開表示を全決定することは、自然な問題として挙げられる。本講演では、$\sqrt{m}$ の整数係数連分数展開表示について、特定の型を持つものを全決定したので、その結果を紹介する。また、今回決定し

た $\sqrt{m}$ の整数係数連分数展開表示から、ペル方程式の最小解が正則連分数展開の場合と同様に得られることについても紹介したい。本講演は東京理科大学の吉崎彪雅氏との共同研究に基づくものである。

Periodic integer continued fractions (PICFs) are generalization of the regular periodic continued fractions (RPCFs). It is classical that a RPCF expansion of an irrational number is unique. However, it is no longer unique for a PICF expansion. Hence it is a natural problem to determine all PICF expansions of irrational numbers. In this talk, we determine certain type PICF expansions of square roots of positive square-free integers. As an application of these results, we obtain fundamental solutions of the Pell equations from PICF expansions of square roots of positive square-free integers as well as the RPCF expansions. This is joint work with Hyuga Yoshizaki (Tokyo Univ. of science).

December 15 (Thu)

10:00 – 10:50 **杉山 真吾 / Shingo Sugiyama** (Nihon University)
モジュラー形式のヘッケ固有値の代数的整数性
Integrality of Hecke eigenvalues of modular forms

楕円モジュラー形式のヘッケ固有値が代数的整数であることはよく知られており、重さやレベルが限定されたヒルベルトモジュラー形式やジーゲルモジュラー形式に対しても、同様の現象が考察されてきた。本講演では、一般の重さとレベルを持つヒルベルトモジュラー形式とジーゲルモジュラー形式のヘッケ固有値が代数的整数であることを証明する。応用として、GL($2d$)(ただし d は素数) と Sp($2n$) のカスピダル保型表現のヘッケ体の拡大次数の増大度の評価を与える。これは佐久川憲児氏 (信州大学) との共同研究である。

It is well known that Hecke eigenvalues of elliptic modular forms are algebraic integers. Similar phenomena have been observed for Hilbert modular forms and for Siegel modular forms under certain constraints on weights and levels. In this talk, we prove that Hecke eigenvalues of Hilbert modular forms and of Siegel modular forms are algebraic integers in a general setting. As an application, we estimate the growth of the degrees of Hecke fields of cuspidal automorphic representations of GL($2d$) with a prime number d and of Sp($2n$). This talk is based on a joint work with Kenji Sakugawa (Shinshu University).

11:10 – 12:00 **山名 俊介 / Shunsuke Yamana** (Osaka Metropolitan University)
On p-adic L-functions for $\mathrm{U}(3) \times \mathrm{U}(2)$

The Ichino-Ikeda conjecture is an explicit relation between the central L-value and squares of a certain period of automorphic forms. This conjecture has been established by Beuzart-Plessis, Yifeng Liu, Wei Zhang, Xinwen Zhu, Chaudouard and Zydor for unitary groups. I will report on my joint work in progress with Michael Harris and Ming-Lun Hsieh on the construction of p-adic L-functions for $\mathrm{U}(3) \times \mathrm{U}(2)$ via the Ichino-Ikeda conjecture.

13:30 – 14:20 **Ken Ono** (University of Virginia)
Sato-Tate type distributions for hypergeometric varieties

Studying the statistical behavior of number theoretic quantities is presently in vogue. The proof of the Sato-Tate Conjecture on point counts of a fixed elliptic curve over finite fields by Richard Taylor (and collaborators) is one of the most significant recent results in the field. Here we discuss point counts in another aspect, for “hypergeometric families” of elliptic curves and $K3$ surfaces. We obtain Sato-Tate distributions for these families, which turn out to be of $SU(2)$ type (a.k.a. semicircular) and of O_3 type (a.k.a. Batman type).

14:40 – 15:30 **Tuan Ngo Dac** (CNRS / University of Caen Normandy)
On Zagier-Hoffman's conjectures in positive characteristic

Multiple zeta values (MZV's) are positive real numbers investigated by Euler in the late eighteenth century. Surprisingly, these numbers are ubiquitous in many mathematical and physical theories such as number theory, mixed Tate motives, and quantum field theory. They were studied by Broadhurst, Brown, Deligne–Goncharov, Goncharov, Hoffman, Ihara-Kaneko–Zagier, Tsumura, Yamamoto, Zagier among others.
By a well-known analogy between the arithmetic of number fields and that of function fields of positive characteristic, Thakur introduced a theory of positive characteristic multiple zeta values associated to the projective line in 2004. In this talk, we study Todd-Thakur's analogues of Zagier-Hoffman's conjectures in positive characteristic. We first establish the algebraic part of these conjectures which is the analogue of Brown's theorem and those of Deligne-Goncharov and Terasoma. We then give some results towards the transcendental part of these conjectures.

15:50 – 16:50 **金子 昌信 / Masanobu Kaneko** * (Kyushu University)
フルヴィッツ型多重ゼータ値の正規化と川島関係式
Regularization of Hurwitz type multiple zeta values and Kawashima's relation

多重ゼータ値は，与えられた自然数の組に対して，ある無限級数で定義される実数であるが，級数が発散するような場合にも考察を広げて有限値を取り出す，正規化と呼ばれる理論がある．これを，フルヴィッツ型の多重ゼータ値に一般化すると，ニュートン級数を使った独特な方法で導き出されていた川島関係式と呼ばれる多重ゼータ値の大きな関係式族を，より自然と思われる方法で証明することが出来る．今回は概説講演ということなので，専門家ではない人に向けて，背景を重点的にお話しすることにしたい．

We present a generalization of the theory of regularization of multiple zeta values to Hurwitz type multiple zeta values. Through this generalization, we obtain a different, seemingly more natural approach to Kawashima's relation, which is conjectured to be one of the largest families of relations among multiple zeta values. This talk is largely expository.

December 16 (Fri)

09:30 – 10:20 **大井 理生 / Masao Oi**
Beilinson の l-adic Eisenstein class について
On l-adic Eisenstein classes

Beilinson の Eisenstein symbol の etale realization（この講演では、l-adic Eisenstein class と呼ぶ）の再構成を universal elliptic curve の場合に行う。Beilinson は Eisenstein symbol の構成を、具体的に元を作ることにより行っているが、本講演では、etale cohomology with support in a closed subscheme の元をまず構成し、その元の Gysin map による像が 0 であることを、直接、計算により示すことにより行われる（この方法では、Eisenstein symbol ではなく、l-adic Eisenstein class のみ構成出来ることに注意する）。同様のアイデアにより、講演者の２０１１年のシンポジウム「代数的整数論とその周辺」で述べた予想（二つの modular curve の直積の関数体の Milnor K_3 の Euler system の存在に関する予想）への新たなアプローチが得られることについても述べる。

Beilinson has constructed the Eisenstein symbol as element of K-groups of powers of the universal elliptic curve. In our lecture, we construct the etale realization of Beilinson's Eisenstein symbol (we refer to it as l-adic Eisenstein classes) in the case of the universal elliptic curve by another method. Beilinson has constructed the Eisenstein symbol by the explicit construction, but we construct l-adic Eisenstein classes by 2 steps. At first, we construct the elements of the etale cohomology with support in a closed subscheme, next we show, by explicit calculations, the image by the Gysin map is zero. Note that we can only construct l-adic Eisenstein classes, not Eisenstein symbols. The same idea gives the new approach to the conjecture we stated in the symposium "Algebraic Number Theory and related topics 2011" concerning the existence of Euler systems in the Milnor K_3 of the function fields of the self-product of modular curves.

10:35 – 11:25　**中村 健太郎 / Kentaro Nakamura** (Saga University)
Coleman-Mazur 固有値曲線上の p 進 L 関数
Construction of a p-adic L-function over the Coleman-Mazur eigencurve

Coleman-Mazur 固有値曲線とは、p 成分が非超カスプ表現となる（過収束）保型形式をパラメトライズするリジッド解析的曲線である。本講演では、発表者の以前の研究で構成した普遍変形に対するゼータ元を用いて固有値曲線上のガロア表現の族に対してゼータ元を構成し、局所イプシロン予想での計算から得られる Perrin-Riou 理論の一般化をこのゼータ元に適用することで、固有値曲線上の p 進 L 関数を構成する。本研究は Chan-Ho Kim 氏 (KIAS) との共同研究である。

The Coleman-Mazur eigencurve is a rigid analytic curve which parametrizes (overconvergent) modular forms which are non-supercuspidal at p. In this talk, we first construct a zeta element for the Galois representation over the eigencurve using the zeta element for the corresponding universal deformation which was constructed by the speaker. Applying a generalization of Perrin-Riou theory to this zeta element, we construct a p-adic L-function over the eigencurve. This is joint work with Chan-Ho Kim (KIAS).

11:40 – 12:40　**坂本 龍太郎 / Ryotaro Sakamoto** * (RIKEN / University of Tsukuba)
Euler systems for GSp(4) and its applications

The theory of Euler systems has played a crucial role for studying the arithmetic of global Galois representations. However, there are very few known examples of Euler systems and it is difficult to construct them. In this talk, we will survey the construction of an Euler system for GSp(4) by Loeffler and Zerbes and their co-authors Pilloni and Skinner (the construction of the Euler system of GSp(4), including an explicit reciprocity law, has been done in three papers). Applications to the BSD conjecture for modular abelian surfaces and the Iwasawa main conjecture for quadratic Hilbert modular forms will also be explained.

14:10 – 15:10　**伊藤 哲史 / Tetsushi Ito** * (Kyoto University)
保型形式の対称積持ち上げの構成 (after James Newton and Jack Thorne)
Construction of symmetric power liftings of modular forms (after James Newton and Jack Thorne)

2021 年に James Newton と Jack Thorne は、すべての n について、複素上半平面上の正則保型形式に伴う n 番目の対称積 L 関数が、$\mathbb{Q}$ 上の $\mathrm{GL}(n+1)$ の保型表現に伴う標準 L 関数と一致することを証明しました。この画期的な成果は「Langlands 関手性予想」を

対称積の場合に解決したものであり、今後は様々な応用が期待されます。Newton-Thorne による証明には、剰余表現が可約な場合の「保型性の持ち上げ定理」や、保型形式の p 進解析族を用いた巧妙な議論 (Ping Pong)、Bloch-加藤の随伴 Selmer 群の消滅定理などの最新の結果が用いられます。講演では Newton-Thorne の結果とその周辺についての解説を行いたいと思います。

In 2021, James Newton and Jack Thorne proved that, for every n, the n-th symmetric power L-function associated with a holomorphic modular form on a complex upper half plane coincides with the standard L-function associated with an automorphic representation of $\mathrm{GL}(n+1)$ over $\mathbb{Q}$. This groundbreaking result solves “the Langlands functoriality conjecture” for symmetric powers, and many applications are expected now. In their proof, they used many recent results such as “the automorphy lifting theorems” for residually reducible representations, an ingenious argument (Ping Pong) using p-adic analytic families of modular forms, and a vanishing theorem for Bloch-Kato adjoint Selmer groups. In this talk, we survey the results of Newton-Thorne and related topics.

15:25 – 16:15 **大坪 紀之 / Noriyuki Otsubo** (Chiba University)
Gross-Deligne CM 周期予想の l 進類似と p 進類似
l-adic and p-adic analogues of the Gross-Deligne CM period conjecture

アーベル体に CM (虚数乗法) を持つモチーフの複素周期がそのモチーフの Hodge 数とガンマ関数の特殊値で表されるだろう，というのが Gross-Deligne の CM 周期予想 (1978) である．本講演では，この予想の l 進類似および p 進類似について述べる (l, p はともに素数)．前者は CM モチーフの l 進エタール・コホモロジーへの Frobenius 作用に関するものであり，後者は p 進 Hodge 理論の比較定理によって定義される Fontaine 環に値を取る p 進周期に関するものである．本研究は Bruno Kahn 氏 (IMJ-PRG) との共同研究である．

The Gross-Deligne CM period conjecture (1978) asserts that the complex period of a motive having complex multiplication in an abelian field can be expressed in terms of the Hodge numbers of the motive and special values of the gamma function. In this talk, we explain its l-adic and p-adic analogues, where l and p are prime numbers. The former concerns the Frobenius action on the l-adic étale cohomology of the CM motive, and the latter is about the p-adic periods taking values in a Fontaine ring defined by the comparison theorem in p-adic Hodge theory. This is a joint work with Bruno Kahn (IMJ-PRG).

* Invited speakers

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