Resummed gamma-ray spectrum from electroweakly interacting spin-1 dark matter

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Based on MF, M. Vollmann [arXiv:2502.XXXXX] (work in progress)

See also
T. Abe, MF, J. Hisano, K. Matsushita, JHEP 07 (2020) 136 [arXiv:2004.00884]
T. Abe, MF, J. Hisano, K. Matsushita, JHEP 10 (2021) 163 [arXiv:2107.10029]

Tokyo University of Science @Noda February 27, 2025

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Dark Matter in our universe

Dark Matter (DM) = Invisible gravitational source in our universe

- Indirect evidence: rotational curves of galaxy, bullet cluster, gravitational lensing
  [Rubin et al. (1980)] [Markevich et al. (2002)] [Clowe et al. (2006)] [Oguri et al. (2018)]

→ **astrophysical observation suggests missing mass coherently**

Features of DM

- Electrically neutral
- Stable, or long-lived particle
- Non-relativistic property @structure formation
- Occupy ~1/4 of total energy density in current universe (Cosmic Microwave Background) [N. Aghanim, et al. [Planck Collaboration] (2020)]
- **No candidate in the Standard Model**

DM identification = Problem in Particle phys./Astrophys./Cosmology

Particle Dark Matter

Hypothesis: **Dark matter (DM) $\stackrel{?}{=}$ New particle**

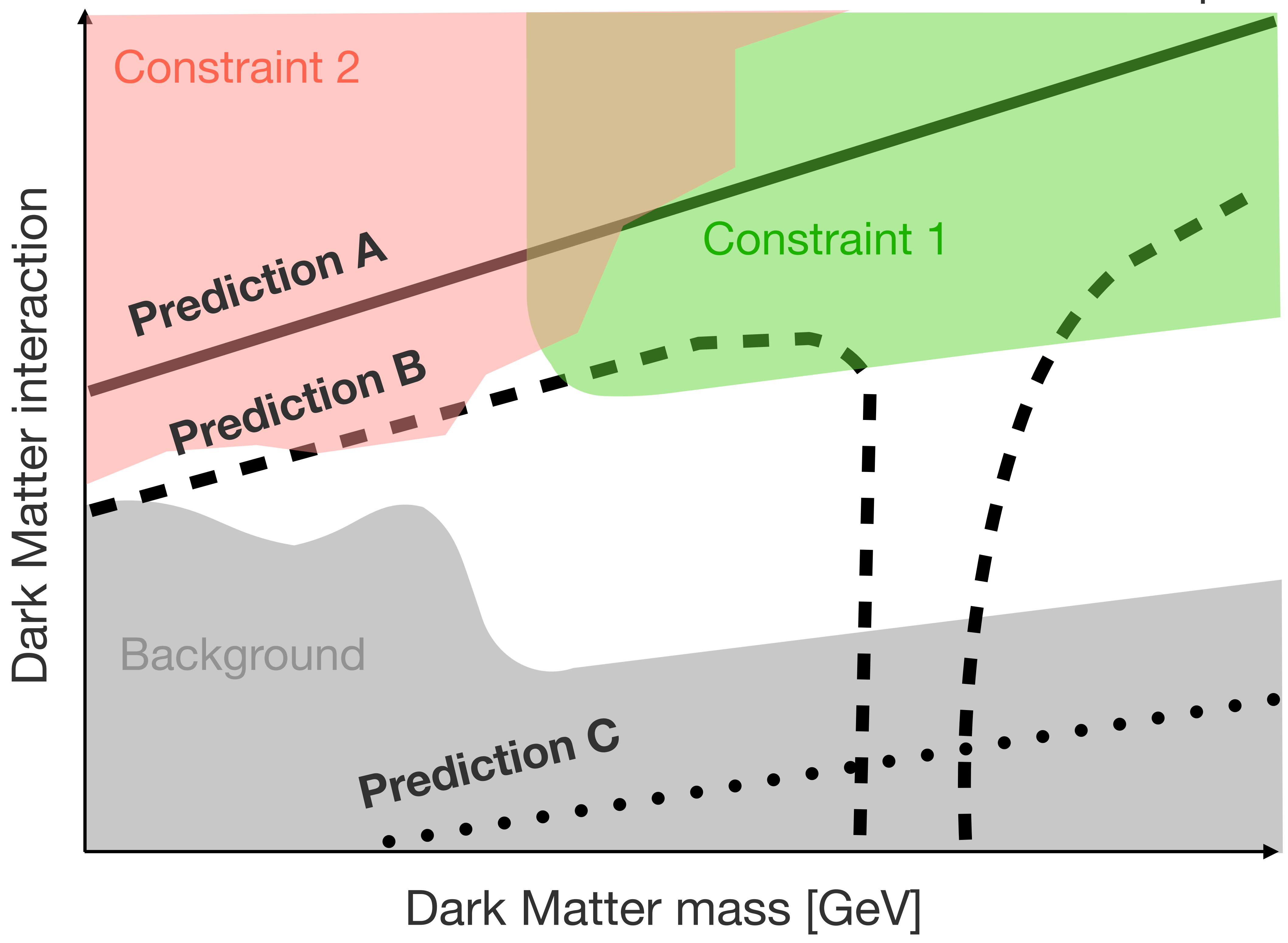
Our goal: **Identification of DM interaction theory**

- How to identify "interaction theory for DM"?
  → Identify DM Quantum Number
  - DM spin
  - DM Mass
  - Interaction btw DM and SM particles, etc

Efforts toward DM identification

- Constructing possible DM models
- Derive precise predictions & compare w/ exp. results
- Develop new methods to probe DM

※ sketch of concept



Precision in theoretical prediction is crucial to reconstruct DM properties form exp.

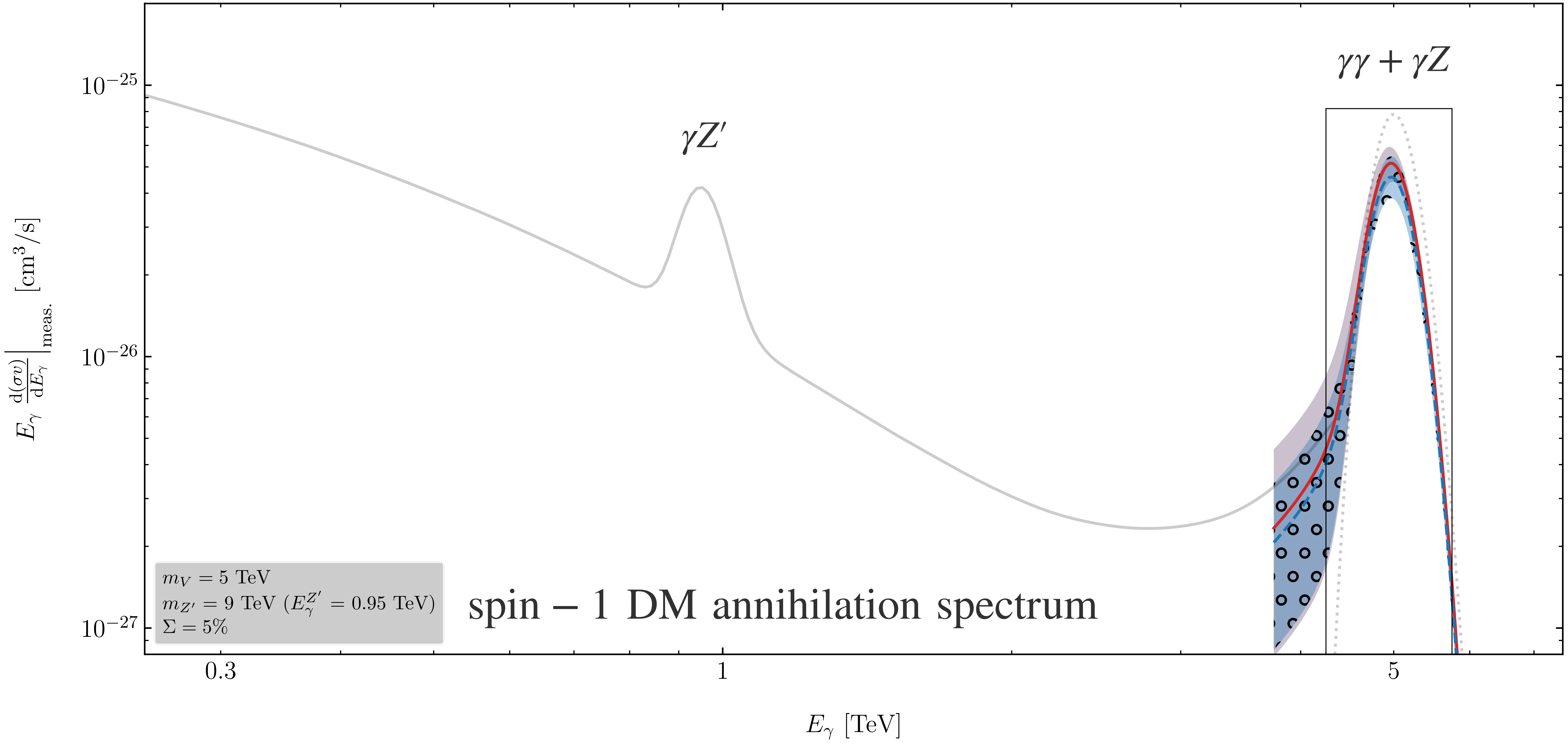
Today's Talk

spin	triplet
0	✓
1/2	✓
1	

Question : **How to distinguish DM spin among DM candidates w/ the same int.?** (e.g. $SU(2)_L$ triplet DM)

EFT approach : **Achieve high precision systematically (Key: Resummation of large log)**

: **Framework is largely Universal (= spin independent) w/ separable features**



[MF, T. Abe, J. Hisano, K. Matsushita (2020)]

Electroweakly interacting spin-1 Dark Matter

Extended EW Symme: $\mathrm{SU}(2)_0 \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_\mathrm{Y} \longrightarrow U(1)_{em}$

$\langle \Phi_i \rangle \neq 0 \ (i = 1{,}2)$
$\langle H \rangle \neq 0$

Exchange Symme. ($\mathrm{SU}(2)_0 \leftrightarrow \mathrm{SU}(2)_2$)

Vacuum expectation values:

$$\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Phi & 0 \\ 0 & v_\Phi \end{pmatrix} \quad : v_\Phi \sim \mathcal{O}(1)\ \mathrm{TeV}$$

$\gg$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad : v \sim \mathcal{O}(100)\ \mathrm{GeV}$$

field	spin	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_0$	$\mathrm{SU}(2)_1$	$\mathrm{SU}(2)_2$	$\mathrm{U}(1)_Y$
q_L	$\frac{1}{2}$	3	1	2	1	$\frac{1}{6}$
u_R	$\frac{1}{2}$	3	1	1	1	$\frac{2}{3}$
d_R	$\frac{1}{2}$	3	1	1	1	$-\frac{1}{3}$
ℓ_L	$\frac{1}{2}$	1	1	2	1	$-\frac{1}{2}$
e_R	$\frac{1}{2}$	1	1	1	1	-1
Φ_1	0	1	2	2	1	0
Φ_2	0	1	1	2	2	0
H	0	1	1	2	1	$\frac{1}{2}$

Electroweakly interacting spin-1 Dark Matter

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$\langle \Phi_i \rangle \neq 0 \ (i = 1,2)$

$\langle H \rangle \neq 0$

Exchange Symme.

Z_2 parity from exchange symmetry: $W^a_{0\mu} \leftrightarrow W^a_{2\mu}$

Mass	Vector	Scalar	Z_2 parity		Mass
↑	Z' $W'^\pm$	h'	**even**	$\sim v_\Phi$	$\mathcal{O}(1)$ TeV
	V^0 $V^\pm$	h_D	**odd**		
	Z $W^\pm$	h	**even**	$\sim v$	$\mathcal{O}(100)$ GeV
	γ		**even**		massless

$$V^0 = \frac{W^3_{0\mu} - W^3_{2\mu}}{\sqrt{2}} \quad \text{(neutral)}$$

$$V^\pm = \frac{W^\pm_{0\mu} - W^\pm_{2\mu}}{\sqrt{2}} \quad \text{(charged)}$$

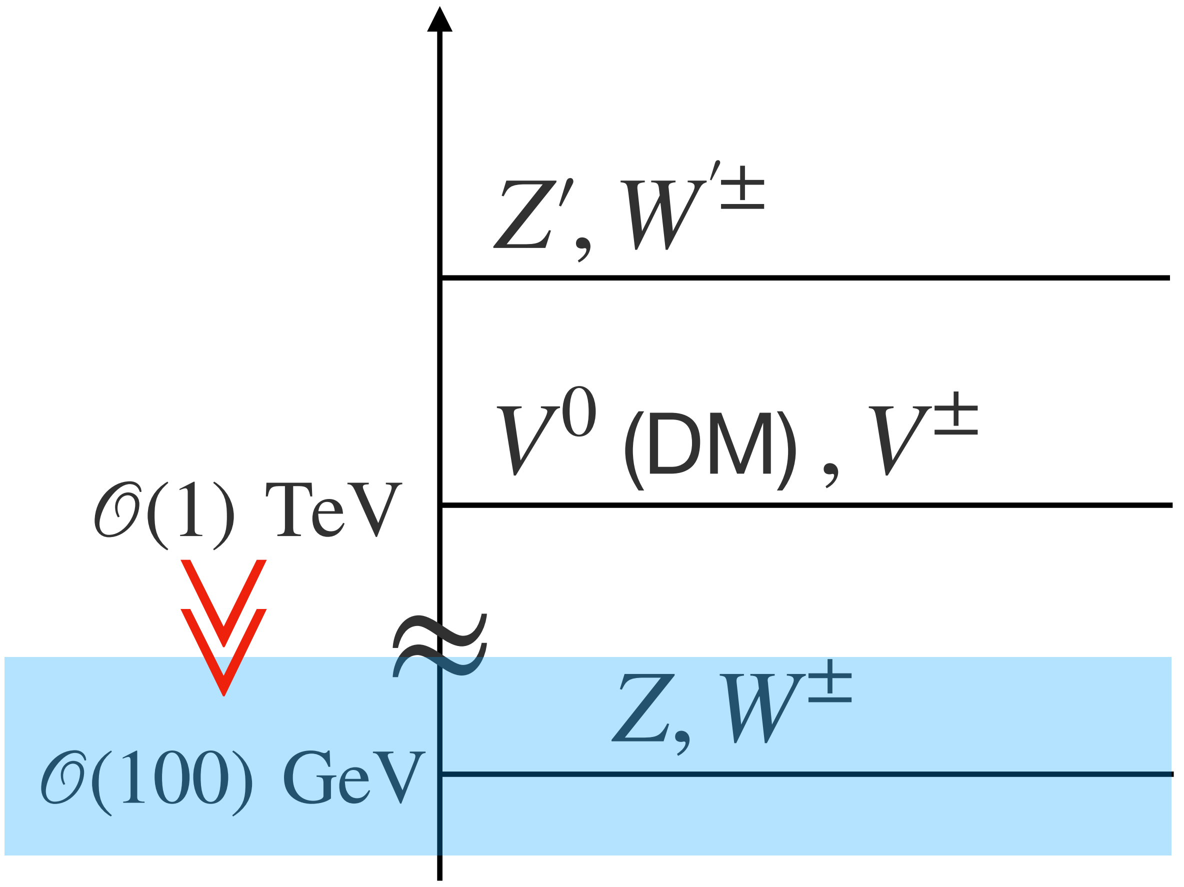
$$\left[W^\pm_{n\mu} = \frac{W^1_{n\mu} \mp i W^2_{n\mu}}{\sqrt{2}} \quad (n = 0,1,2) \right]$$

$V^A (A = 1,2,3) \supset V^0, V^\pm$: **SU(2)$_\mathrm{L}$ triplet-like spin-1 particles, incl. stable DM candidate**

γ-ray spectrum from DM Annihilation

Non-perturbative corrections

- Sommerfeld effects → should solve Schrödinger eq. ✔
- Sudakov log → should be resummed (today's topic)



Q. How to sum corrections systematically? → A. **Soft Collinear Effective Theory**

Current status	SU(2)$_L$ odd	SU(2)$_L$ even
spin-0	✔ Framework is completed M. Bauer, et al. (2015) [triplet]	
spin-1/2	✔ G. Ovanesyan, et al. (2018) [triplet] M. Beneke, et al. (2019) [triplet] M. Baumgart, et al. (2024) [quintuplet]	✔ M. Beneke, et al. (2020) [doublet]
spin-1	**This work! [triplet]**	

Aim: Precise gamma-ray spectrum from spin-1 DM annihilation, Key: Universality in EFT construction

Contents

- **Introduction + Model setup**
- **Soft Collinear Effective Theory (SCET)** [short review]
- **Construction of Soft Effective Theory** [practice to construct EFT]
- **SCET for spin-1 DM**
- **Results**
- **Summary & Discussions**

Soft Collinear Effective Theory

[Short review]

Original: [C. W. Bauer, S. Fleming, M. Luke (2000)] in the context of $B \to X_s \gamma$
Review: [T. Becher, A. Broggio, A. Ferroglia (2015) [arXiv: 1410.1892]]
Review: [T. Becher's lecture note @Les Houches 2017 [arXiv: 1803.04310]]

Large log in QED [review]

$Q^2 \equiv -q^2$

m : electron mass

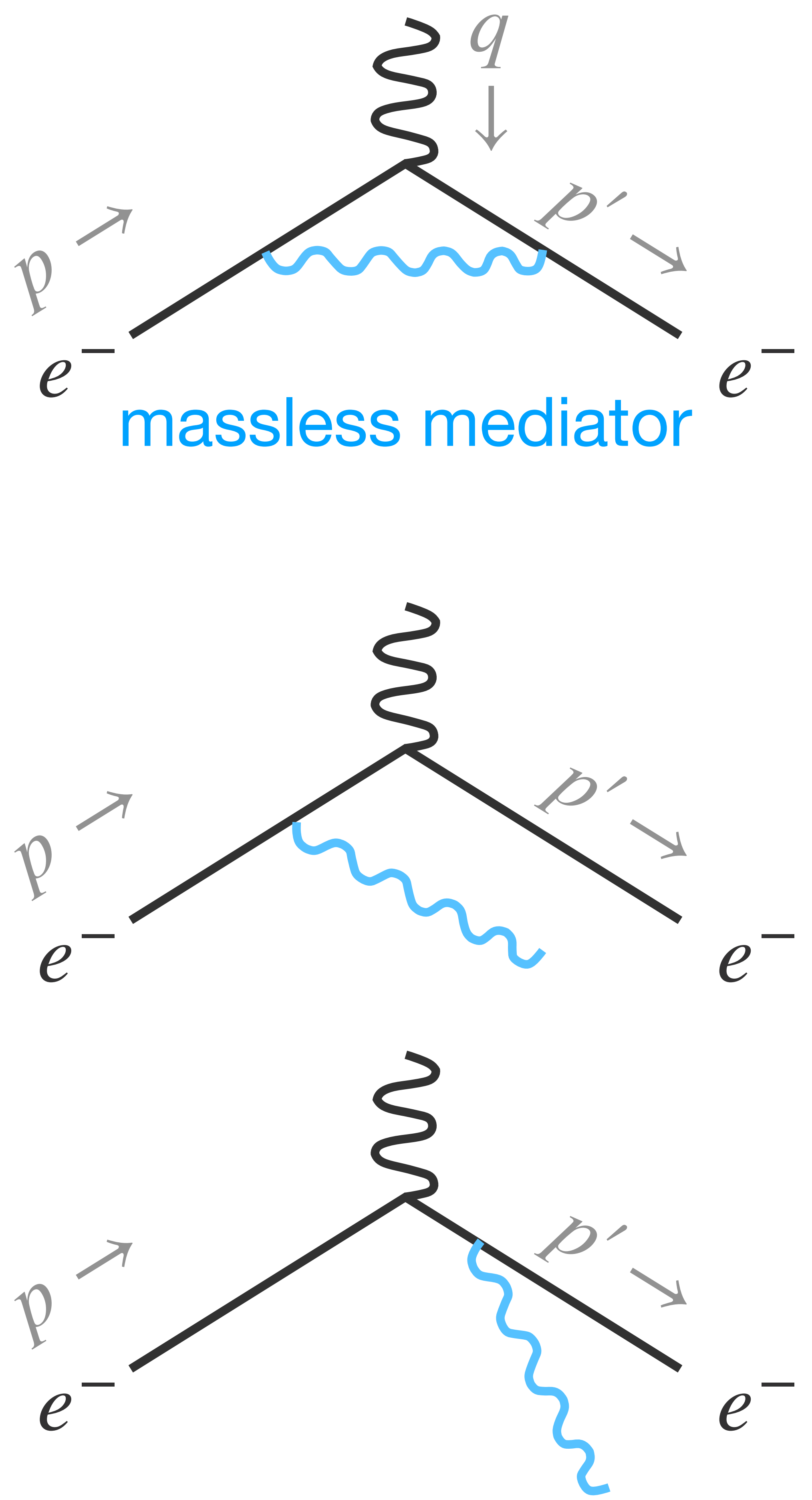
μ : artificial cutoff

Sudakov double log [Peskin-Schröder's textbook, Ch. 6, Eq. (6.66)]

$$\frac{d\sigma}{d\Omega}(p \to p') = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi}\log\left(\frac{Q^2}{m_e^2}\right)\log\left(\frac{Q^2}{\mu^2}\right) + \mathcal{O}\left(\alpha^2\right)\right]$$

IR divergence @$\mu \to 0$

$$\frac{d\sigma}{d\Omega}(p \to p' + \gamma) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[+\frac{\alpha}{\pi}\log\left(\frac{Q^2}{m_e^2}\right)\log\left(\frac{Q^2}{\mu^2}\right) + \mathcal{O}\left(\alpha^2\right)\right]$$



- What is observed cross section in the realistic experiments?
  Answer: $e^-(p_1) \to e^-(p_2)$ + (soft photon) [∵ finite energy resolution]
- μ-dependence cancels in the total amplitude **including soft photons**
- **Double log corrections** ($\log\#\ \log\#$) appear w/ typical energy scale in loop amplitude

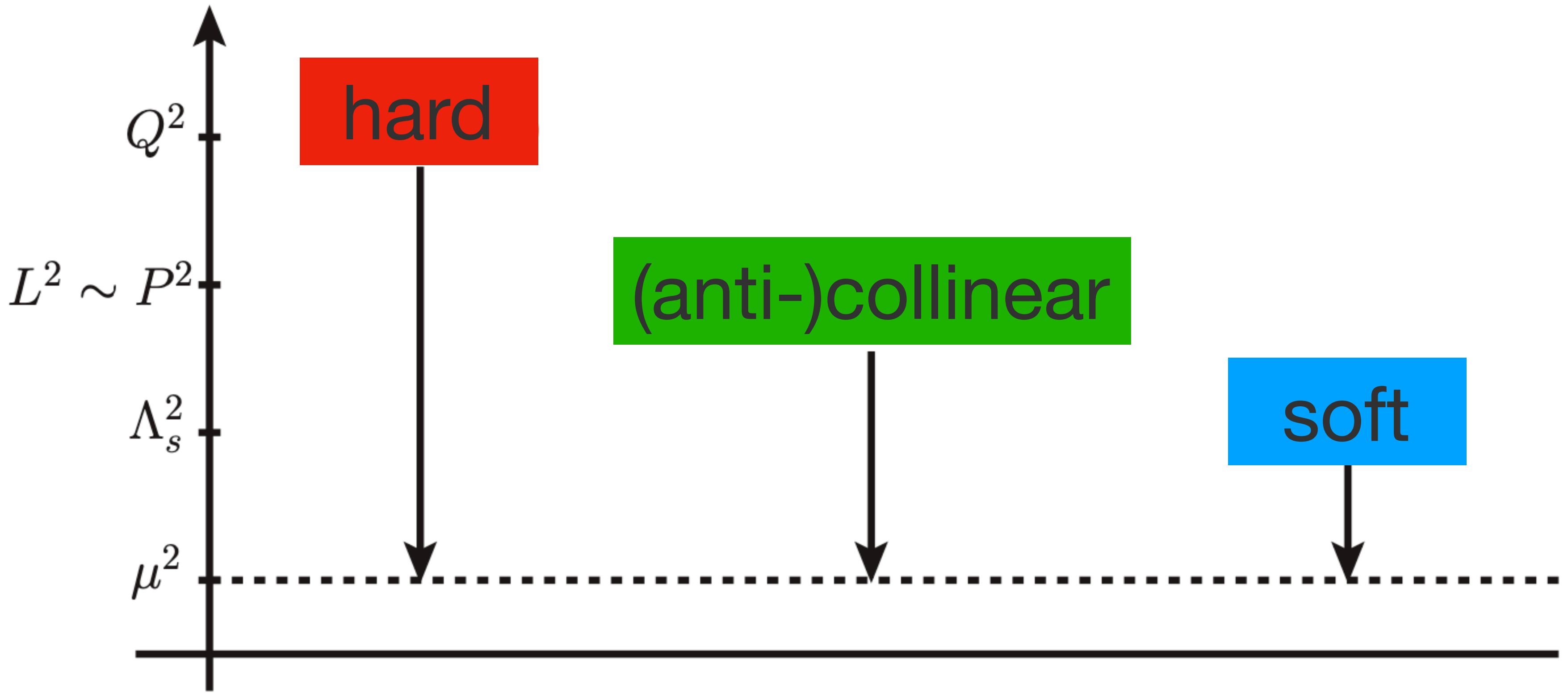
Multi-energy system

Kinematics of Heavy DM annihilation

- Hard initial state : non-relativistic DM particles $q^\mu \sim (m_{\rm DM}, \mathbf{0})$
- Anti-collinear/Collinear particles : EW bosons $p^\mu \sim m_{\rm DM} n^\mu$, $\ell^\mu \sim m_{\rm DM} \bar{n}^\mu$
- Soft radiation : EW radiation Λ_s^2

Perturbative parameter:

$$\lambda = \frac{m_W}{2m_{\rm DM}} \quad \text{or} \quad \frac{m_{\rm DM} - E_\gamma}{2m_{\rm DM}} \simeq 10^{-1} \text{ (for TeV DM)}$$



e.g. see [M. Beneke, A. Broggio, C. Hasner, K. Urban, M. Vollmann (2019)]

Multi-energy system

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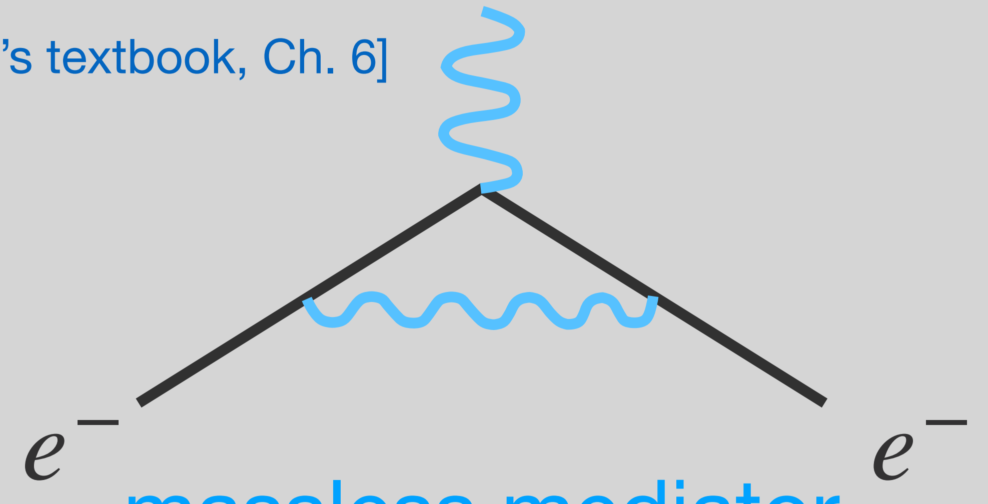
Amplitude: $\supset \# \log\left(\frac{Q^2, P^2, L^2, \Lambda_s^2}{\mu^2}\right) + \# \left[\log\left(\frac{Q^2, P^2, L^2, \Lambda_s^2}{\mu^2}\right)\right]^2 + \cdots$

affecting RGEs

cf. Sudakov form factor in QED

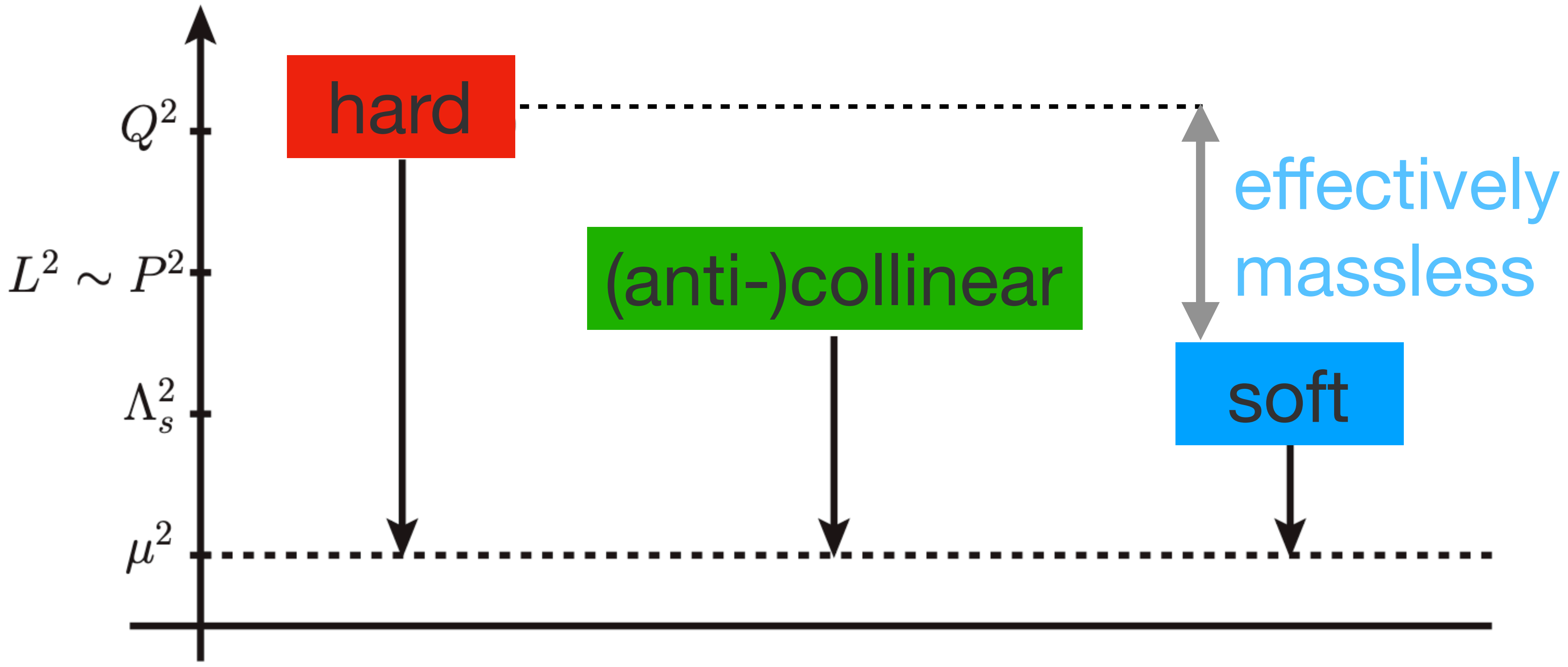
[Peskin-Schröder's textbook, Ch. 6]



Problem: The amplitude involves various scales $(Q^2, P^2, L^2, \Lambda_s^2)$

e.g. If we take $\mu = \Lambda_s$, $\quad \log(Q^2/\mu^2) \gg 1$

(Large log) appears → Perturbative calc. breaks down



Solution: Factorization

$$F(x,y,z) = -\frac{3y^4z^4}{x^2} + \frac{6y^4z^2}{x^2} - \frac{21y^2z^4}{x^2} + 5x^2y^2z^2 + \frac{42y^2z^2}{x^2}$$
$$-10x^2y^2 + 35x^2z^2 - 70x^2 + y^4z^4 - 2y^4z^2$$
$$+7y^2z^4 - 29y^2z^2 + 30y^2 - 105z^2 + 210$$

↓ Factorization

$$= (x^2-3) \times (y^2+7) \times (z^2-2) \times (y^2z^2/x^2+5)$$

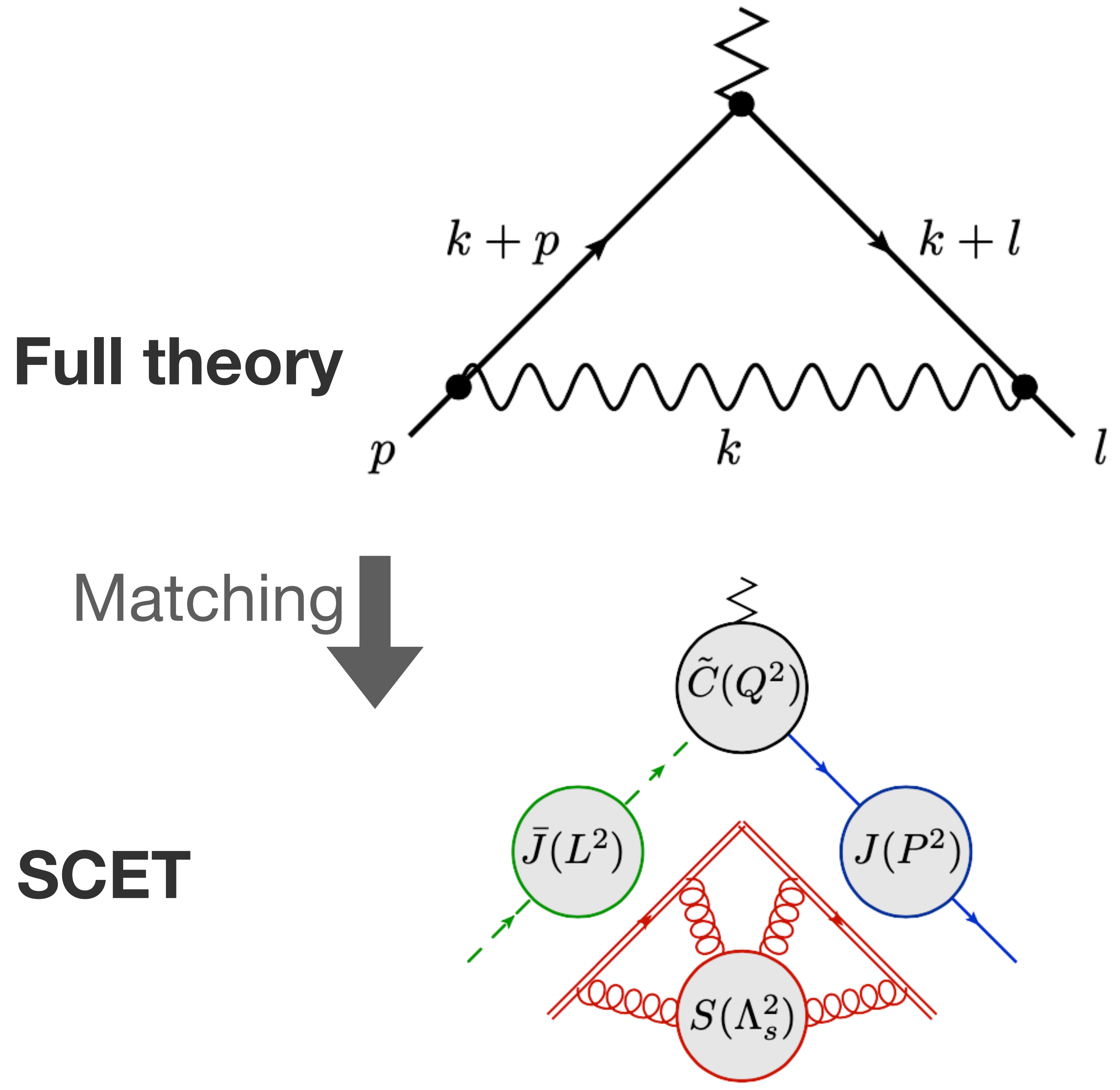
e.g. Factorization of (x, y, z)- polynomials

Solution: Factorization

$$i\mathcal{M} = \left(\text{complicated func. of } \{Q^2, P^2, L^2, \Lambda_s^2\}\right)$$

Method of region + dim. regularizaion
→ Separate scales of loop momentum

$$= \underbrace{C(Q^2,\mu)}_{\text{hard}} \times \underbrace{J(P^2,\mu)}_{\text{anti-collinear}} \times \underbrace{\bar{J}(L^2,\mu)}_{\text{collinear}} \times \underbrace{S(\Lambda_s^2,\mu)}_{\text{soft}}$$



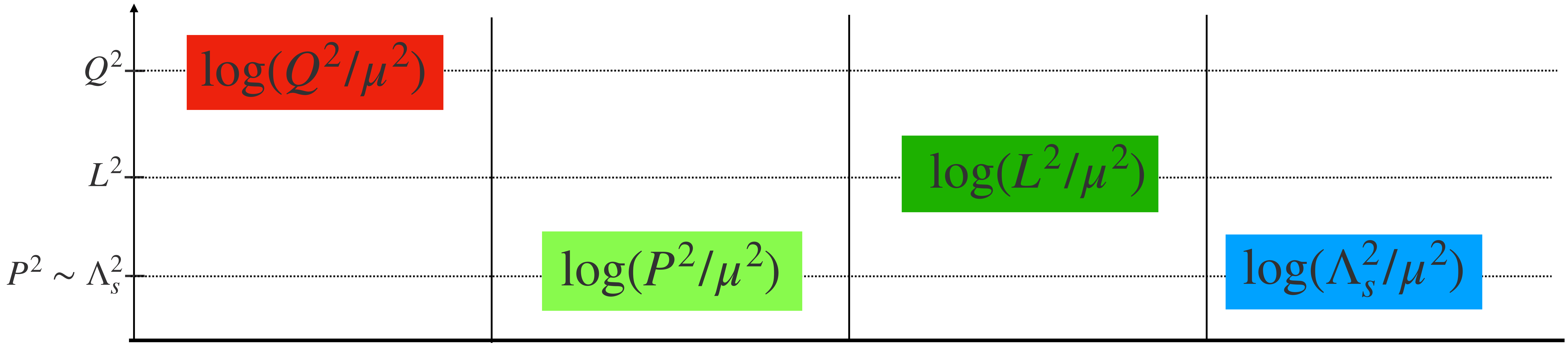
Our strategy

We “construct” the effective theory (EFT) to reproduce the same physical result as the full theory

- In the EFT, **the physics @each energy scale (hard, collinear, anti-collinear, soft) is separately described**

(Multi-energy scale system) $\Leftarrow$ matching $\Rightarrow$ (single-energy system) $\times 4$

Benefits of Factorization [1/3]

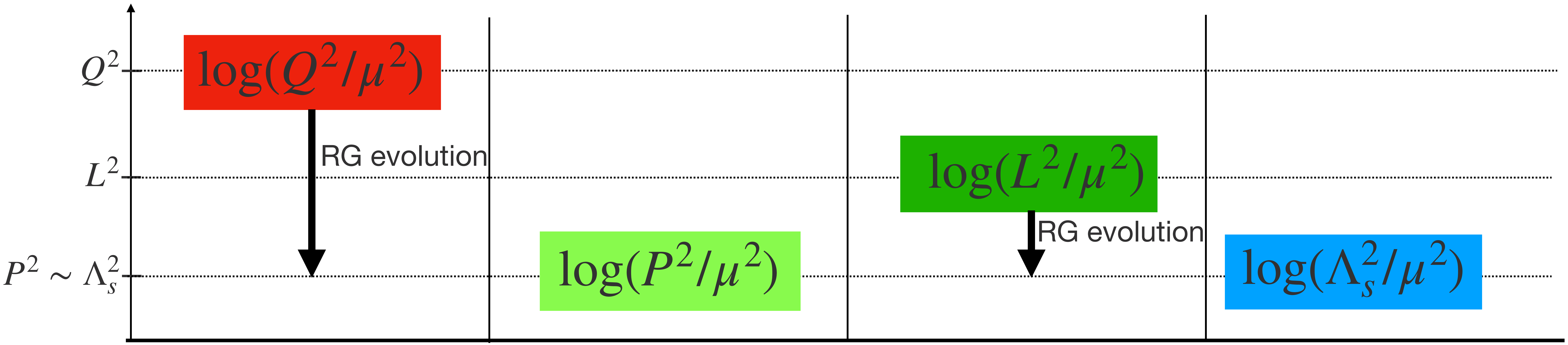


$$\langle \gamma(p) X(\ell) |\ \Upsilon^A_\alpha\ \Upsilon^B_\beta\ \mathscr{A}^C_\mu\ \mathscr{A}^D_\nu\ T^{ABCD} U^{\alpha\beta\mu\nu} | V^0(p_1) V^0(p_2) \rangle$$

$$= \langle 0 | \Upsilon\Upsilon | V^0 V^0 \rangle \times \langle \gamma | \mathscr{A} | 0 \rangle \times \langle X_c | \mathscr{A} | 0 \rangle \times \langle X_s | YY\mathscr{Y}\mathscr{Y} | 0 \rangle$$

- Amplitude / Cross section is **automatically factorized**, and each part is calculated @natural scale

Benefits of Factorization [2/3]



$$\mu \frac{d}{d\mu} F_i(p_i^2, \mu) = \Gamma_i \times F_i(p_i^2, \mu)$$

$$\Gamma_i = \gamma_{i,1} \log\left(\frac{p_i^2}{\mu^2}\right) + \gamma_{i,2}$$

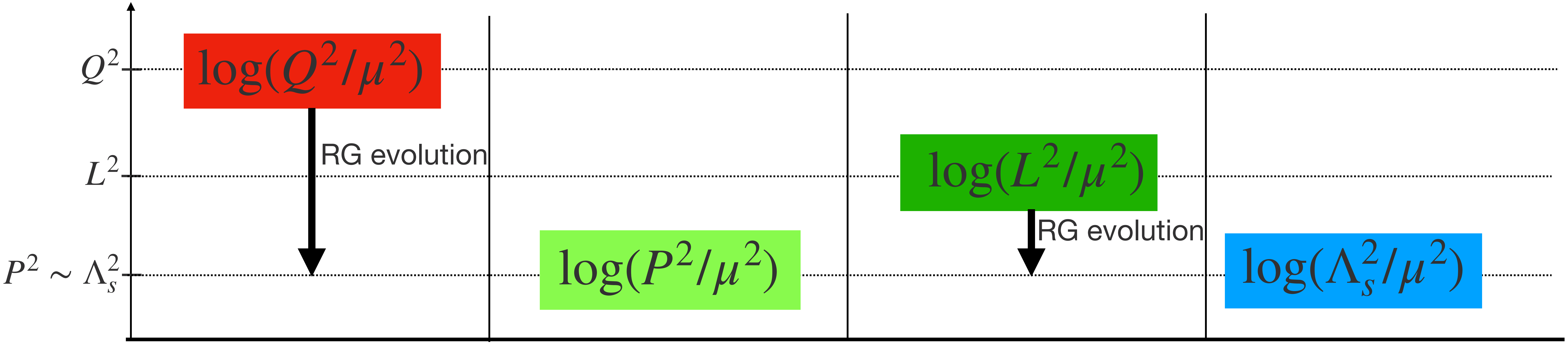
Anomalous dim.

$i = \{\text{hard, (anti)collinear, soft}\}$

- Energy evolution follows Renormalization Group (RG) equation w/ only single scale

  → Chose μ to suppress large log, then **perturbative calculation is again viable!**
  (RG improved perturbation theory)

Benefits of Factorization [3/3]



$$\mu \frac{d}{d\mu} F_i(p_i^2, \mu) = \Gamma_i \times F_i(p_i^2, \mu)$$

Solution →

$$F_i(p_i^2, \mu) = \int_{\mu_i}^{\mu_{\rm ref}} d\mu \frac{1}{\mu} \left[\gamma_{i,1} \log\left(\frac{p_i^2}{\mu^2}\right) + \gamma_{i,2} \right]$$

$$\Gamma_i = \gamma_{i,1} \log\left(\frac{p_i^2}{\mu^2}\right) + \gamma_{i,2}$$

Anomalous dim.

$i = \{\text{hard}, \text{(anti)collinear}, \text{soft}\}$

- Via RG evolution, we can **systematically "resum"** log corrections + finite corrections

  → Reduction of μ-scale dependence & uncertainty in final prediction

Construction of Soft Effective Theory (SET)

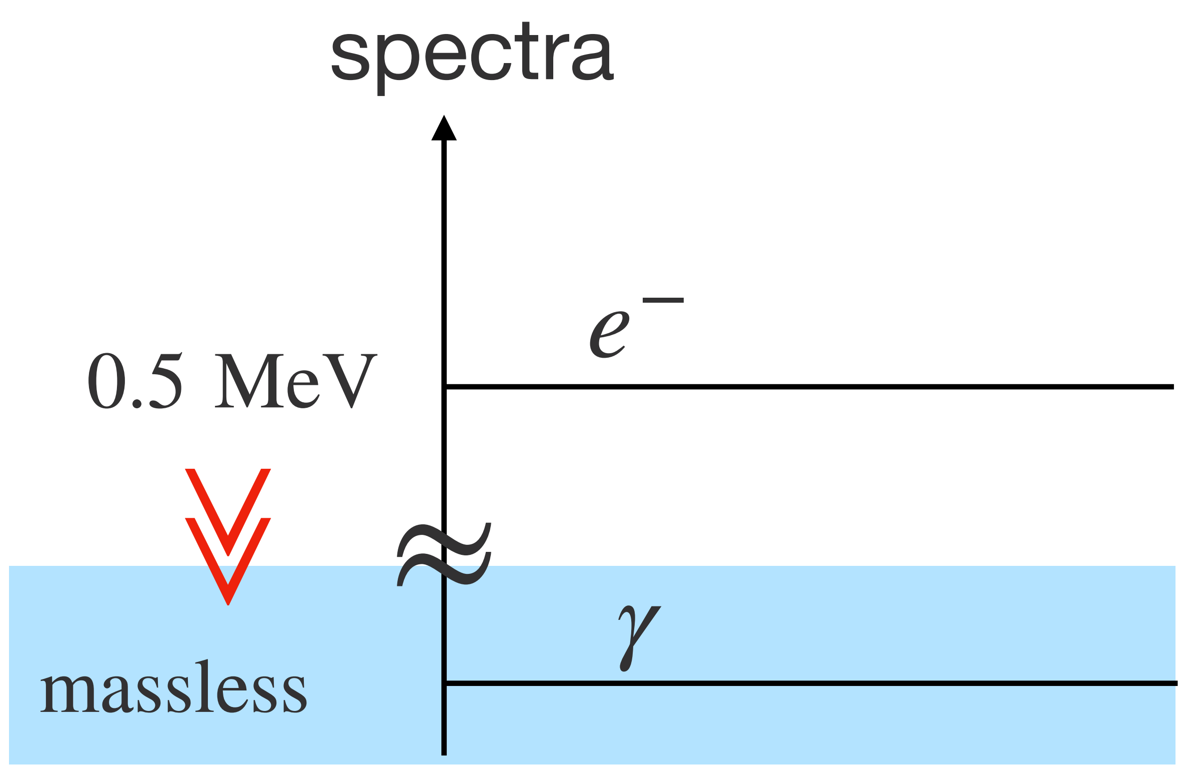
[let's practice w/o collinear particles]

[T. Becher's lecture note @Les Houches 2017 [arXiv: 1803.04310]]

e.g. Semi-relativistic electron scattering

Kinematics

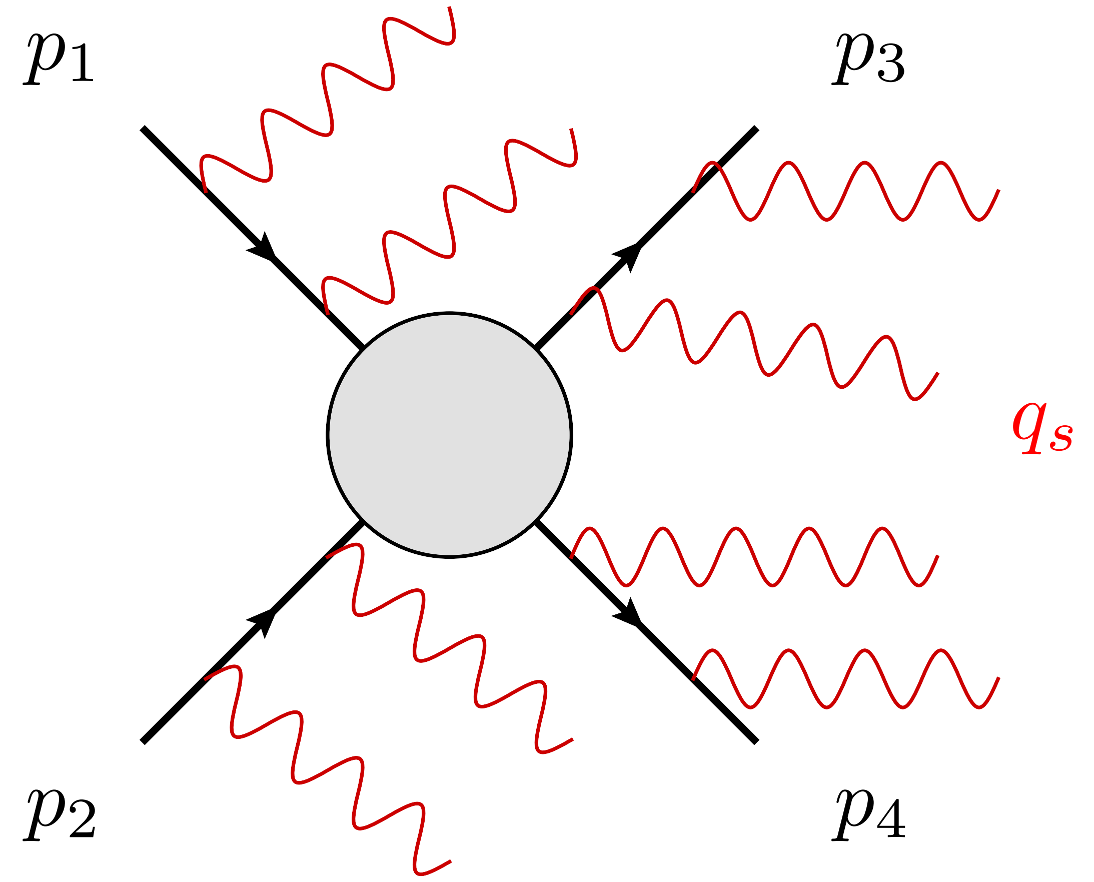
$$e^-(p_1) + e^-(p_2) \to e^-(p_3) + e^-(p_4) + X_s(q_s)$$



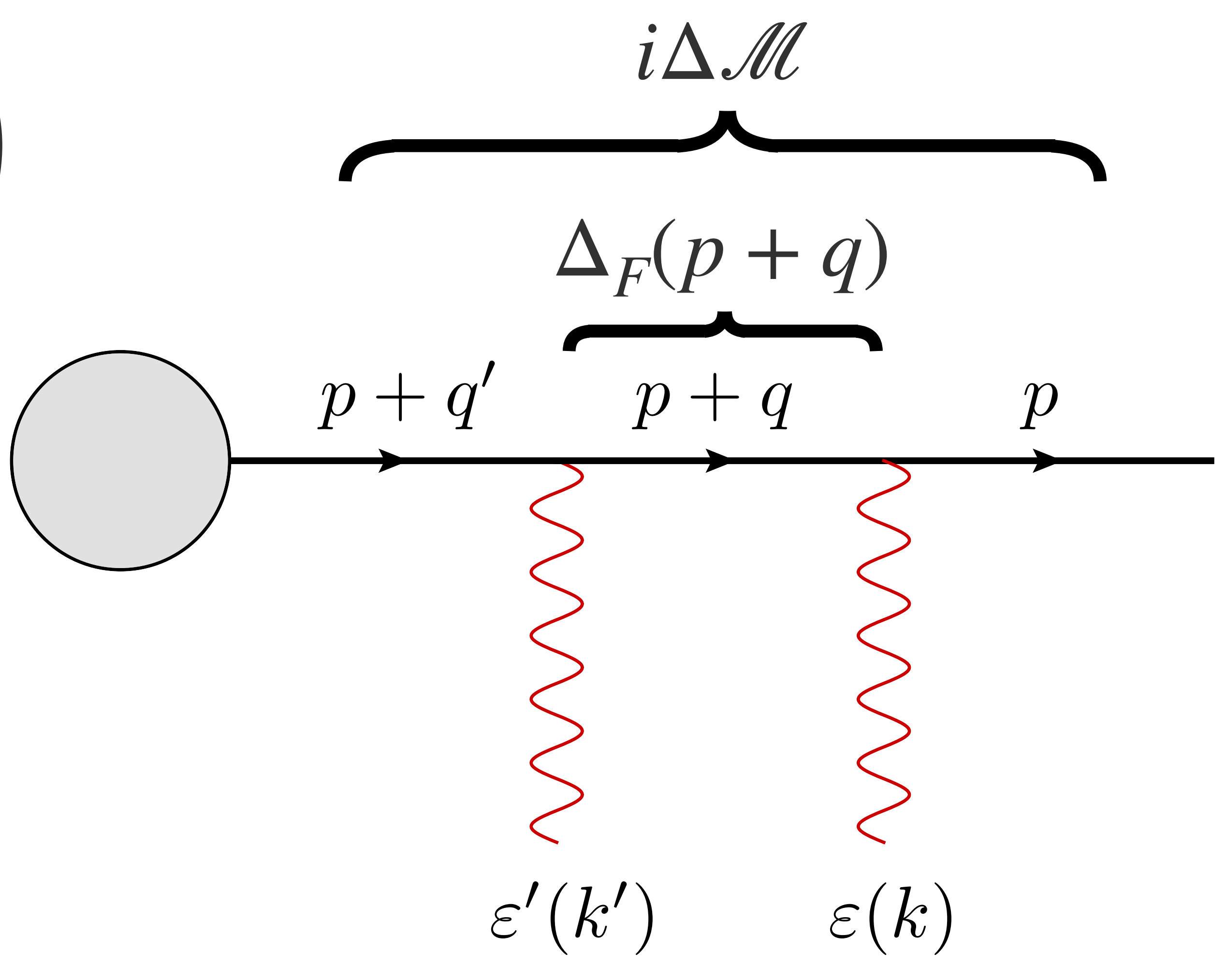
- Hard initial/final state : electrons: $p_1, p_2 \to p_3, p_4$
- ~~Collinear/Anti-collinear particles~~ : No collinear particles (as long as electron is semi-relativistic: $E_e \sim m_e$)
- Soft photon radiation : arbitrary # of soft photons w/ $E_s \ll m_e$, $X_s(q_s) = \gamma(q_1) + \gamma(q_2) + \cdots + \gamma(q_n)$

How to construct Soft Effective Theory (SET)

- Write down amplitude w/ soft photons in full theory (=QED)
- Find Feynman rules for soft photon processes
- Construct effective Lagrangian to reproduce derived amplitude
- Factorize soft correction from the electron field
  → **Factorization of Hard physics & Soft physics**



Amplitude in full theory (=QED)



Soft photons

Amplitude w/ soft photons

$$\begin{aligned}
\Delta_F(p+q) &= \frac{i(\not{p}+\not{q}+m_e)}{(p+q)^2-m_e^2+i\epsilon} \\
&\simeq \frac{i(\not{p}+m_e)}{2p\cdot q+i\epsilon} \qquad (\because p^2=m_e^2,\ q^2\simeq 0,\ |p|\gg|q|) \\
&= \frac{m_e\not{v}+m_e}{2m_e}\frac{i}{v\cdot q+i\epsilon} \\
&= \frac{\not{v}+1}{2}\frac{i}{v\cdot q+i\epsilon} \\
&= P_v\frac{i}{v\cdot q+i\epsilon},
\end{aligned}$$

$P_v \equiv \frac{1+\not{v}}{2}$ (Projection operator)

$\not{v}\,P_v = P_v\,,\qquad P_v^2 = P_v\,,\qquad P_v\not{\varepsilon}\,P_v = P_v\,\varepsilon\cdot v\,.$

Eikonal approximation

- Neglecting $\mathscr{O}(\lambda = E_s/m_e)$
- On-shell condition for soft photons

➡ Simplified Feynman rules

$$i\Delta\mathcal{M} = \bar{u}(p)\underbrace{P_v}_{\text{projection}}\underbrace{(-ie\epsilon\cdot v)}_{\text{vertex}}\underbrace{\frac{1}{v\cdot q+i\epsilon}}_{\text{propagator}}(-ie\epsilon'\cdot v)\frac{1}{v\cdot q'+i\epsilon}.$$

[T. Becher's lecture note @Les Houches 2017 [arXiv: 1803.04310]]

Effective Lagrangian for SET

Introduction auxiliary field

※ $P_v h_v = h_v$

$$\mathscr{L}_{\text{eff}} = \sum_{i=0}^{4} \bar{h}_{v_i}(x) i v_i \cdot D h_{v_i}(x) - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \sum_j C_j(v_1, v_2, v_3, v_4, m_e) \bar{h}^{\gamma}_{v_3}(x)\Gamma_j h^{\alpha}_{v_1}(x) \bar{h}^{\delta}_{v_4}(x) \Gamma_j h^{\beta}_{v_2}(x)$$

- Auxiliary fermions for each velocity direction (fermions living in limited momentum region: $p_i^\mu \sim m_e v_i^\mu$)
- Single pole for fermion propagator
  ∵ Fermion in SET is very close to on-shell condition $p^\mu \sim m_e v^\mu + q^\mu$
  Anti-particle cannot appear from external lines (→ absorbed in higher dim. operators)
- All we need to do: Determine the Wilson coefficient by matching btw full-theory amplitude & SET operator
- No loop corrections to this matching (∵ No dimension-full quantity in SET → loop correction = scaleless integral = 0)

$$i\Delta\mathcal{M} = \bar{u}(p) \underbrace{P_v}_{\text{projection}} \underbrace{(-ie\epsilon\cdot v)}_{\text{vertex}} \underbrace{\frac{1}{v\cdot q + i\epsilon}}_{\text{propagator}} (-ie\epsilon'\cdot v) \frac{1}{v\cdot q' + i\epsilon}.$$

[T. Becher's lecture note @Les Houches 2017 [arXiv: 1803.04310]]

Factorization of soft photon

Wilson lines

$$S_i(x) = \exp\left[-ie\int_{-\infty}^{0} dsv_i \cdot A(x+sv_i)\right].$$

- Charged particle traveling along $y^\mu = x^\mu + sv_i^\mu$
  → External electrons (no recoils due to soft photons, $E_e \gg E_s$)
- Matrix element for single photon reproduces Eikonal amplitude
- Wilson line is the solution of SET's EoM: $v_i \cdot DS_i(x) = 0$.
- Wilson line is useful representation to make gauge singlet combination

$$\begin{aligned}\langle \gamma(k)\,|\,S_i(0)\,|\,0\rangle &= -ie\int_{-\infty}^{0} dsv_i \cdot \langle \gamma(k)\,|\,A(x+sv_i)\,|\,0\rangle \\ &= -ie\,(v_i\cdot\epsilon(k))\int_{-\infty}^{0} dse^{-is(v_i\cdot k - i\epsilon)} \\ &= -ie\,(v_i\cdot\epsilon(k))\left[\frac{e^{-is(v_i\cdot k-i\epsilon)}}{-i\,(v_i\cdot k - i\epsilon)}\right]_{-\infty}^{0} \\ &= e\frac{v_i\cdot\epsilon(k)}{v_i\cdot k - i\epsilon}.\end{aligned}$$

Decoupling transformation: $h_{v_i}(x) = S_i(x)h_{v_i}^{(0)}(x)$,

$$\mathcal{M} = \sum_i C_i \bar{u}(v_3)\Gamma_i u(v_1)\bar{u}(v_4)\Gamma_i u(v_2)\left\langle X_s(k)\,\middle|\,\bar{S}_3^\dagger S_1 \bar{S}_4^\dagger S_2\,\middle|\,0\right\rangle = \underbrace{\mathcal{M}_{ee}}_{\text{Hard phys.}} \times \underbrace{\left\langle X_s(k)\,\middle|\,\bar{S}_3^\dagger S_1 \bar{S}_4^\dagger S_2\,\middle|\,0\right\rangle}_{\text{Soft phys.}}.$$

[T. Becher's lecture note @Les Houches 2017 [arXiv: 1803.04310]]

Relation btw Hard & Soft phys.

Scale dependence

$$\sigma = \mathcal{H}(m_e, \{\underline{v}\})\, \mathcal{S}(E_s, \{\underline{v}\}) \,,$$

Renormalization

Hard: $\mathcal{H}(m_e, \{\underline{v}\}) = \frac{1}{2E_1 2E_2 |\vec{v}_1 - \vec{v}_2|} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} |\mathcal{M}_{ee}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_3) \,,$

Soft: $\mathcal{S}(E_s, \{\underline{v}\}) = \sum\!\!\!\!\!\!\int_{X_s} \left| \langle X_s | \bar{S}_3^\dagger\, S_1\, \bar{S}_4^\dagger\, S_2 | 0 \rangle \right|^2 \theta(E_s - E_{X_s}) \,.$

$$\sigma = \mathcal{H}(m_e, \{\underline{v}\}, \mu)\, \mathcal{S}(E_s, \{\underline{v}\}, \mu)$$

$\frac{d}{d\mu}\sigma = 0$: Physics should be μ-independent $\Leftrightarrow$ $\left[\left(\mu \frac{d}{d\mu} \mathscr{H} \right) \frac{1}{\mathscr{H}} + \left(\mu \frac{d}{d\mu} \mathscr{S} \right) \frac{1}{\mathscr{S}} \right] \mathscr{H}\mathscr{S} = 0$

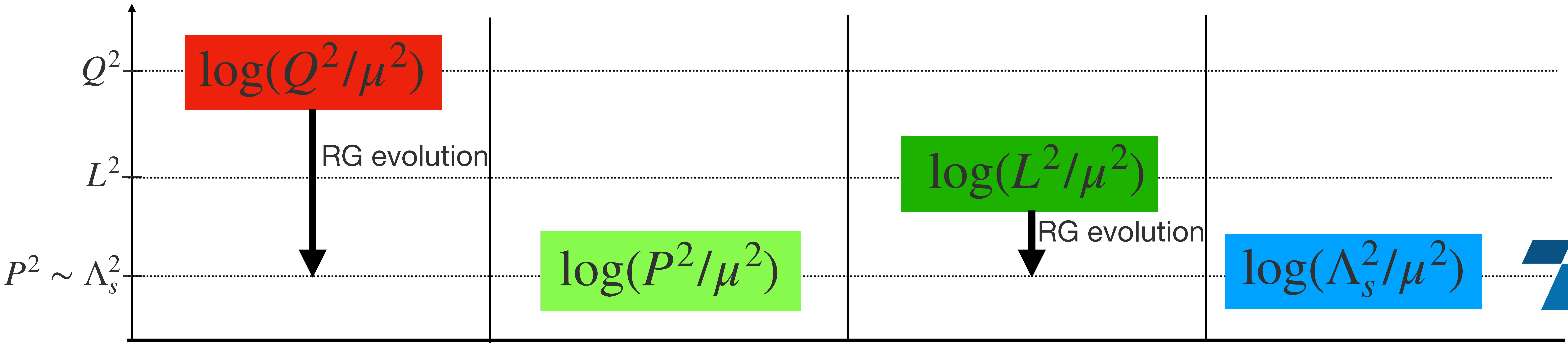
Sum of anomalous dim.

RGE properties of Hard physics is determined by Soft physics (or vice versa)

SET

Soft Collinear Effective Theory for spin-1 DM

Soft Collinear Effective Theory for spin-1 DM



[MF, M. Vollmann (work in progress)]

SCET construction

Regime

- Perturbative expansion w.r.t. $\{\ \alpha_2,\ \lambda\ \}$
- Factorize amplitude for each scale (to suppress large logs)

For TeV DM:

$$\lambda = \frac{m_W}{2m_{\rm DM}} \simeq \frac{m_{\rm DM} - E_\gamma}{2m_{\rm DM}} \simeq 10^{-1}$$

Degrees of freedom

Full theory

- Spin-1 DM : $V_\mu^A\ (A = 1,2,3)$ →
- EW boson : $W_\mu^A\ (A = 1,2,3)$ →

SCET

- Heavy NR spin-1 DM : Υ_v
- Wilson lines
  - (Anti-)collinear EW boson : $\{\mathscr{A}_{\perp n}, \mathscr{A}_{\perp \bar{n}}\}$ (Wilson line)
  - Soft EW radiation : S_v factorized from DM
    : Y_n factorized from (anti-)collinear particles

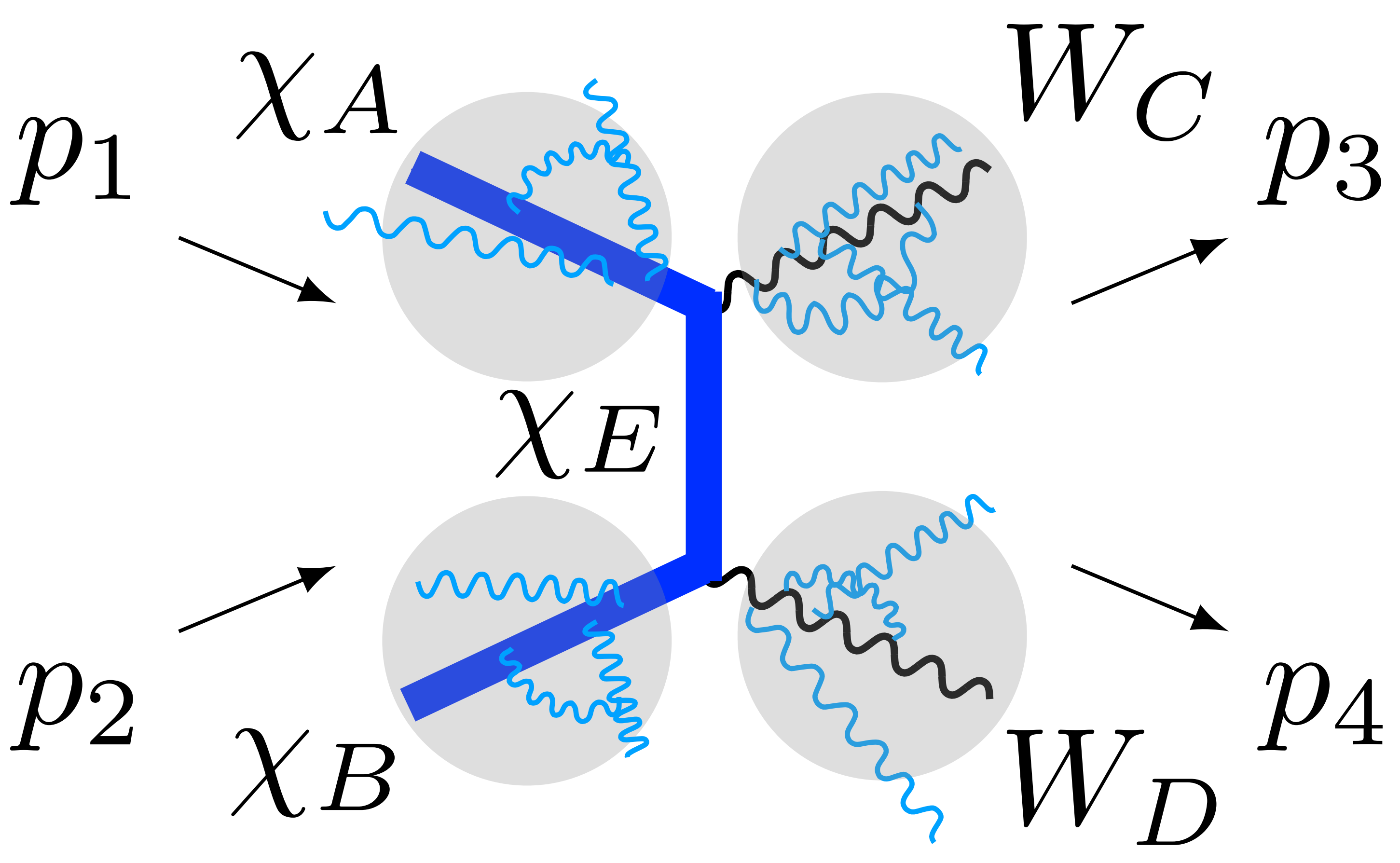
Matching btw "Amplitudes in full theory" & "SCET operators for DM annihilation"
→ Factorize amplitude for each energy scale: hard, (anti-)collinear, soft

[MF, M. Vollmann (work in progress)]

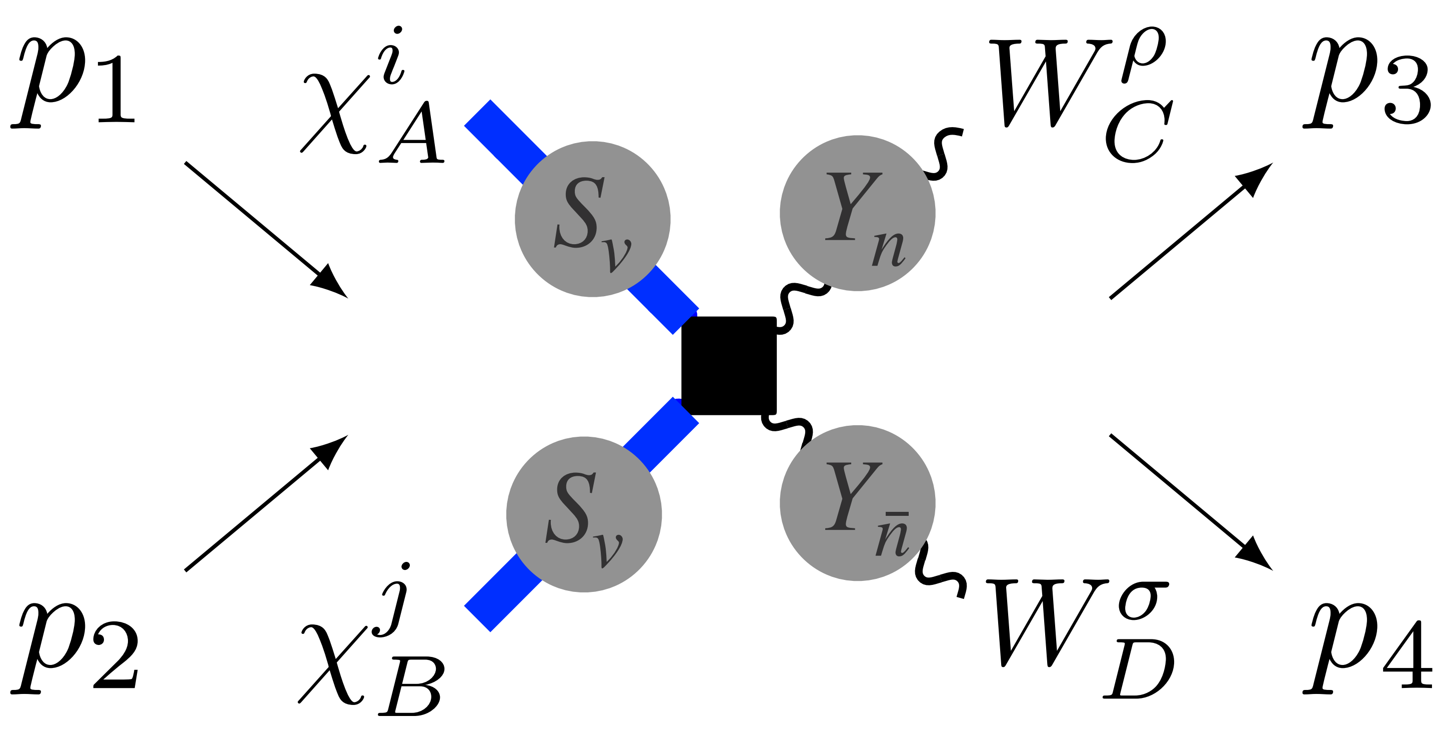
Tree-level matching

Full theory setup

- Initial state: two NR DM particles
- Final state: massless EW bosons (transverse modes only)



Matching



EFT operators

- LO potential is spherically symmetric → total spin is conserved
- Final state: massless EW bosons (transverse modes only)

$$\mathcal{O}_{\mathcal{J}}^{(S)_i} = \int ds \int dt \tilde{\Upsilon}_\alpha^A \tilde{\Upsilon}_\beta^B \tilde{\mathscr{A}}_\mu^C(sn_+) \tilde{\mathscr{A}}_\nu^D(tn_-) \times T_{\mathcal{J}}^{ABCD} U_{(S)_i}^{\alpha\beta\mu\nu}$$

	$SU(2)_L$ charge	Lorentz spin	Orbital wave func.
State :	$\mathcal{J} = 0,2$	$S = 0,2$	s-wave (LO)
Under $\Upsilon_\mu^A \Leftrightarrow \Upsilon_\nu^B$:	Symmetric	Symmetric	Symmetric
	$\mathcal{J} = 1$	$S = 1$	p-wave (suppressed)
	Anti-symme.	Anti-symme.	Anti-symme.

Operators for Spin-1 DM

$$\mathcal{L}_{\rm int} = \frac{1}{2m_V} \sum_{S=0}^{2} \sum_{\mathcal{J}=0}^{2} \sum_i \int \mathrm{d}s\, \mathrm{d}t\, \tilde{C}^{(S)_i}_{\mathcal{J}}(s,t,\mu) \mathcal{O}^{(S)_i}_{\mathcal{J}}(t,s,\mu) \,,$$

Spin SU(2)L charge

Spin-0: $$\mathcal{O}^{(0)}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha \eta_u^{\alpha\beta} \tilde{\Upsilon}^B_\beta T^{ABCD}_{\mathcal{J}} \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+) \eta_\perp^{\mu\nu} \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-) \,,$$

Spin-2 (1): $$\mathcal{O}^{(2)_1}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha \tilde{\Upsilon}^B_\beta T^{ABCD}_{\mathcal{J}} (\eta_\perp^{\alpha\mu}\eta_\perp^{\beta\nu} + \eta_\perp^{\alpha\nu}\eta_\perp^{\beta\mu}) \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+) \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-) - \frac{2}{3}\mathcal{O}^{(0)}_{\mathcal{J}} \,,$$

Spin-2 (2): $$\mathcal{O}^{(2)_2}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha (n_+ - n_-)^\alpha \tilde{\Upsilon}^B_\beta (n_+ - n_-)^\beta T^{ABCD}_{\mathcal{J}} \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+) \eta_\perp^{\mu\nu} \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-) + \frac{4}{3}\mathcal{O}^{(0)}_{\mathcal{J}}$$

$$\begin{cases} T^{ABCD}_{\mathcal{J}=0} = \delta^{AB}\delta^{CD} \,, \\ T^{ABCD}_{\mathcal{J}=2} = \delta^{AC}\delta^{BD} + \delta^{AD}\delta^{BC} - \frac{2}{3}\delta^{AB}\delta^{CD} \,. \end{cases}$$

SU(2)L charge: $\mathscr{J} = 0$

SU(2)L charge: $\mathscr{J} = 2$

[MF, M. Vollmann (work in progress)]

Factorized Amplitude: $V^0 V^0 \to \gamma X$

$$\mathscr{M}(V_0 V_0 \to \gamma X) = \# \sum_i \sum_S \sum_{\mathscr{J}} C_{\mathscr{J}}^{(S)_i}(\mu)$$

$$\times \langle 0 \,|\, [\Upsilon_\alpha \Upsilon_\beta U^{\alpha\beta\mu\nu}_{(S)_i}]_I \,|\, [VV]_{00} \rangle \times K_I^{A'B'}$$ ~ hard

$$\times \langle \gamma \,|\, \mathscr{A}^{D'}_{\bar{c},\nu}(0) \,|\, 0 \rangle$$ ~ anti-coll.

$$\times \langle X_c \,|\, \mathscr{A}^{C'}_{c,\nu}(0) \,|\, 0 \rangle$$ ~ collinear

$$\times \langle X_s \,|\, [Y_u^{AA'} T_{\mathscr{J}}^{ABCD} Y_u^{BB'}] \mathscr{Y}^{CC'} \mathscr{Y}^{DD'} \,|\, 0 \rangle$$ ~ soft

Factorized cross section

$$\frac{d(\sigma v)}{dE_\gamma} = 2\sum_{I,J} S_{IJ} \sum_{i,j} \sum_{S} \sum_{\mathcal{I},\mathcal{J}} \frac{1}{(\sqrt{2})^{n_{id}}} \frac{1}{9} \frac{1}{2\pi} \frac{1}{m_V}$$

$$\times\, C^{(S)_i}_{\mathcal{I}}(\mu) C^{(S)_j}_{\mathcal{J}}(\mu)^* \quad \sim \text{(hard function)}$$

$$\times\, Z^{33}_{\gamma} \quad \sim \text{(photon jet function)}$$

$$\times \int d(n_- \cdot k) J_{int}\Big(4m_V(m_V - E_\gamma - n_- \cdot k/2)\Big) \quad \sim \text{(collinear jet function)}$$

$$\times\, W_{\mathcal{I}\mathcal{J},IJ}(n_- \cdot k) \quad \sim \text{(soft function)}$$

Once EFT operators are specified, amplitude/cross-section are automatically factorized for each energy scale

→ Renormalization Group Equation for each function? (next page)

M. Bauer, et al. (2015)] [M. Beneke, A. Broggio, C. Hasner, K. Urban, M. Vollmann (2019)]

Renormalization group equation

$$\frac{d}{d\ln\mu}F_i(p_i^2,\mu) = \Gamma_i \times F_i(p_i^2,\mu)$$

$$\Gamma_i = \gamma_{i,1}\log\left(\frac{p_i^2}{\mu^2}\right) + \gamma_{i,2}$$ Anomalous dim.

$$i = \{\text{hard, collinear, anticollinear, soft}\}$$

$\frac{d}{d\mu}(\text{factorized cross section}) = 0$: Physics should be μ-independent

※Sketch of proof taking $\Lambda_s^2 \sim \left(\frac{Q^2}{P^2L^2}\right)^{-1}$

$$\Leftrightarrow \quad \sum_i \Gamma_i \overset{!}{=} 0$$

$$= \left[\gamma_{h,1}\log\left(\frac{Q^2}{\mu^2}\right) + \gamma_{c,1}\log\left(\frac{L^2}{\mu^2}\right) + \gamma_{\bar{c},1}\log\left(\frac{P^2}{\mu^2}\right) + \gamma_{s,1}\log\left(\frac{\Lambda_s^2}{\mu^2}\right)\right] + \sum_i \gamma_{i,2}$$

log (Q, P, L, Λ_s) part: $\gamma_{\mathrm{h},1} = -\gamma_{\mathrm{c},1} = -\gamma_{\bar{\mathrm{c}},1} = \gamma_{\mathrm{s},1}$

log-indep. part: $\gamma_{\mathrm{h},2} = -\left(\gamma_{\mathrm{c},2} + \gamma_{\bar{\mathrm{c}},2} + \gamma_{\mathrm{s},2}\right)$

Hard properties   Determined by SU(2) properties

Anomalous dim. is spin independent

We can recycle the same γ as spin-1/2 case

Universality of LL resummation was first pointed out by [M. Bauer+(2015)]

Constraint on anomalous dimensions [BU]

Physics should be μ-independent ⇔ Sum of anomalous dimensions cancel

※Sketch of proof taking $\Lambda_s^2 \sim \left(\frac{Q^2}{P^2L^2}\right)^{-1}$

$$\sum_i \Gamma_i \overset{!}{=} 0$$

$$= \left[\gamma_{h,1}\log\left(\frac{Q^2}{\mu^2}\right) + \gamma_{c,1}\log\left(\frac{L^2}{\mu^2}\right) + \gamma_{\bar{c},1}\log\left(\frac{P^2}{\mu^2}\right) + \gamma_{s,1}\log\left(\frac{\Lambda_s^2}{\mu^2}\right)\right] + \sum_i \gamma_{i,2}$$

$$= -\left(\gamma_{h,1}+\gamma_{c,1}+\gamma_{\bar{c},1}+\gamma_{s,1}\right)\log\mu^2 + \gamma_{h,1}\log Q^2 + \gamma_{c,1}\log L^2 + \gamma_{\bar{c},1}\log P^2 + \gamma_{s,1}\log\Lambda_s^2 + \sum_i \gamma_{i,2}$$

① ② Most non-trivial part to cancel ③

Only one possibility to cancel different scales

→ log (products of momenta) = log 1 = 0

Note: $\frac{Q^2}{P^2L^2}\cdot\Lambda_s^2 = \frac{Q^2}{P^2L^2}\cdot\frac{P^2L^2}{Q^2} = 1$

→ Universal anomalous dim. (up to sign)

① log μ part: $\gamma_{\mathrm{h},1} = -\left(\gamma_{\mathrm{c},1}+\gamma_{\bar{\mathrm{c}},1}+\gamma_{\mathrm{s},1}\right)$ included in ②

② log (Q, P, L, Λ_s) part: $\gamma_{\mathrm{h},1} = -\gamma_{\mathrm{c},1} = -\gamma_{\bar{\mathrm{c}},1} = \gamma_{\mathrm{s},1}$

③ log-indep. part: $\gamma_{\mathrm{h},2} = -\left(\gamma_{\mathrm{c},2}+\gamma_{\bar{\mathrm{c}},2}+\gamma_{\mathrm{s},2}\right)$

[MF, M. Vollmann (work in progress)]

"Exact" universality of spectra (spin-0, 1/2, 1)

$$\mu \frac{d}{d\mu} C^{(S)_i}_{\mathcal{J}} = \Gamma_{\mathcal{J}} C^{(S)_i}_{\mathcal{J}}, \qquad \Gamma_{\mathcal{J}} = \gamma_{\rm cusp}\left[2\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{i\pi}{2}\left(\mathcal{J}(\mathcal{J}+1) - 4\right)\right] + 2\gamma_{\rm adj} + \gamma_V$$

Spin-0: $\mathcal{O}^{(0)}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha \eta^{\alpha\beta}_u \tilde{\Upsilon}^B_\beta T^{ABCD}_{\mathcal{J}} \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+)\eta^{\mu\nu}_\perp \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-)$,

Spin-2 (1): $\mathcal{O}^{(2)_1}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha \tilde{\Upsilon}^B_\beta T^{ABCD}_{\mathcal{J}} (\eta^{\alpha\mu}_\perp \eta^{\beta\nu}_\perp + \eta^{\alpha\nu}_\perp \eta^{\beta\mu}_\perp) \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+) \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-) - \frac{2}{3}\mathcal{O}^{(0)}_{\mathcal{J}}$,

Spin-2 (2): $\mathcal{O}^{(2)_2}_{\mathcal{J}} = \tilde{\Upsilon}^A_\alpha (n_+ - n_-)^\alpha \tilde{\Upsilon}^B_\beta (n_+ - n_-)^\beta T^{ABCD}_{\mathcal{J}} \tilde{\mathcal{A}}^C_{\perp c,\mu}(sn_+)\eta^{\mu\nu}_\perp \tilde{\mathcal{A}}^D_{\perp \bar{c},\nu}(tn_-) + \frac{4}{3}\mathcal{O}^{(0)}_{\mathcal{J}}$

$$\sum_{\mathcal{J}} C^{(S)_i}_{\mathcal{J}} \mathcal{O}^{(S)_i}_{\mathcal{J}} = \text{(Lorentz structure)} \times C^{(S)_i}_{\mathcal{J}=2} \left(\delta^{AC}\delta^{BD} + \delta^{AD}\delta^{BC} - 2\delta^{AB}\delta^{CD}\right) \qquad \left(\because C^{(S)_i}_{\mathcal{J}=0} = -\frac{4}{3} C^{(S)_i}_{\mathcal{J}=2}\right)$$

$\propto \{T^A, T^B\}_{CD}$ in triplet rep.

$$\mathcal{O}_{\rm spin-0} = \phi^{c\dagger}_v \{T^A, T^B\} \phi_v\, \mathcal{A}^A_{\perp c,\mu}(sn_+) \mathcal{A}^B_{\perp \bar{c},\mu}(tn_-)$$

$$\mathcal{O}_{\rm spin-\frac{1}{2}} = \chi^{c\dagger}_v \{T^A, T^B\} \chi_v \mathcal{A}^A_{\perp c,\mu}(sn_+)\, \mathcal{A}^B_{\perp \bar{c},\nu}(tn_-) \epsilon^{\mu\nu}$$

M. Bauer, et al. (2015)] [M. Beneke, A. Broggio, C. Hasner, K. Urban, M. Vollmann (2019)]

→ **"Exactly" universal spectra for all DM spin (up to overall factor)**

※ Universality breaks down by loop corrections

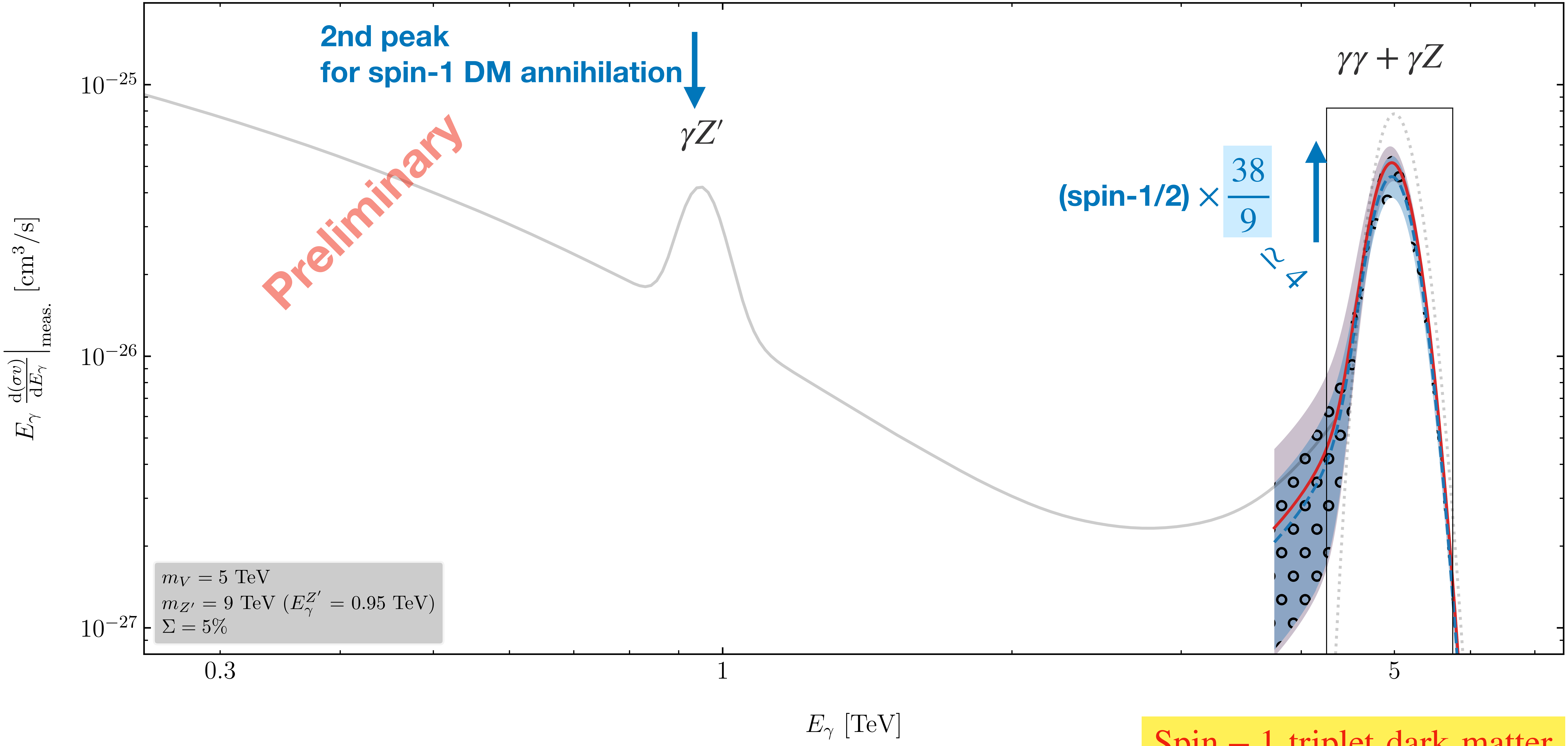
Results

[MF, M. Vollmann (work in progress)]

γ-ray spectrum (1/2)

Spin-dependence: (1) peak location = DM mass
(2) overall cross section

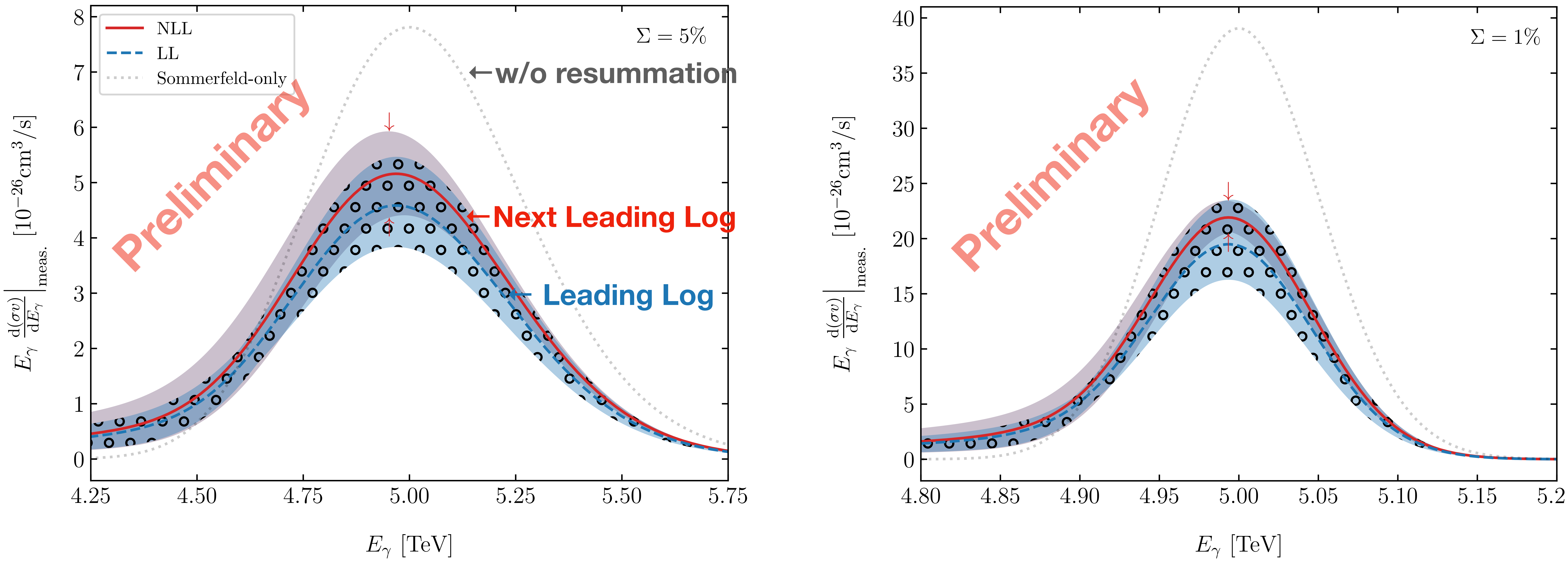
Spin-independence: shape of spectra (Next Leading log)



Spin – 1 triplet dark matter

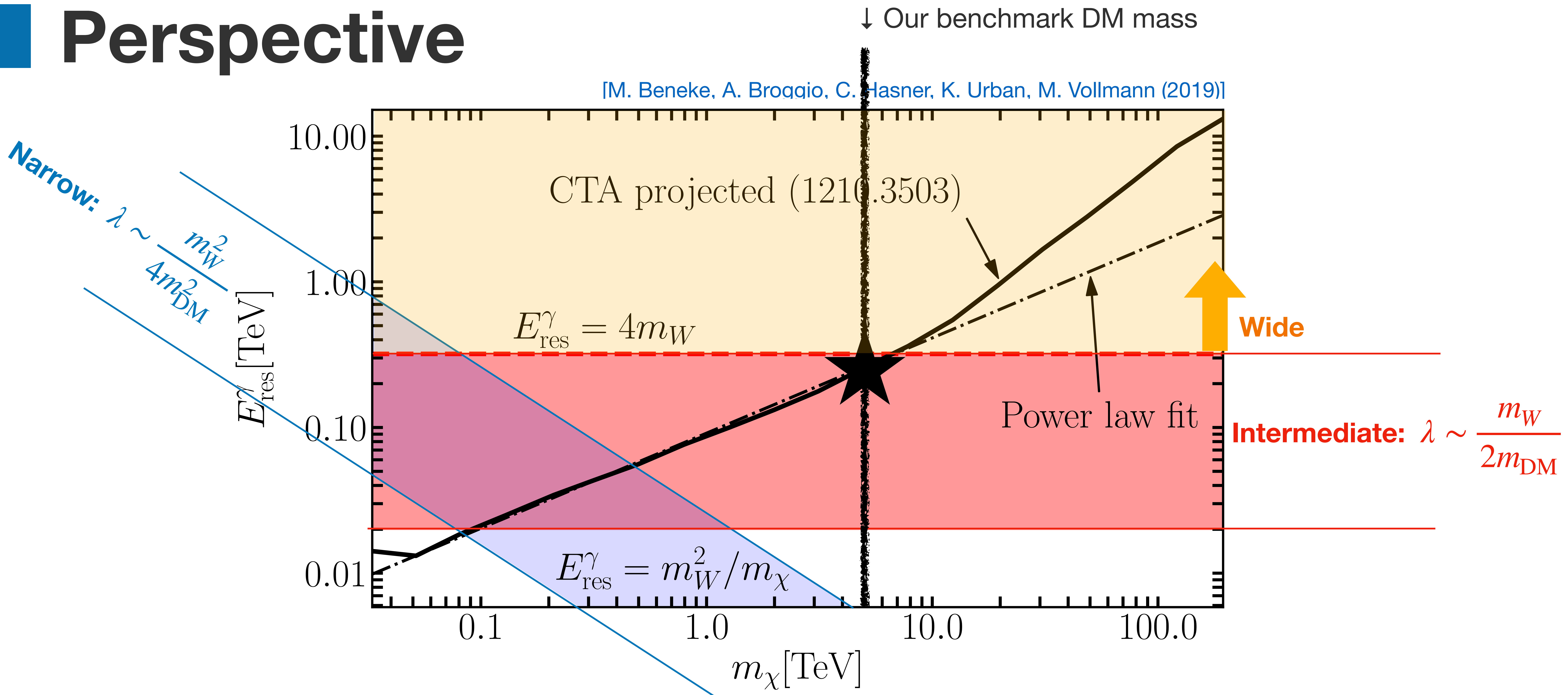
[MF, M. Vollmann (work in progress)]

γ-ray spectrum (2/2)



Comparison: { Sommefeld factor only, Leading Log, Next Leading Log }
Uncertainty is decreased depending on DM mass / energy resolution (next page)

Perspective



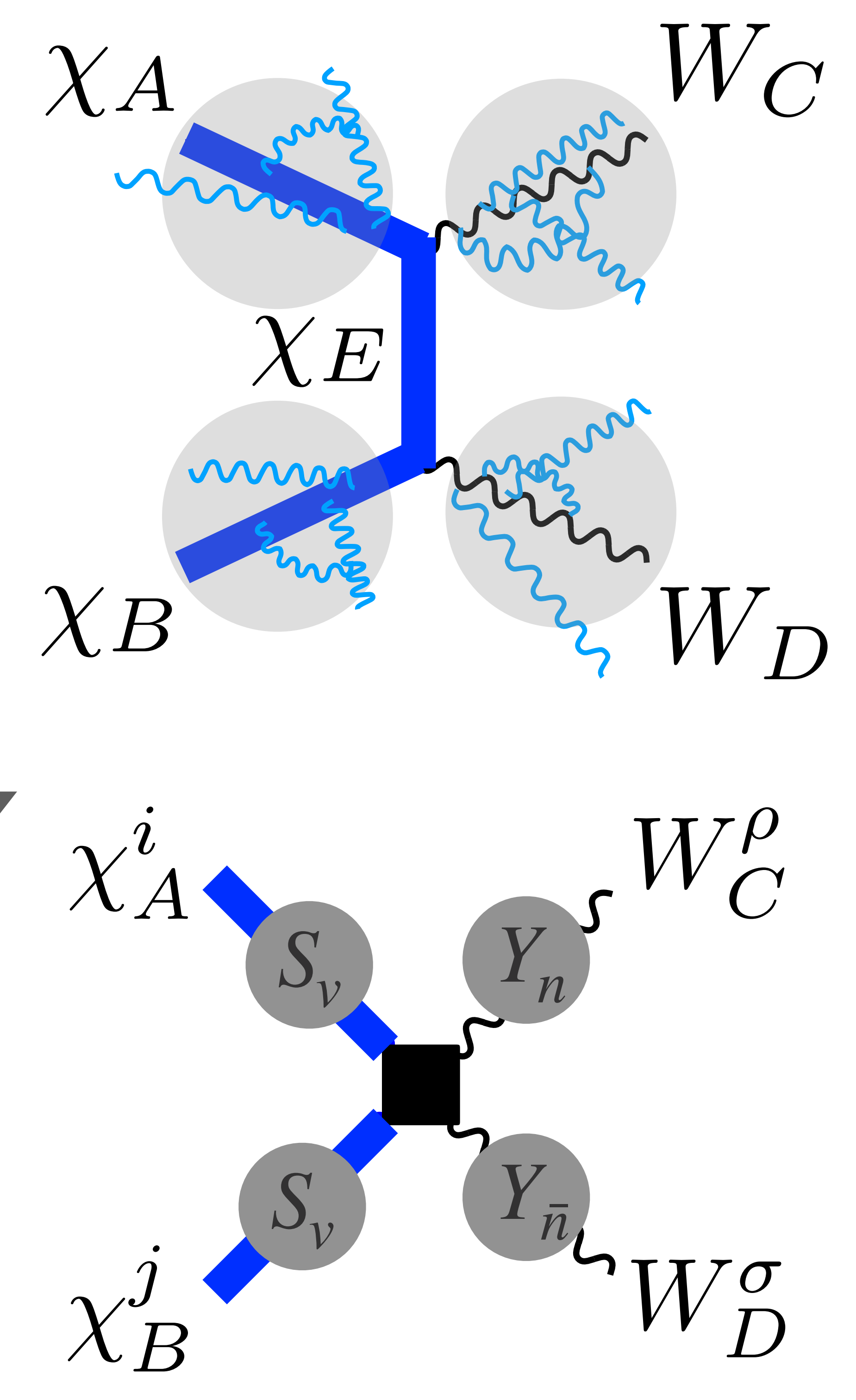
Lighter DM: **Power suppression (λ) gets weak** → **1-loop correction is important**

Heavier DM: **Realistic energy resolution @CTA** → **SCET of wide resolution regime is necessary**

Summary

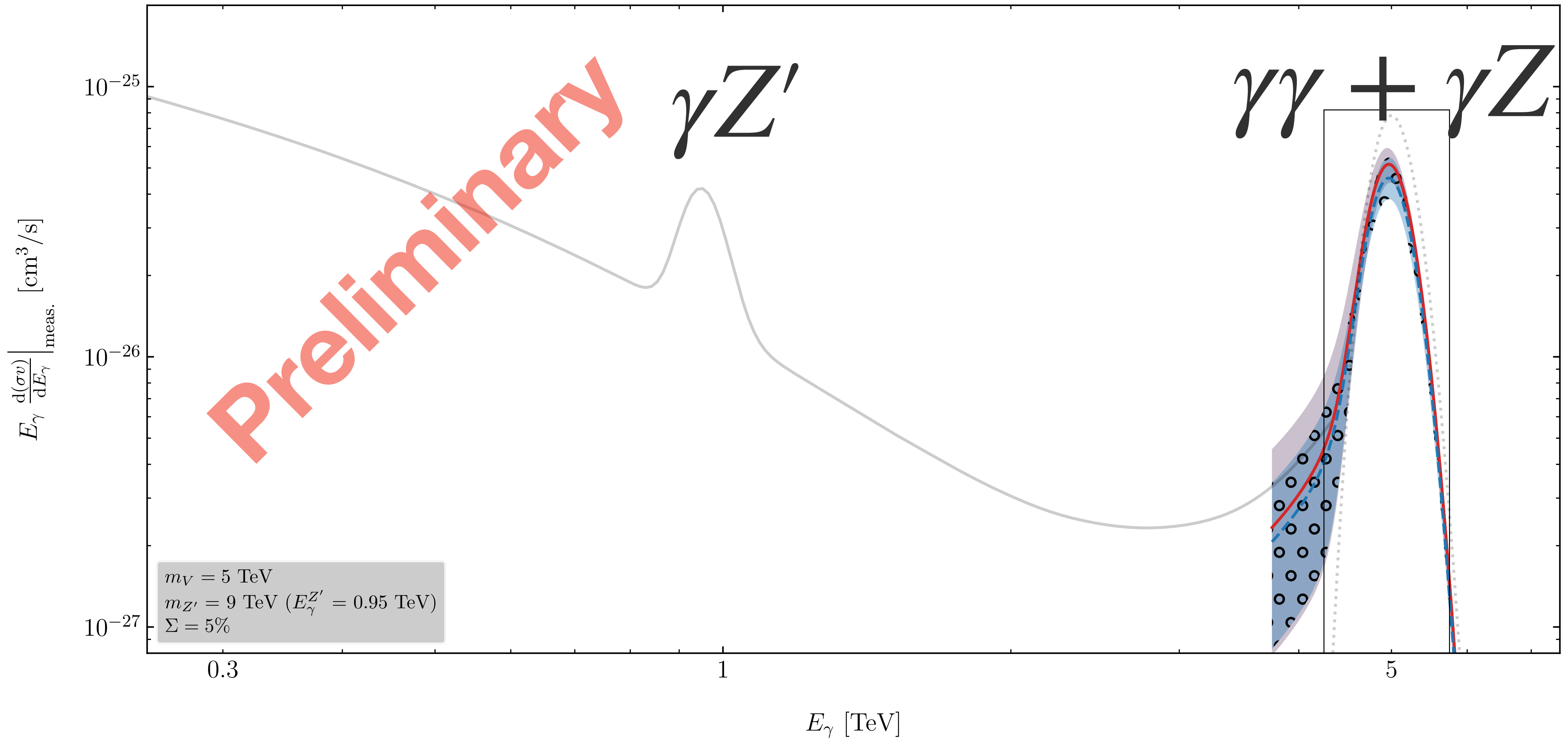
SCET for EW interacting spin-1 DM

- We construct SCET for EW spin-1 DM for the first time
- Tree-level matching → Specify operators in SCET
- Factorization formula is derived (best benefit in SCET formalism)
- We directly prove **universal shape of energy spectra for all DM spin**
  Universality of LL resummation was first pointed out by [M. Bauer+(2015)]



Resumed γ-ray spectrum

- γ-ray energy spectrum ($\gamma\gamma, \gamma Z, \gamma Z'$) @NLL accuracy
- Perspectives
  - Extension to other regime for heavier DM
  - 1-loop matching (cf. [M. Beneke+(2019)] for spin-1/2 DM)



The first step for completion of EW int. DM for all the spin
Detailed analysis is ongoing (thermal relic DM mass, etc)

Acknowledgement

To Dr. Martin Vollmann

Key Moments in our project : The 2nd DMNet International Symposium @Heidelberg, 9/2022
: Visiting in University of Tübingen, 5/2024
: Mainz Institute for Theoretical Physics (MITP) workshop @Mainz, 9/2024

[http://www3.u-toyama.ac.jp/theory/index-e.html]

Visit us! (U. Toyama, Theory Group)

Topic: Particle Cosmology

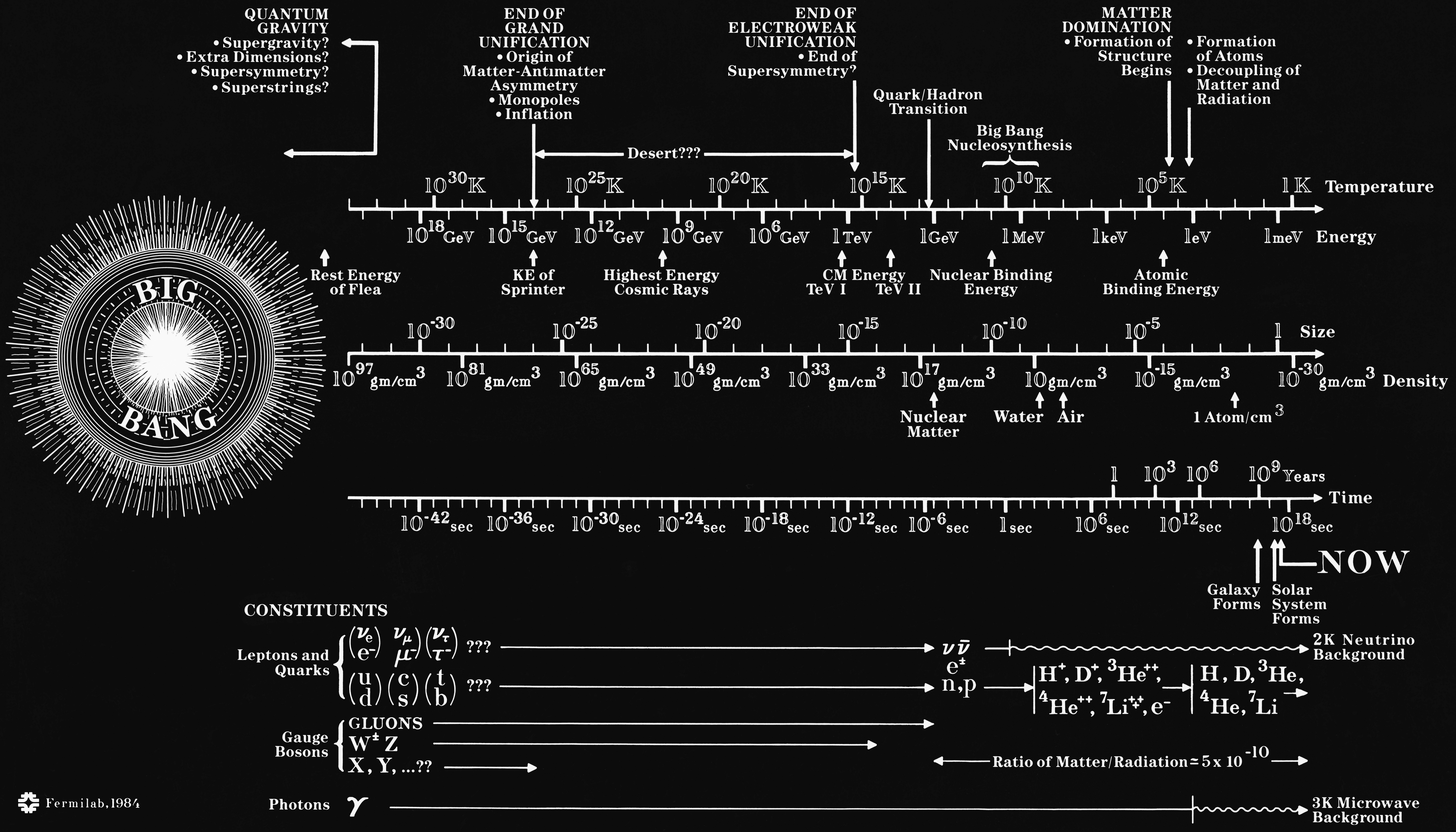
- Baryon asymmetry
- Neutrino
- Dark Matter
- Higgs Physics
- Phase transition
- Gravitational wave … (See our Web site for more info)

If you stay at the dormitory in U. Toyama, you can stay 5 days within travel support per day (~10,000 yen)

Food in Toyama is AMAZING 🍣🦐🦑🦀🍢🍜🍚🍶

Giving seminar / Hosting JSPS fellow / Visiting student
You are always welcome!!!

Backup



QUANTUM GRAVITY
• Supergravity?
• Extra Dimensions?
• Supersymmetry?
• Superstrings?
END OF GRAND UNIFICATION
• Origin of Matter-Antimatter Asymmetry
• Monopoles
• Inflation
END OF ELECTROWEAK UNIFICATION
• End of Supersymmetry?
Desert???
Quark/Hadron Transition
Big Bang Nucleosynthesis
MATTER DOMINATION
• Formation of Structure Begins
• Formation of Atoms
• Decoupling of Matter and Radiation
10^30 K
10^25 K
10^20 K
10^15 K
10^10 K
10^5 K
1 K
Temperature
10^18 GeV
10^15 GeV
10^12 GeV
10^9 GeV
10^6 GeV
1 TeV
1 GeV
1 MeV
1 keV
1 eV
1 meV
Energy
Rest Energy of Flea
KE of Sprinter
Highest Energy Cosmic Rays
CM Energy
TeV I
TeV II
Nuclear Binding Energy
Atomic Binding Energy
BIG BANG
10^-30
10^-25
10^-20
10^-15
10^-10
10^-5
1
Size
10^97 gm/cm^3
10^81 gm/cm^3
10^65 gm/cm^3
10^49 gm/cm^3
10^33 gm/cm^3
10^17 gm/cm^3
10 gm/cm^3
10^-15 gm/cm^3
10^-30 gm/cm^3
Density
Nuclear Matter
Water
Air
1 Atom/cm^3
1
10^3
10^6
10^9 Years
Time
10^-42 sec
10^-36 sec
10^-30 sec
10^-24 sec
10^-18 sec
10^-12 sec
10^-6 sec
1 sec
10^6 sec
10^12 sec
10^18 sec
NOW
Galaxy Forms
Solar System Forms
CONSTITUENTS
Leptons and Quarks
(νe e-) (νμ μ-) (ντ τ-) ???
(u d) (c s) (t b) ???
ν ν̄
e±
n,p
H+, D+, 3He++, 4He++, 7Li+++, e-
H, D, 3He, 4He, 7Li
2K Neutrino Background
Gauge Bosons
GLUONS
W± Z
X, Y, ...??
Ratio of Matter/Radiation ≈ 5 x 10^-10
Photons γ
3K Microwave Background
Fermilab, 1984

[MF, T. Abe, J. Hisano, K. Matsushita (2020)]

Matter contents

$$SU(2)_0 \times SU(2)_1 \times SU(2)_2 \times U(1)_Y \xrightarrow{\langle\Phi_i\rangle \neq 0\ (i=1,2)} SU(2)_L \times U(1)_Y \xrightarrow{\langle H\rangle \neq 0} U(1)_{em}$$

Exchange Symme.

- Each fermion corresponds to SM fermion
- Scalar field to realize $U(1)_{em}$ in low energy

$$\Phi_j = \mathbf{1}\sigma_j + \tau^a \pi_j^a \qquad \left[\text{s.t. } \Phi_j = -\epsilon \Phi_j^* \epsilon \quad (j=1,2)\right]$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi^1 - \pi^2 \\ \sigma - i\pi^3 \end{pmatrix}$$

4 real degrees of freedom for each

- Symmetry transformation
  - Gauge trans. (for scalars)

$$\begin{cases} \Phi_1 \mapsto U_0 \Phi_1 U_1^\dagger \\ \Phi_2 \mapsto U_2 \Phi_2 U_1^\dagger \\ H \mapsto U_1 H \end{cases}$$

$U_n = \exp[i\theta_n(x)] \quad (n = 0, 1, 2)$

  - Exchange trans.

$$\Phi_1 \leftrightarrow \Phi_2, \qquad W_{0\mu}^a \leftrightarrow W_{2\mu}^a$$

※ $g_0 = g_2$ ($\neq g_1$)

- Vacuum expectation values

$$\langle\Phi_1\rangle = \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\Phi & 0 \\ 0 & v_\Phi \end{pmatrix}$$

$$\langle H\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$(v_\Phi \gg v)$

v_Φ: $\mathcal{O}(1)$ TeV, v: $\mathcal{O}(100)$ GeV

field	spin	$SU(3)_c$	$SU(2)_0$ ($W_{0\mu}^a$)	$SU(2)_1$ ($W_{1\mu}^a$)	$SU(2)_2$ ($W_{2\mu}^a$)	$U(1)_Y$
q_L	$\frac{1}{2}$	3	1	2	1	$\frac{1}{6}$
u_R	$\frac{1}{2}$	3	1	1	1	$\frac{2}{3}$
d_R	$\frac{1}{2}$	3	1	1	1	$-\frac{1}{3}$
ℓ_L	$\frac{1}{2}$	1	1	2	1	$-\frac{1}{2}$
e_R	$\frac{1}{2}$	1	1	1	1	-1
Φ_1	0	1	2	2	1	0
Φ_2	0	1	1	2	2	0
H	0	1	1	2	1	$\frac{1}{2}$

Comment: Why Wilson line?

Problem: Derivative operator is not suppressed

How to express $\forall n$-th derivative op. systematically?

$$\partial\phi_n|_{\text{collinear}} \sim \lambda^0 Q\phi_n \text{ (No suppression)}$$

ϕ_n : collinear field

$n^\mu = (1,0,0,1)$

$\bar{n}^\mu = (1,0,0,-1)$

Solution: Representing as non-local interactions

$$\int dsC(s)\bar{\phi}_n(x+s\bar{n})[\cdots]\phi_n(x) = \sum_{n=0}^{\infty} C_n \frac{1}{n!}\left((\bar{n}\cdot\partial)^n\bar{\phi}_n(x)\right)[\cdots]\phi_n(x)$$

$$\begin{cases} \phi_n(x+s\bar{n}) = \sum_{n=0}^{\infty}\frac{s^n}{n!}(\bar{n}\cdot\partial)^n\phi_n(x) \\ C_n = \int dsC(s)s^n \end{cases}$$

$C_n = \int dsC(s)s^n \Leftrightarrow$ coeff. for higher derivative op.

Gauge covariant form → Wilson line

$$\mathscr{A}_n(x) \equiv \mathscr{P}\exp\left[ig\int_{-\infty}^{x} ds'\bar{n}\cdot A_n(s'\bar{n})\right]$$