Excited bound states and their role in dark matter production

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in collaboration with:

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based on:

arXiv:2303.01336

arXiv:2411.08737

Outline

1. Introduction: Thermal Production and Freeze-out
2. Theory background: bound states in PNREFT
3. Bound state formation: semi-classical picture
4. Intermezzo: Perturbative Unitarity Violation in BSF
5. Toy model: Bound States in dark gauge sectors
6. Realistic model: colored + charged t-channel mediator

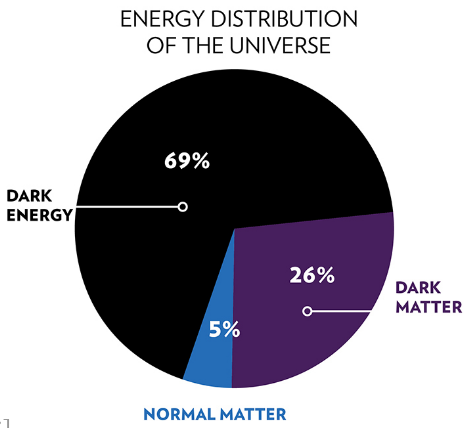
Dark Matter Phenomenology – crash course

Dark Matter:

Key ingredient in modern cosmology

(gravitational evidence from all length scales)

- Dark
- Abundant
- Cold (probably)



- Some strong constraints from cosmology:
  - Cosmic Microwave Background $\Omega_{DM}h^2 = 0.120 \pm 0.001$ [Planck, 2018]
  - Baryogenesis (must not be altered)
  - Structure formation (no warm / hot DM)

  ...

- else: No evidence from SM physics! (in)direct detection / collider searches / ...

→ *How is DM produced in the early Universe?*

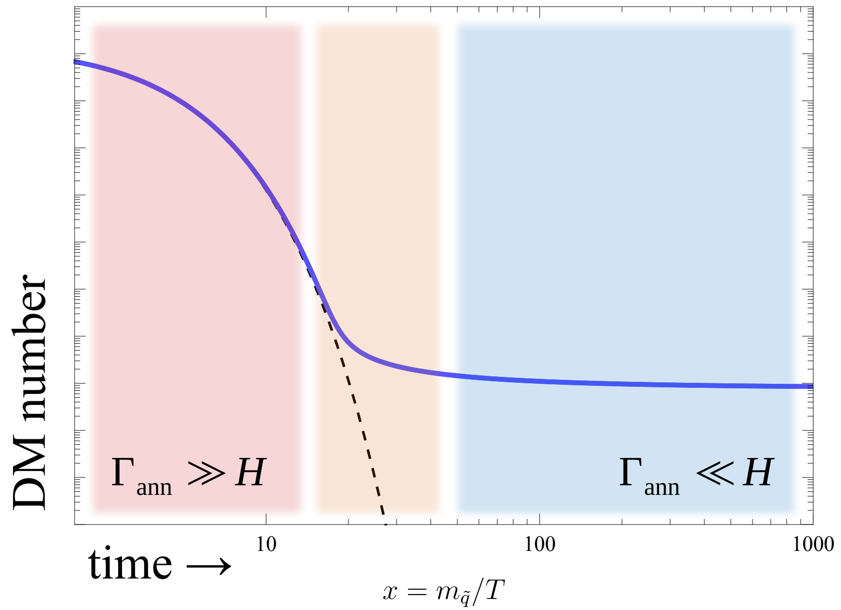
[Bullet Cluster; X-ray: NASA/CXC/CfA/]

Freeze-Out mechanism

Boltzmann equation: $$\frac{dY}{dx} = \frac{1}{3\,H\,s}\frac{ds}{dx}\,\frac{1}{2}\,\langle\sigma v\rangle\left(Y^2 - Y^{eq2}\right)$$

"time" coordinate: $$x = \frac{m_\chi}{T}$$

particle abundance: $$Y = \frac{n}{s} = \frac{\text{particle number}}{\text{entropy}}$$



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Thermally averaged annihilation cross-section

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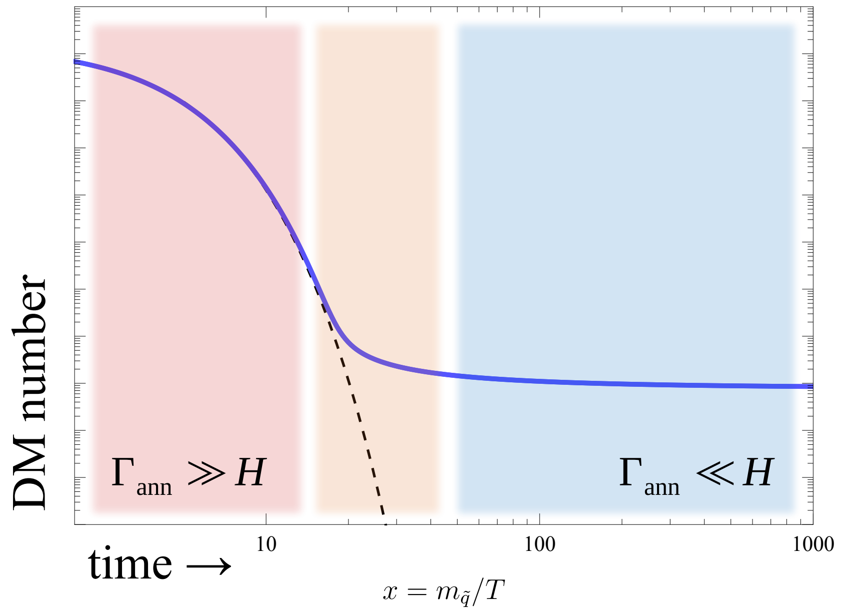

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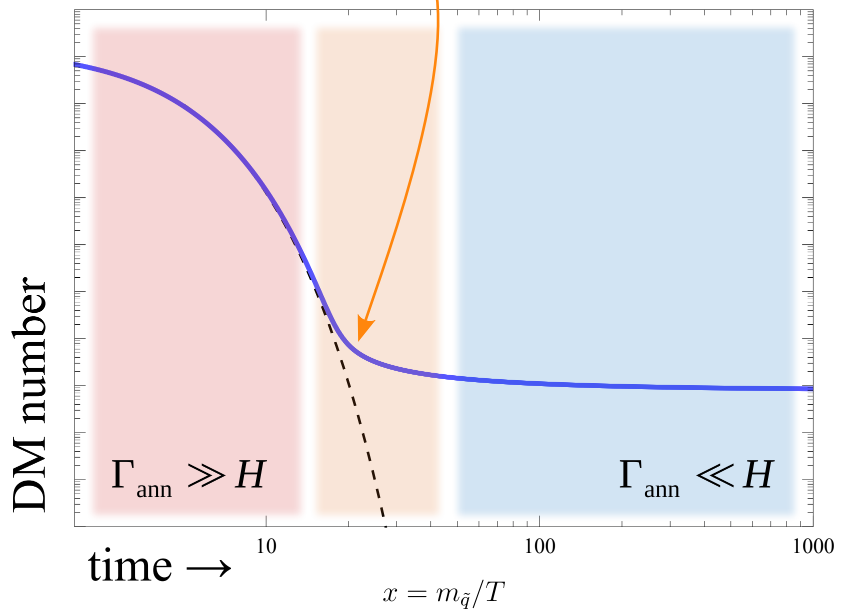
Thermally averaged annihilation cross-section

$$\Rightarrow \frac{\Gamma_{annh}}{H} = \frac{n\cdot\langle\sigma v\rangle}{H} \sim T\cdot\langle\sigma v\rangle \quad \text{with} \quad n\sim T^2,\ H\sim T^3$$

annihilation turns inefficient: „Freeze Out“ when $\langle\sigma v\rangle < T^{-1} \sim x$



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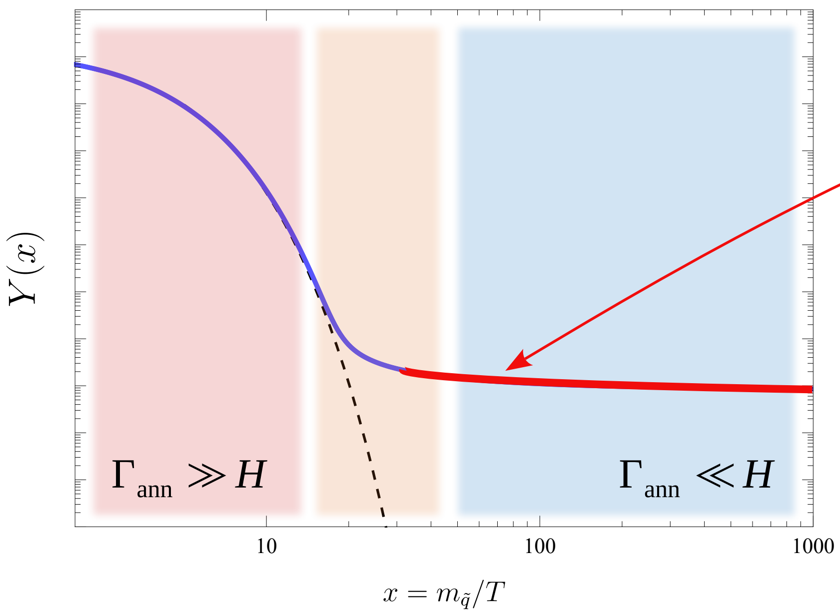
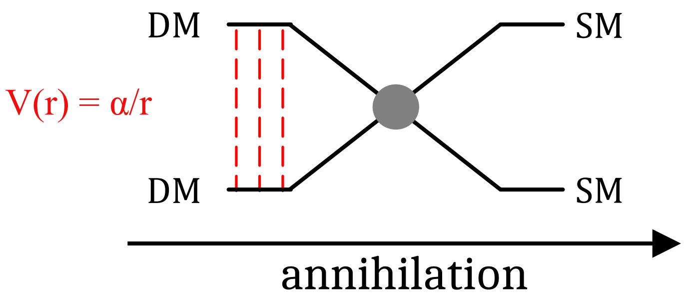


„Sommerfeld Effect” [Hisano: 2005]

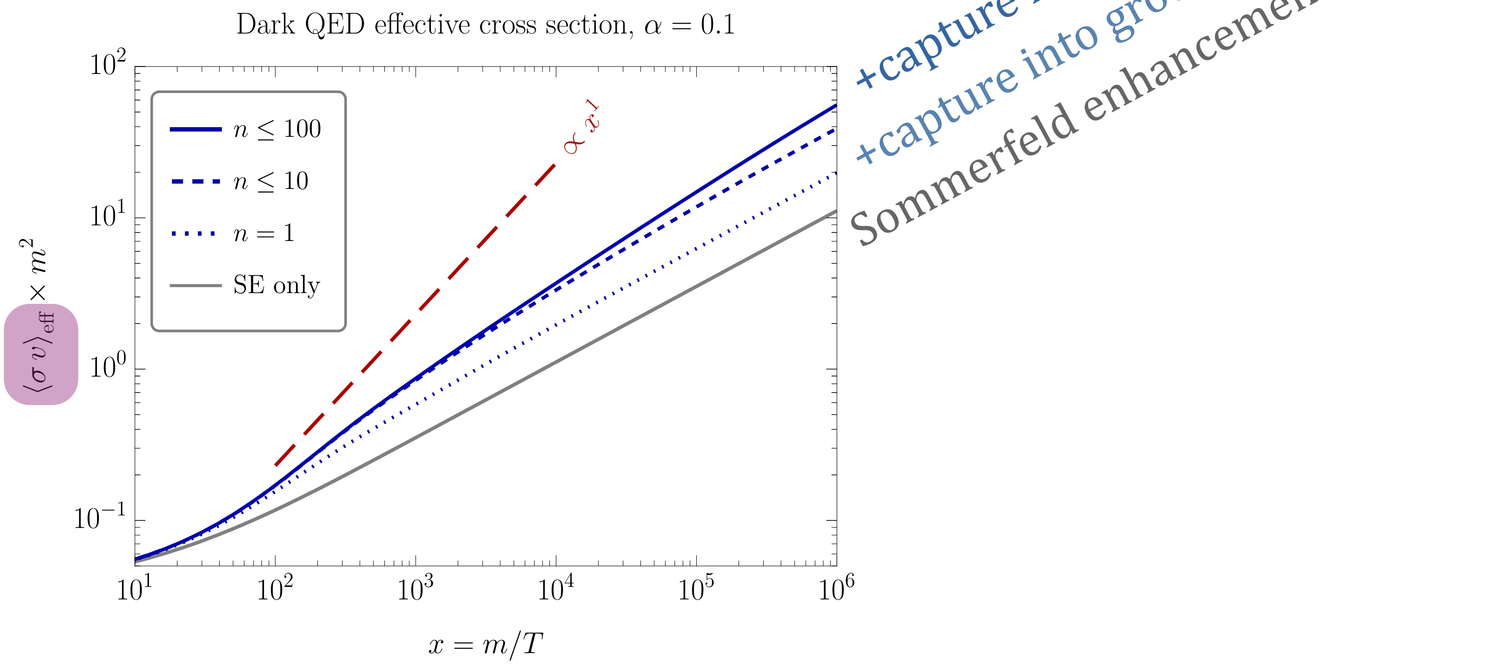
Long range interactions:



Effective cross-section: U(1) vs SU(N)

$$\frac{dY}{dx} = \frac{1}{3\,H\,s}\frac{ds}{dx}\;\frac{1}{2}\langle\sigma\,v\rangle\left(Y^2 - Y^{eq2}\right)$$

dark U(1)



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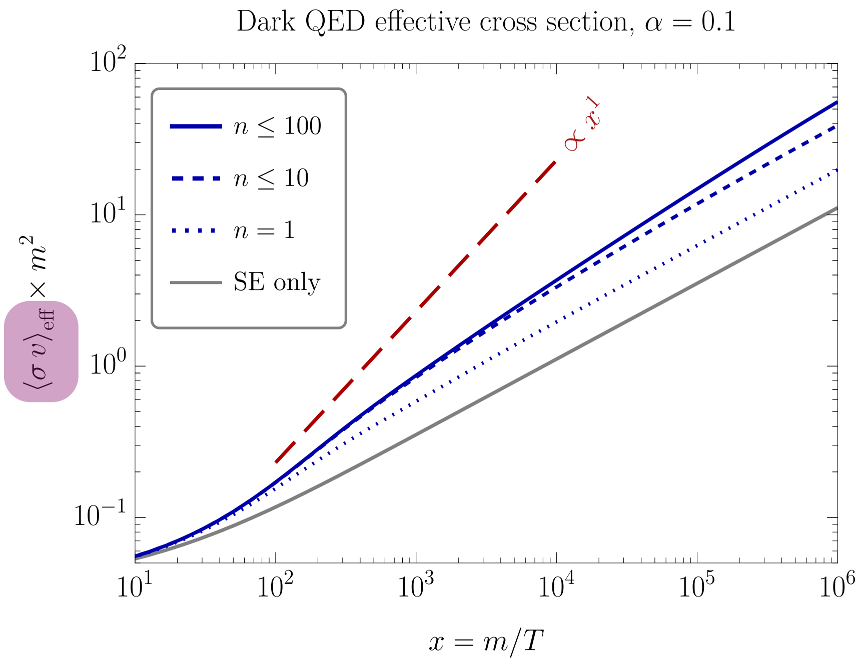
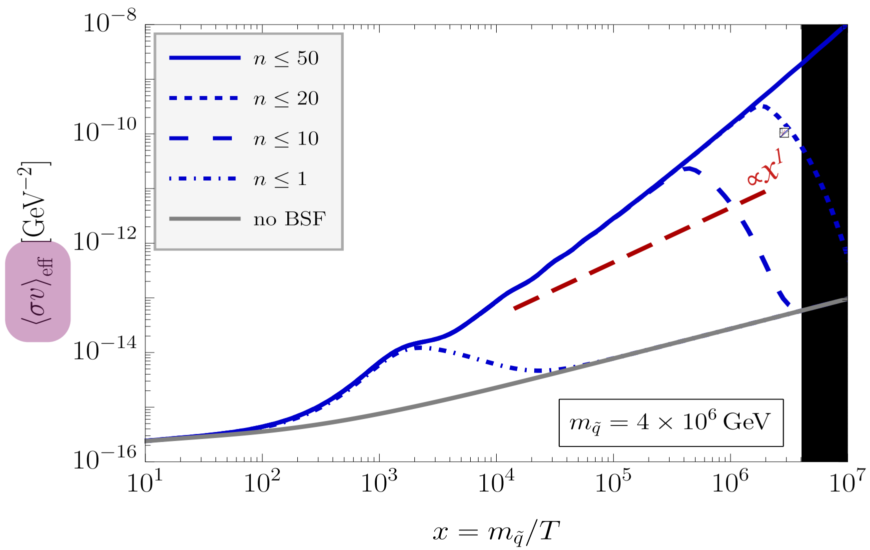


SU(3)$_c$ x U(1)$_Y$



Theory background

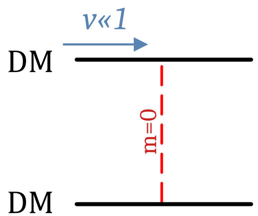
PNREFT

Potential **N**on-**R**elativistic **E**ffective **T**heory

Theory Background: long-range Potentials

For interactions with **light mediators**, perturbation theory breaks down:

- $1 \gg \alpha$ perturbative QFT
- $1 \gg v$ NREFT
- $1 \sim \alpha/v$ PNREFT



$$\sim \frac{\alpha}{p_{DM}} \sim \frac{\alpha}{v} \sim O(1)$$

→ Resume interaction to **all orders**.

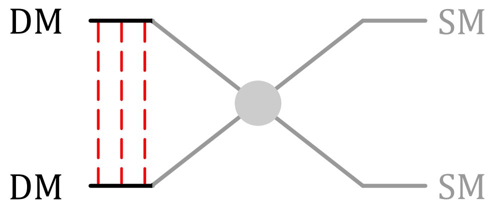
⇒ Coulomb-potential.

Yukawa potential if m≠0.

„Sommerfeld Effect" = long-range potentials between heavy particles

for DM: [Hisano: 2005]

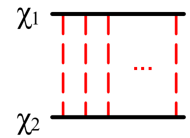
V(r) =α/r



$$(\sigma v) \sim \frac{1}{v_{\text{rel}}} \cdot (\sigma v)_{\text{Born}}$$

Theory background: bound states in SU(N)

$$V(r) = \frac{\alpha_{eff}}{r}$$



If the potential is **attractive**, the spectrum can host **bound states**.

→ What is the potential strength?

QED (photon exchange): simple $\alpha_{eff} = -Q_{\chi_1} Q_{\chi_2} \alpha_{\text{em}}$

QCD (gluon exchange):

quarks = **3** in SU(3)

$$q \otimes \bar{q} \simeq \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\alpha_{[\mathbf{8}]} = -\frac{1}{6}\alpha_3$$

$$\alpha_{[\mathbf{1}]} = +\frac{4}{3}\alpha_3$$

(For DM, representations may differ from $\mathbf{F} \otimes \mathbf{F}^*$.)

→ 2-body states exist in different eigenstates of the potential.

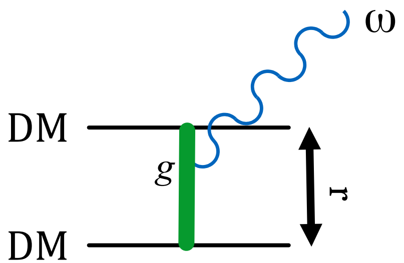
⇒ SU(N) gauge interactions can yield different & repulsive potentials.

Theory background: Multipole interactions

Scales in PNREFT:

- M_χ mass
- $p \sim M v$ momentum
- $E \sim M \alpha^2, \; Mv^2$ energy

„Ultra-soft" emissions, $\omega \sim M_\chi \alpha^2$, can be expanded in multipole orders **L**:



$|\mathbf{r}_1 - \mathbf{r}_2| = r \sim 1/p_{rel} \sim 1/v$,

$\mathbf{r} \cdot \mathbf{p}_\omega \sim r\,\omega \sim \alpha$,

$$\mathscr{L} \supset g\,(\boldsymbol{r} \cdot \boldsymbol{p}_\omega)^L \propto g\,\alpha^L \ll 1$$

These provide **bound-state formation**, Bremsstrahlung & bound-to-bound transitions.

Theory background: PNREFT Lagrangian

We are interested in annihilation and bound states.

$\Rightarrow$ project into 2-particle space:

$\rightarrow$ 2 separate species (S, B) for scattering & scattering states.

$\rightarrow$ independent potentials!

$\rightarrow$ include only the leading multipole operator.

$$\mathcal{L}_{\mathrm{BSF}} = \mathcal{S}^\dagger(R,\vec{r})\left(\mathrm{i}\partial^0 + \frac{\vec{\partial}_{\vec{r}}^2}{M_\chi} - \delta M_{\mathcal{S}} + \frac{\alpha_s}{r}\right)\mathcal{S}(R,\vec{r})$$

think „S = scattering state".
if $\alpha_s > 0$, also bound states S exist

$$+\mathcal{B}^\dagger(R,\vec{r})\left(\mathrm{i}\partial^0 + \frac{\vec{\partial}_{\vec{r}}^2}{M_\chi} - \delta M_{\mathcal{B}} + \frac{\alpha_b}{r}\right)\mathcal{B}(R,\vec{r})$$

think „B = bound state".
also B scattering-states exist

$$+g_a^{\mathrm{eff}}\, p_\phi^a\, r^a P_a(\hat{p}_\phi\cdot\hat{r})\ \mathcal{B}^\dagger(R,\vec{r})\,\phi^\dagger(R)\,\mathcal{S}(R,\vec{r})$$

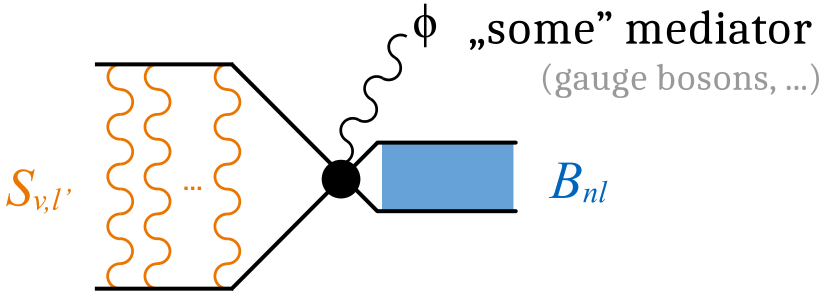
+ „h.c."
+ „iδ(r) operators"

The equations of motion are simply the *Schrödinger equation.*

radiative **B**ound **S**tate **F**ormation

„Seeing the formula“ ≠ „Undestanding the physics“

General **BSF** expression



Some notation

l', l = partial-wave numbers

$$\kappa \equiv \frac{\alpha_s}{\alpha_b} ,$$

$$\zeta_n \equiv \frac{\alpha_b}{n v} , \quad \zeta_s \equiv \frac{\alpha_s}{v}$$

The computation is basically Quantum Mechanics.
General result for all $\{n, l, p, l', L, \alpha_s, \alpha_b\}$:

[Gordon: 1929]

[Beneke, Binder, Garny, **SL**, De Ros: 2024]

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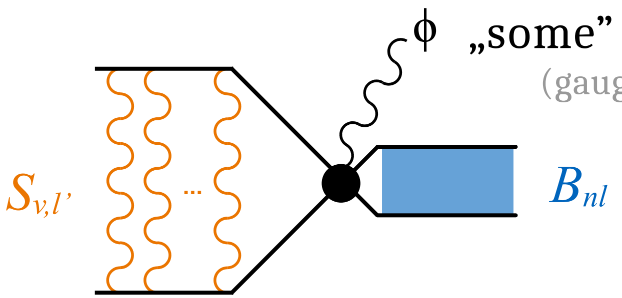

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amplitude

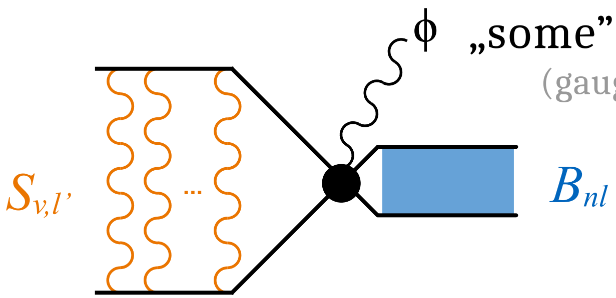
$$|\mathcal{M}|^2 \propto \sum_m \left| \langle nlm | r^L | \vec{p}, l' \rangle \right|^2 \propto I_A \times I_R$$

radial overlap

$$I_R = \frac{2^{4\ell+2}\zeta_n^{2\ell+3}}{(\mu v)^{3+2L}\left(1+\zeta_n^2\right)^{2\ell+4}} \frac{\Gamma(\ell'+1)^2\Gamma(n+\ell+1)}{n\Gamma(2\ell+2)^2\Gamma(n-\ell)} S_{\ell'}(\zeta_s)\, \mathrm{e}^{-4\zeta_s\gamma_n}$$

$$\times \left| \frac{1-\mathrm{e}^{2\mathrm{i}(2(n-\ell)\gamma_n-\gamma_F-\gamma_R)}}{n\kappa\zeta_n\left(\zeta_n^2-1+\frac{2}{\kappa}\right)} \right|^2 |F_+(0)|^2 \left|R^L_{\ell'-\ell}\right|^2 ,$$

General **BSF** expression



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Sommerfeld factor

$$\times \underbrace{\left| \frac{1-e^{2i(2(n-\ell)\gamma_n-\gamma_F-\gamma_R)}}{n\kappa\zeta_n\left(\zeta_n^2-1+\frac{2}{\kappa}\right)} \right|^2}_{\sin^2(\text{phase})} \underbrace{|F_+(0)|^2}_{\text{a single hypergeometric}} \underbrace{|R^L_{\ell'-\ell}|^2}_{\text{a rational polynomial}},$$

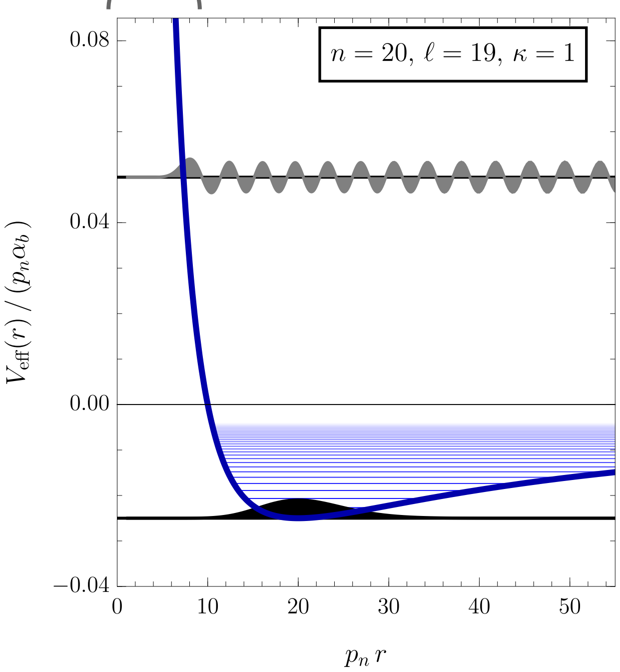
radiative **B**ound **S**tate **F**ormation

„Seeing the formula“ ✔ ≠ *„Undestanding the physics“*

Abelian scenario: $\alpha_b = \alpha_s$

Attractive initial state: $V_{\text{initial}}(r) = V_{\text{final}}(r)$

centrifugal barrier for $l > 0$



$$K = \frac{M_\chi}{4} v^2$$

$$E_n = \frac{M_\chi \alpha_b^2}{4 n^2}$$

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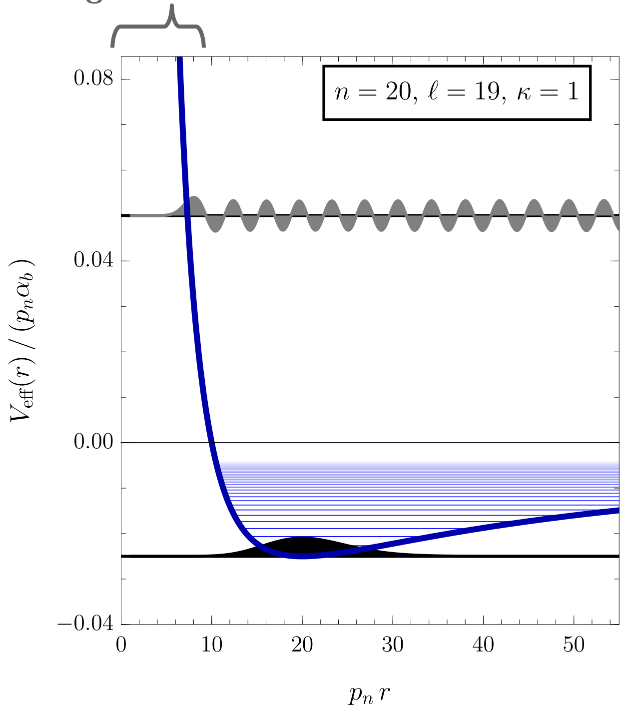

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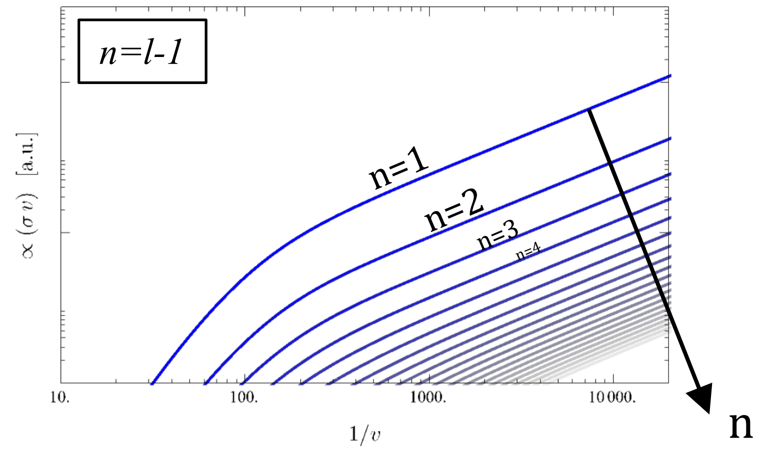
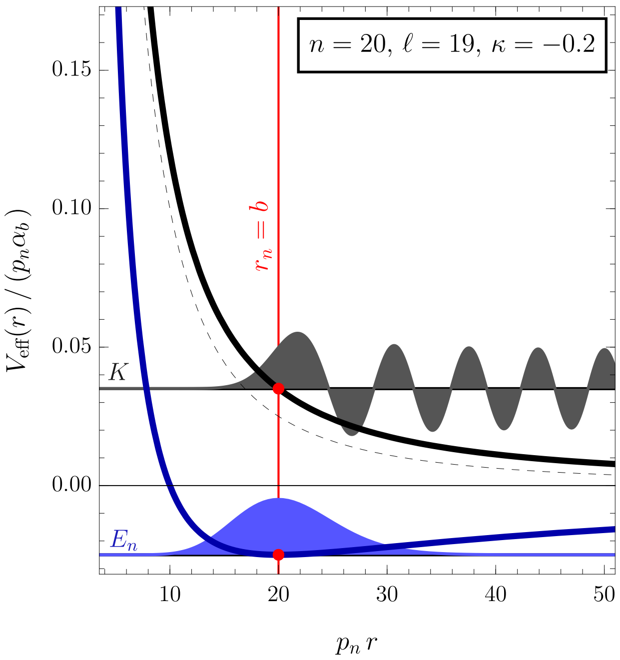


⇒ Higher n are suppressed.

Non-abelian scenario: $\alpha_b \neq \alpha_s$

Repulsive initial state: $V_{\text{initial}}(r) < 0$



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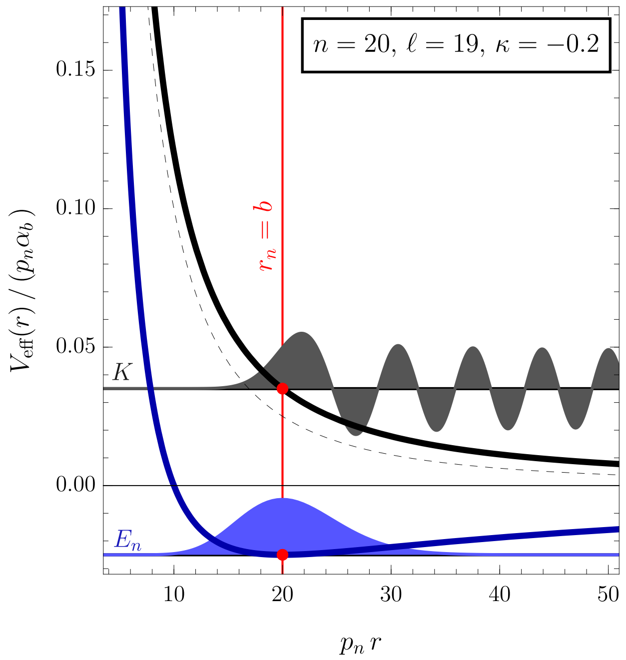

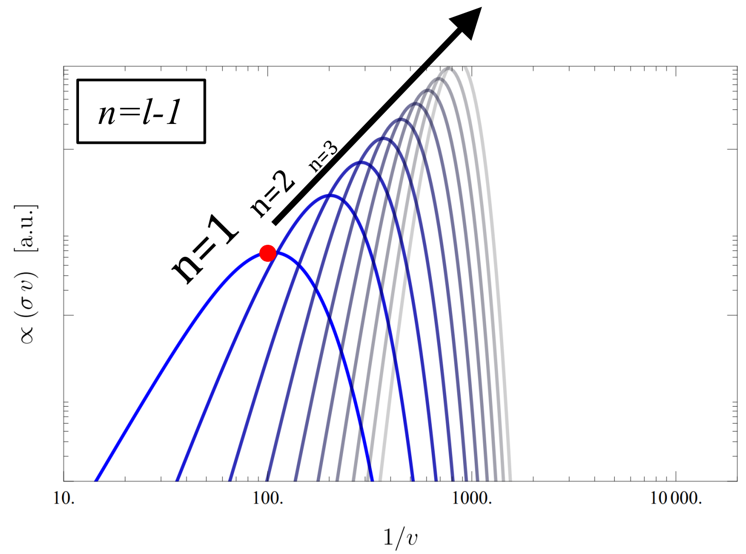

⇒ Higher n are enhanced !
BSF stronger than in the attractive case !

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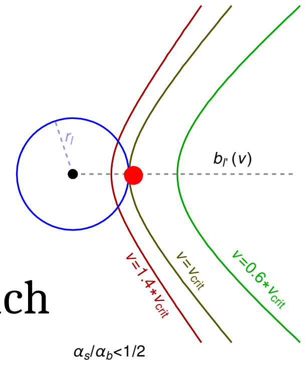
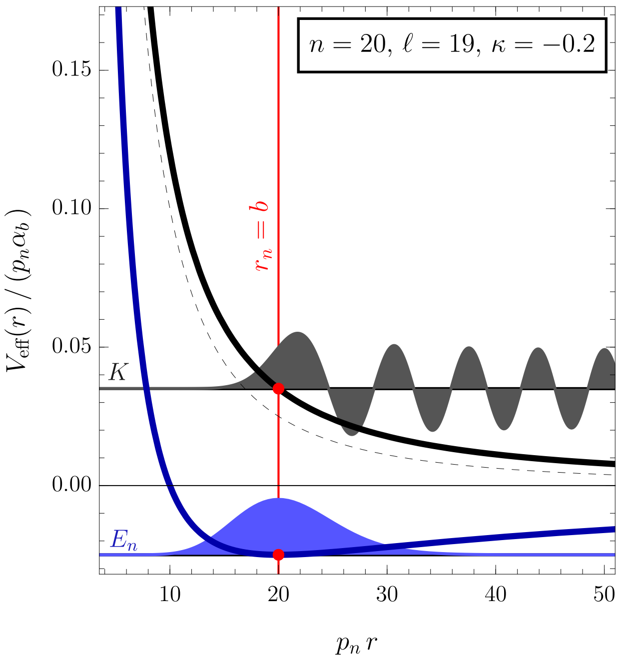
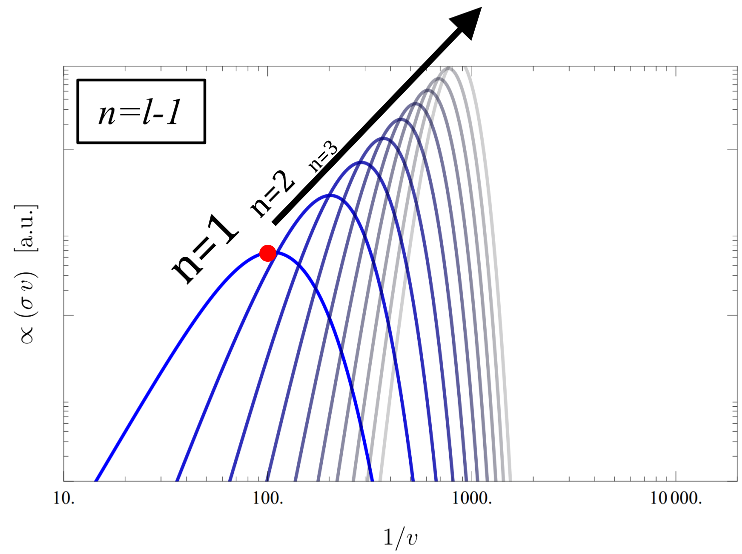


Matching condition:
classical orbits exactly touch





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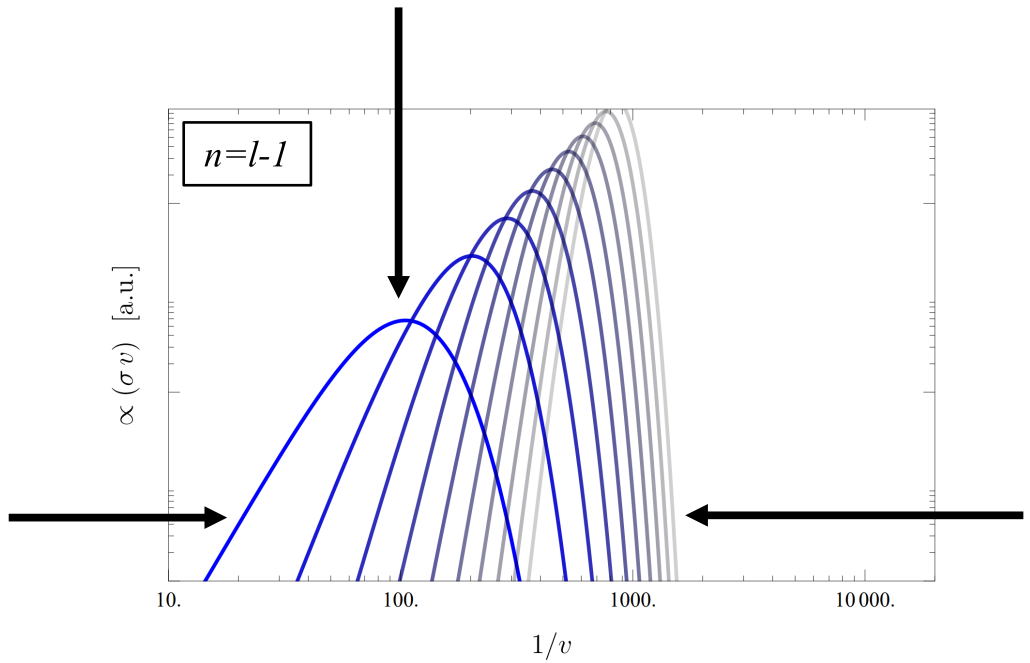
Non-abelian scenario: $\alpha_b \neq \alpha_s$

Anomalous **enhancement over the „Abelian“ case** of identical potentials.

particles fly past each other



particles bounce off each other

The enhancement grows strongly with n !

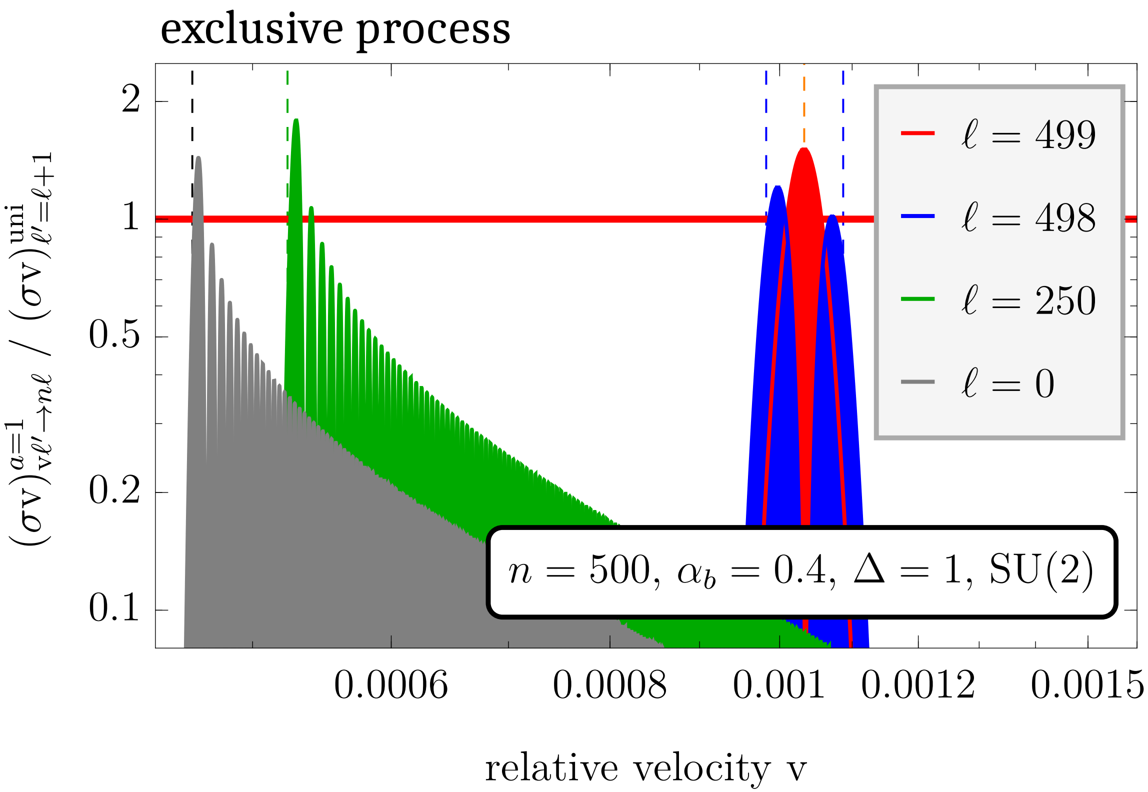
Intermezzo:

perturbative **U**nitarity **Vi**olation

$2 \longrightarrow 2$ scattering bound from S-matrix unitarity:

$$(\sigma_{2\to 2} v)_{l'} \leq (\sigma v)^{uni}_{l'} \equiv \frac{4\pi\,(2l'+1)}{m^2 v} \sim \frac{1}{v}$$

Exemplary Perturbative Unitarity Violation



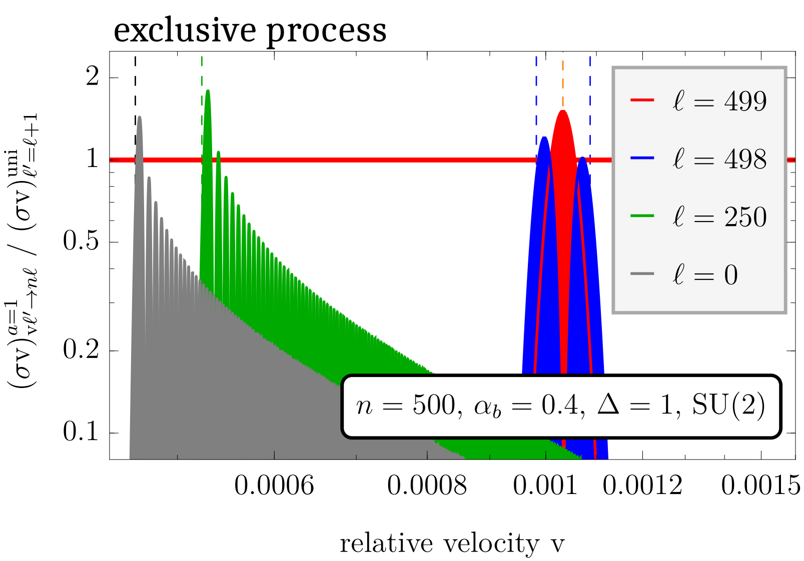
Perturbative Unitarity Violation in BSF

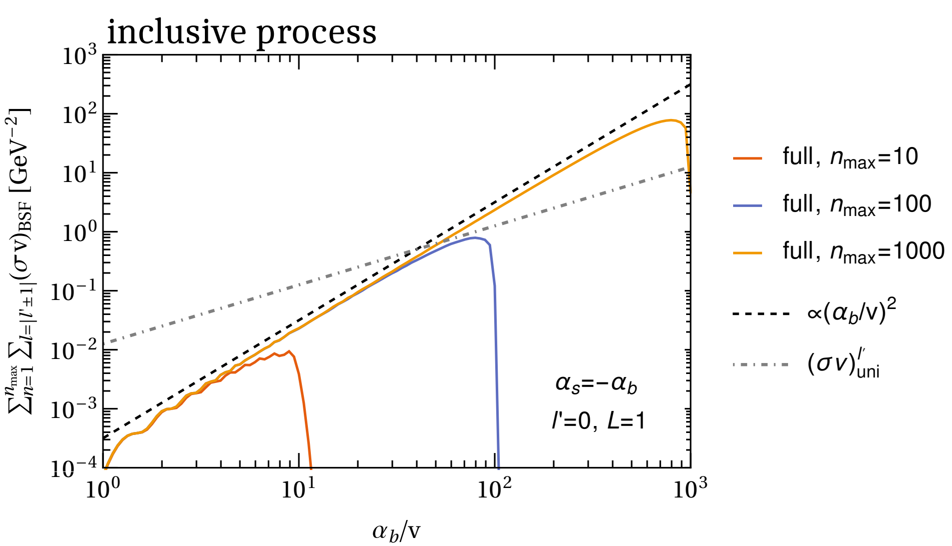
- Proven: for maximal angular momentum $l=n-1$

$$\max_{v}\left\{(\sigma v)^{l'}_{n,l=n-1}\right\} \propto \sqrt{n} \times (\sigma v)^{\mathrm{uni}}_{l'}$$

- Proven: When summed in n, there will always be UVi below some critical velocity.
- Proven: The UVi depends on the coupling ratio $\kappa<1$.

For any small coupling, high n violate unitarity !

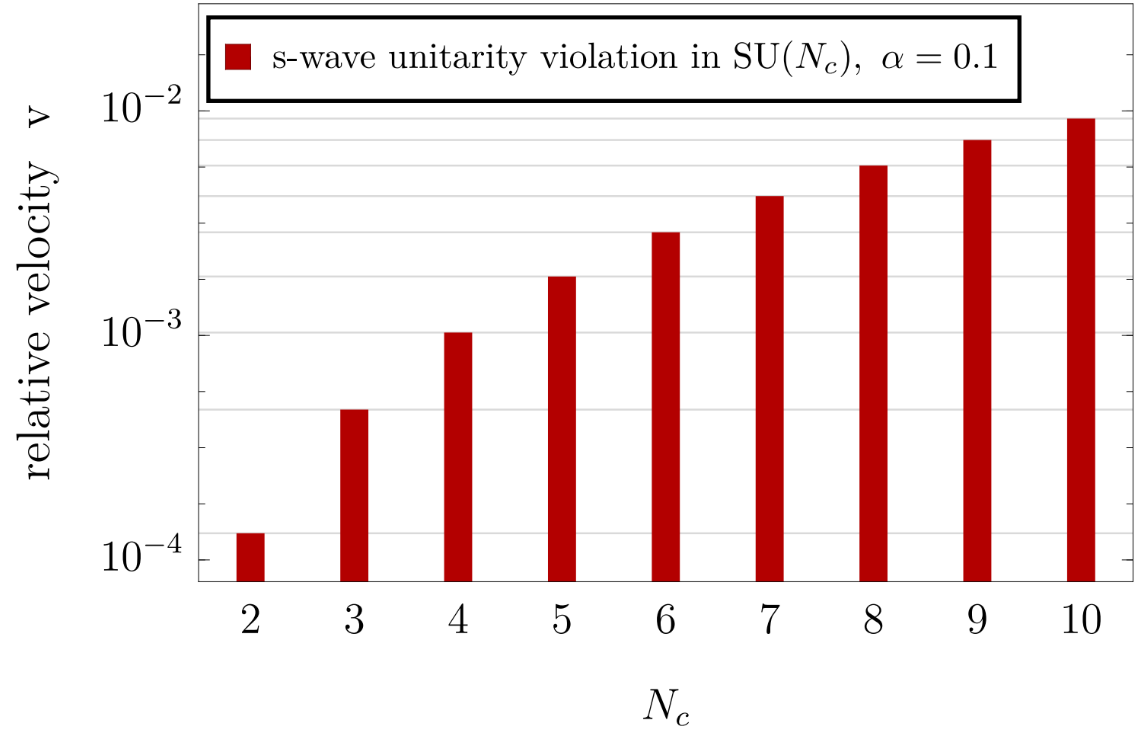




Unitarity violation in dark SU(N)

$$\kappa = \kappa(N_C) = \frac{-1}{N_C^2 - N_C}$$

⇒ for every N_C, at fixed α, find the velocity where UVi first occurs:



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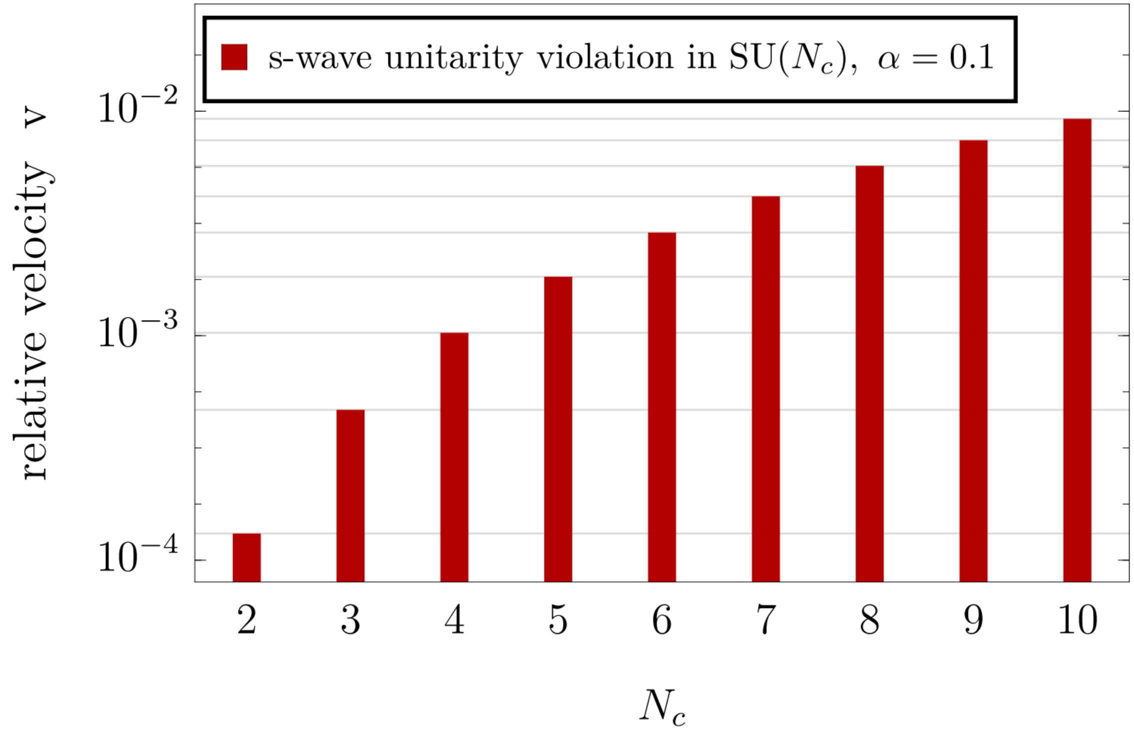
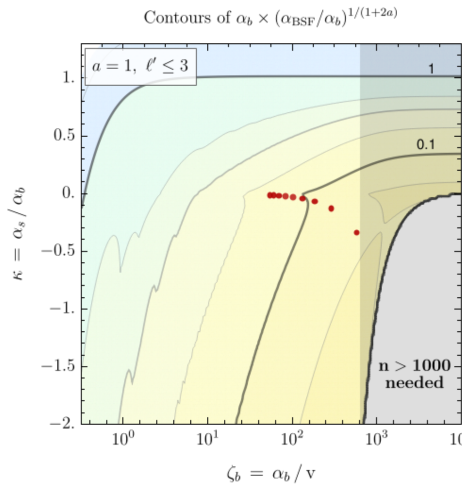




Intermezzo end.

Bound states in thermal production

1. dark-sector toy models

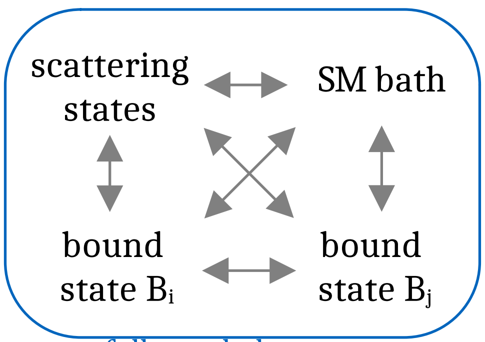
χ = DM: charged under a *new* (dark) symmetry U(1) or SU(N)

Quasi-steady state approximation

Every bound state = one species in the BME. → huge system of equations!

coupled system

$$\begin{cases} \dfrac{dY_\chi}{dx} = \dfrac{1}{3Hs}\dfrac{ds}{dx}\left[\dfrac{1}{2}\langle\sigma^{annh}_{\bar{\chi}\chi}v\rangle\left(Y_\chi^2 - Y_\chi^{eq2}\right) + \sum_i \dfrac{1}{2}\langle\sigma_{BSF,i}v\rangle\left(Y_\chi^2 - Y_\chi^{eq2}\dfrac{Y_i}{Y_i^{eq}}\right)\right] \\[2em] \dfrac{dY_i}{dx} = \dfrac{1}{3Hs}\dfrac{ds}{dx}\left[\Gamma^B_{dec}\left(Y_i - Y_i^{eq}\right) - \sum_{j\neq i}\Gamma^{j\to i}_{trans}\left(Y_j - Y_j^{eq}\dfrac{Y_i}{Y_i^{eq}}\right) + \Gamma_{ion}\left(Y_i - Y_i^{eq}\dfrac{Y_\chi^2}{Y_\chi^{eq2}}\right)\right] \end{cases}$$



full coupled system

[Redi et al.: 1702.01141]
[Petraki et al.: 2112.00042]
[Garny, Heisig: 2112.01499]

Quasi-steady state approximation

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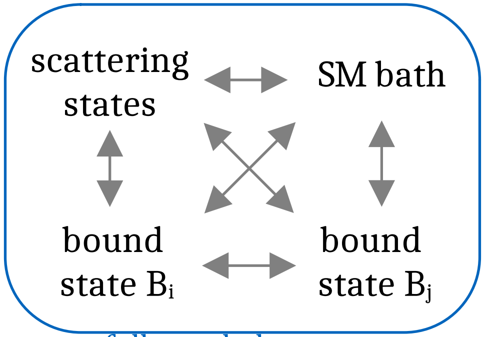
$$0 = \cancel{\frac{dY_i}{dx}} = \frac{1}{3Hs}\frac{ds}{dx}\left[\Gamma^B_{dec}\left(Y_i - Y_i^{eq}\right) - \sum_{j\neq i}\Gamma^{j\to i}_{trans}\left(Y_j - Y_j^{eq}\frac{Y_i}{Y_i^{eq}}\right) + \Gamma_{ion}\left(Y_i - Y_i^{eq}\frac{Y_\chi^2}{Y_\chi^{eq2}}\right)\right]$$

“quasi steady-state“ approximation



full coupled system

$$\frac{dY_\chi}{dx} = \frac{1}{3Hs}\frac{ds}{dx}\frac{1}{2}\underbrace{\langle\sigma v\rangle^{\mathrm{eff}}_{\mathrm{ann}}}\left(Y_\chi^2 - Y_\chi^{\mathrm{eq2}}\right)$$

<u>effective</u> annihilation cross-section

[Redi et al.: 1702.01141]
[Petraki et al.: 2112.00042]
[Garny, Heisig: 2112.01499]

Effective cross-section

$$\frac{dY_\chi}{dx} = \frac{1}{3Hs}\frac{ds}{dx}\frac{1}{2}\langle\sigma v\rangle_{\text{ann}}^{\text{eff}}\left(Y_\chi^2 - Y_\chi^{\text{eq}2}\right)$$

Includes scattering **and** bound states:

$$\langle\sigma v\rangle_{\text{ann}}^{\text{eff}} = \langle\sigma_{\chi\chi}^{\text{ann}} v\rangle + \sum_i R_i \langle\sigma_{\text{BSF,i}} v\rangle$$

direct annihilation

bound state formation

depletion efficiency $\in[0,1]$

(includes transition, ionisation & decay)

recall:

„**Critical scaling**" for freeze-out: $\langle\sigma v\rangle_{\text{ann}}^{\text{eff}} \propto \frac{1}{T} \propto x$

Effective cross-section

$$\frac{dY_\chi}{dx} = \frac{1}{3Hs}\frac{ds}{dx}\frac{1}{2}\langle\sigma v\rangle^{\text{eff}}_{\text{ann}}\left(Y_\chi^2 - Y_\chi^{\text{eq}2}\right)$$

Includes scattering **and** bound states:

$$\langle\sigma v\rangle^{\text{eff}}_{\text{ann}} = \langle\sigma^{\text{ann}}_{\chi\chi} v\rangle + \sum_i R_i \langle\sigma_{\text{BSF,i}} v\rangle$$

direct annihilation

bound state formation

depletion efficiency $\in[0,1]$

(includes transition ✔, ionisation ✔ & decay ✔)

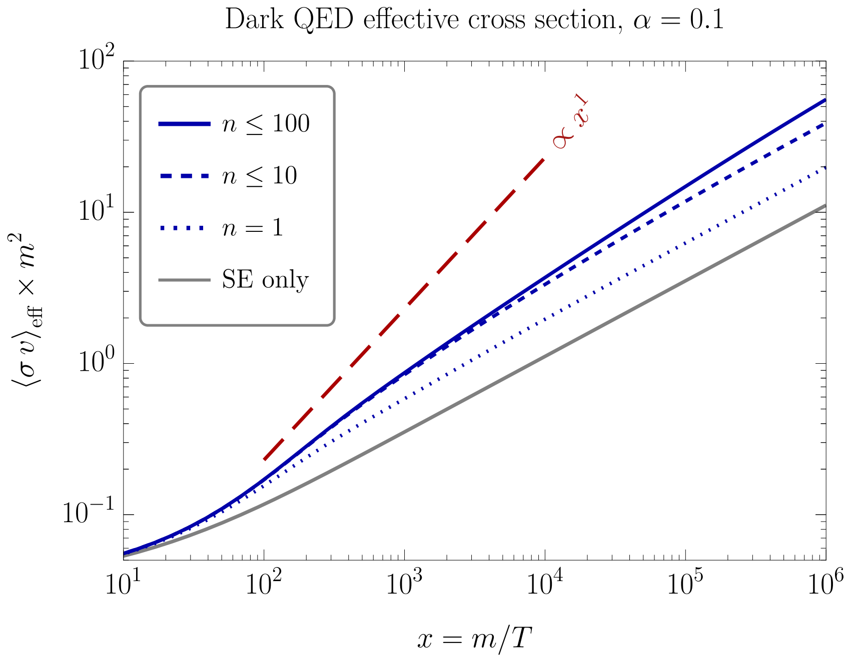
similar *Milne* *known*

recall:

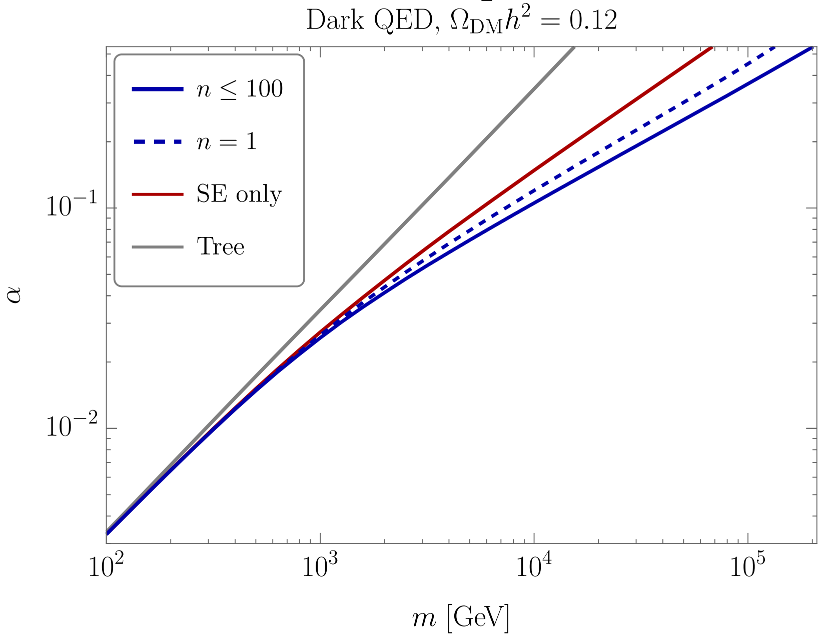
„**Critical scaling**" for freeze-out: $\langle\sigma v\rangle^{\text{eff}}_{\text{ann}} \propto \frac{1}{T} \propto x$

Effective cross-section: **dark U(1)**

effective cross-section



mass-coupling relation
from thermal production

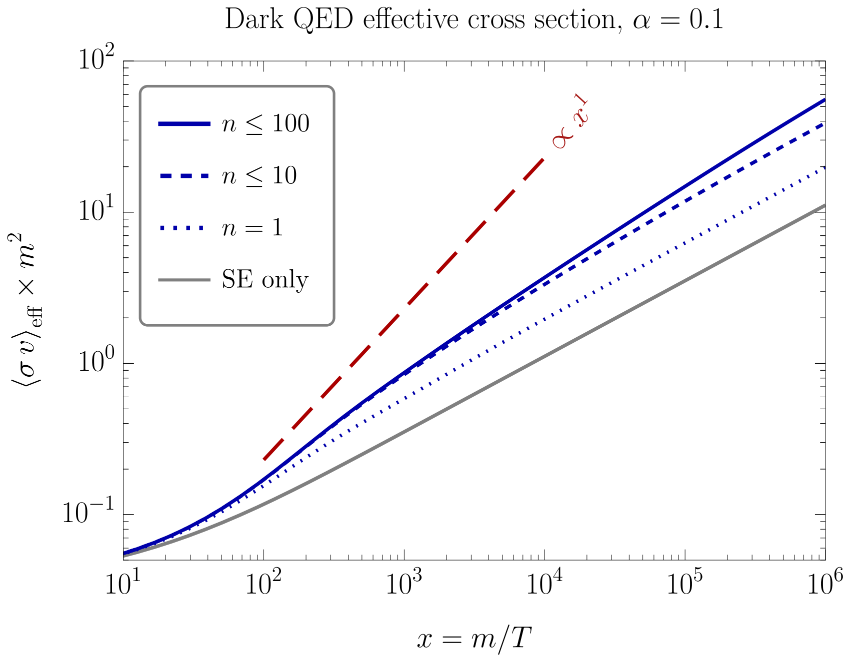


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

Effective cross-section: **U(1)** vs **SU(N)**

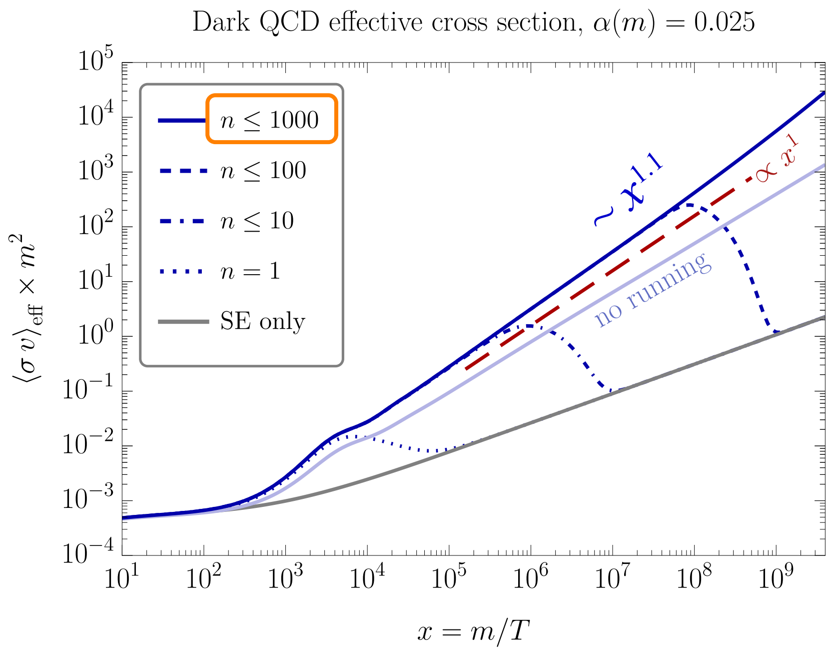
Includes bound-to-bound transitions.

dark U(1)



No bound-to-bound transitions allowed.

dark SU(3)



Super-critical scaling in dark SU(3): no chemical decoupling !

→ contact us for the code to do this.

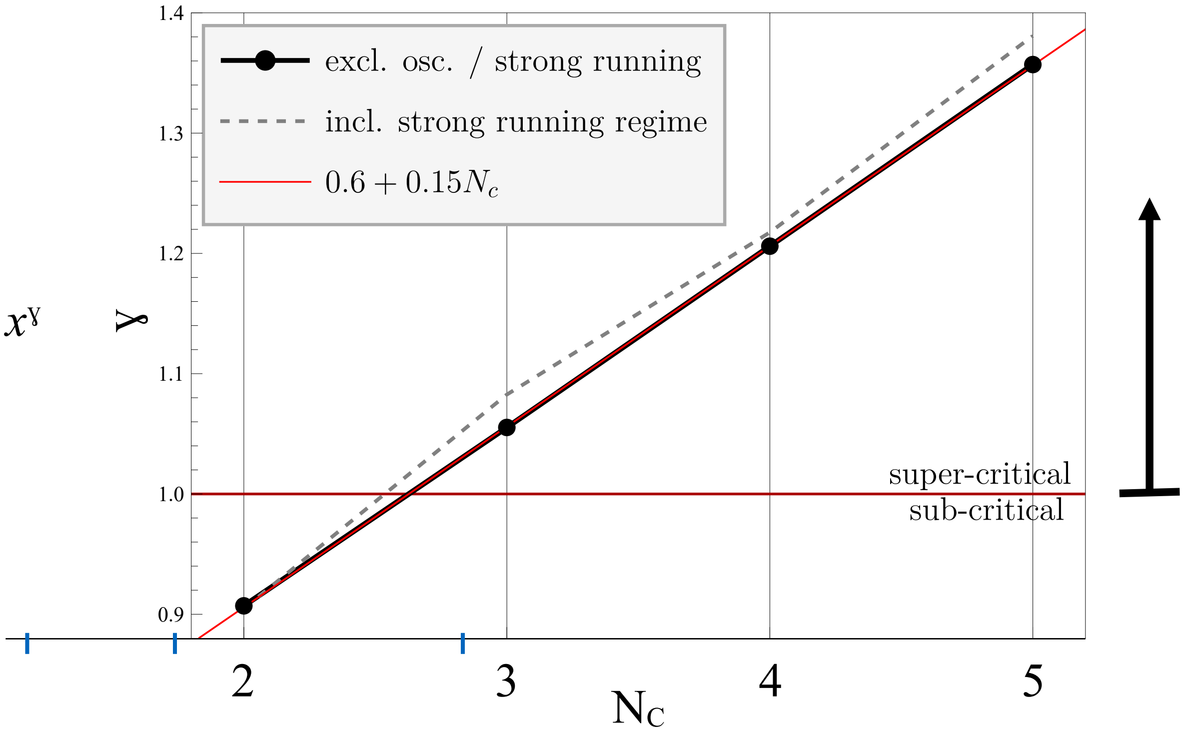
[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

Super-critical scaling in dark SU(N_C)

→ SU(N_C) with no light particles, incl 1-loop running.

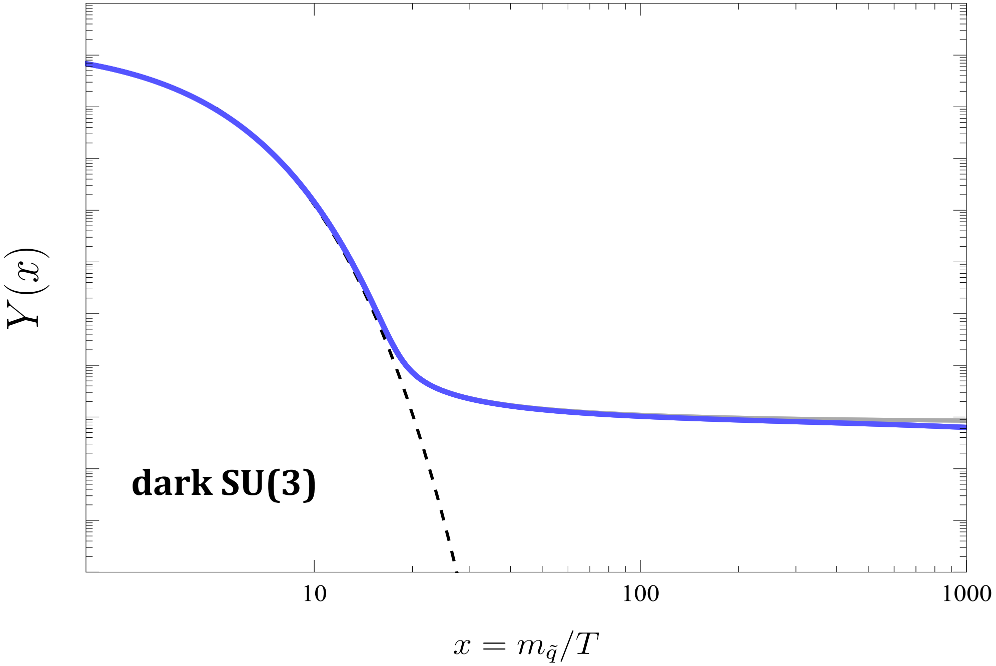
Temperature dependence of the effective cross-section with N_C at low T (large x).

scaling power x^{γ}

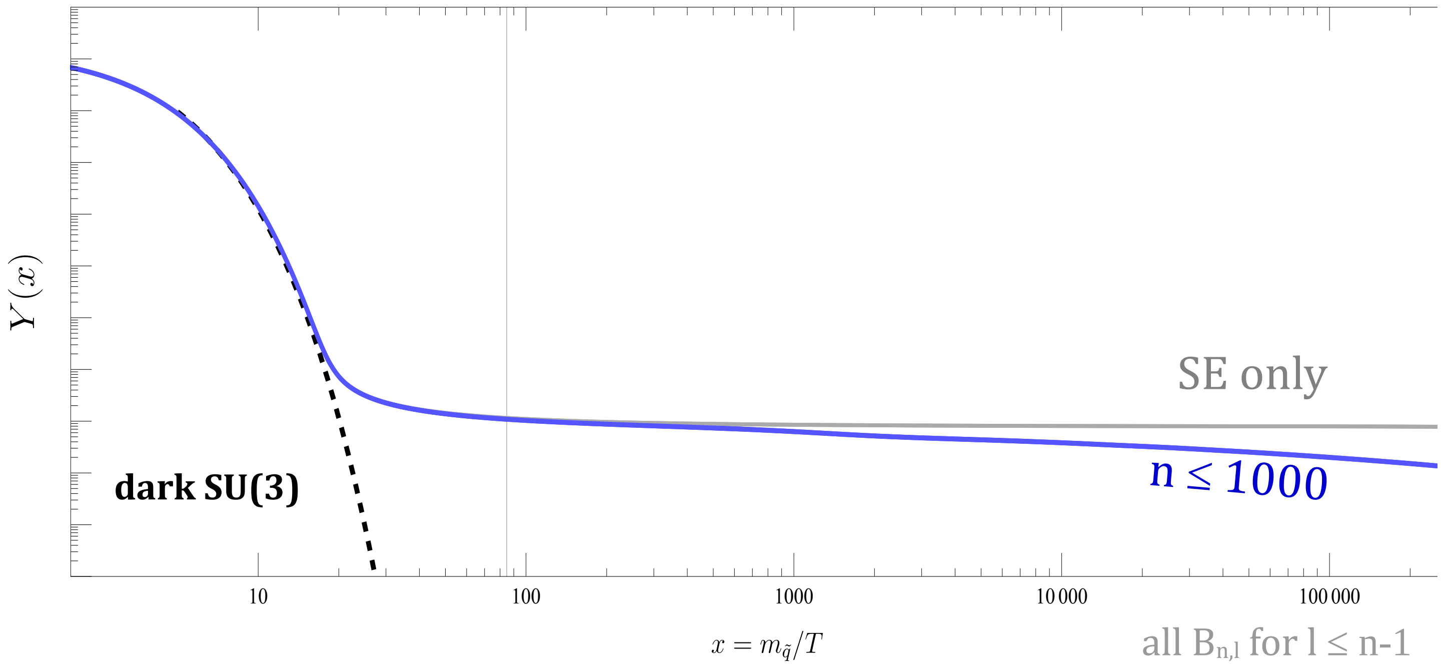


no freeze-out

„Freeze-Out“ including BSF – dark SU(3)



„Freeze-Out“ including BSF – dark SU(3)



all $B_{n,l}$ for l ≤ n-1
(~ 1 million bound states)

Bound states in thermal production

2. t-channel superWIMP

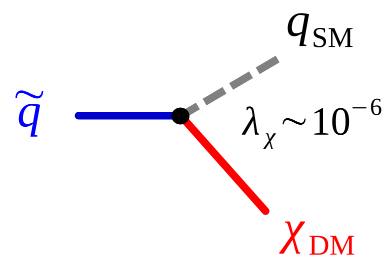
χ = DM: complete gauge singlet

$\tilde{q}$ = mediator: heavy colored & charged scalar

Including transitions: „add an additional U(1)“

Assume a <u>colored</u> **&** <u>charged</u> heavy scalar: $\tilde{q} \in (\mathbf{3},\mathbf{1})_{1/3}$ („b-squark“)

$\mathscr{L} \supset \lambda_\chi \, \bar{q}\,\tilde{q}\,\chi + \text{h.c.}$



→ QCD dominates the potential and BSF $(\tilde{q}^\dagger \tilde{q})^{[\mathbf{8}]} \rightarrow B_i^{[\mathbf{1}]} + g$

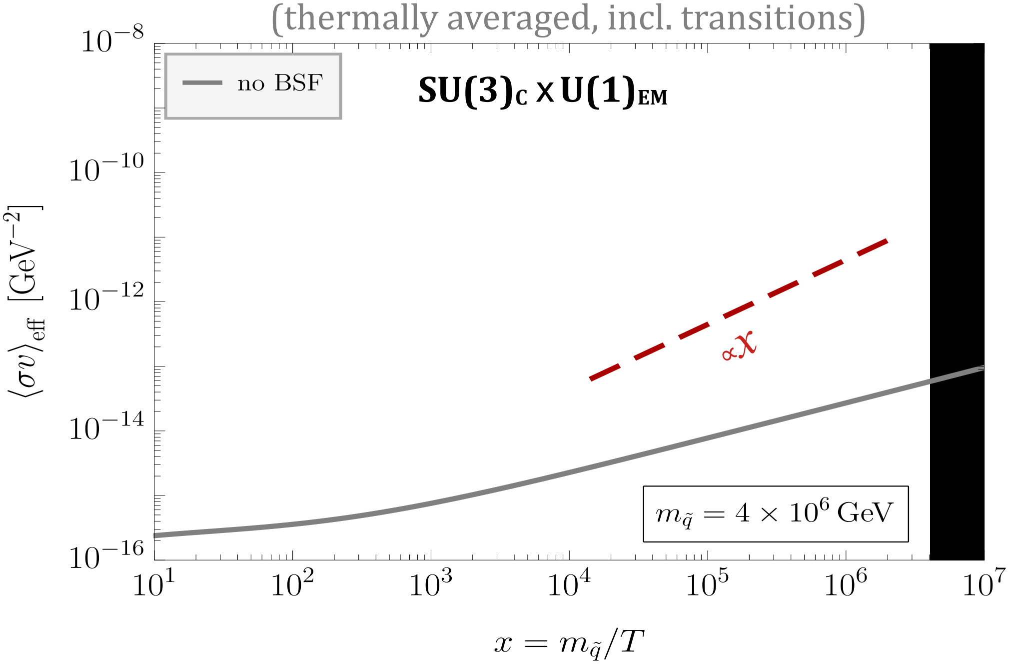
$$\kappa \equiv \frac{V_{[\mathbf{8}]}}{V_{[\mathbf{1}]}} = -\frac{1}{8}$$

→ QED allows transitions $B_i \rightarrow B_j$

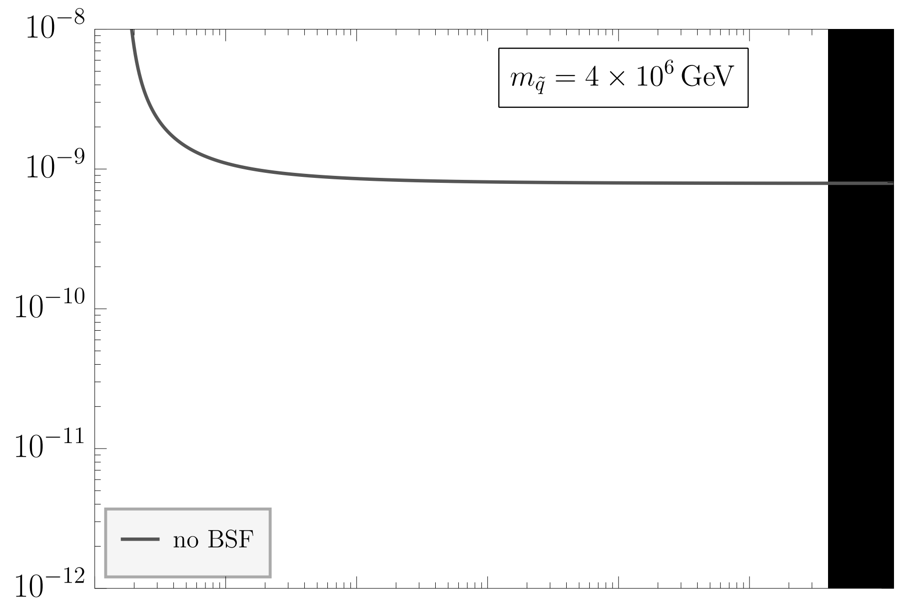
Abundance without excited states

effective cross section

(thermally averaged, incl. transitions)



mediator Yield

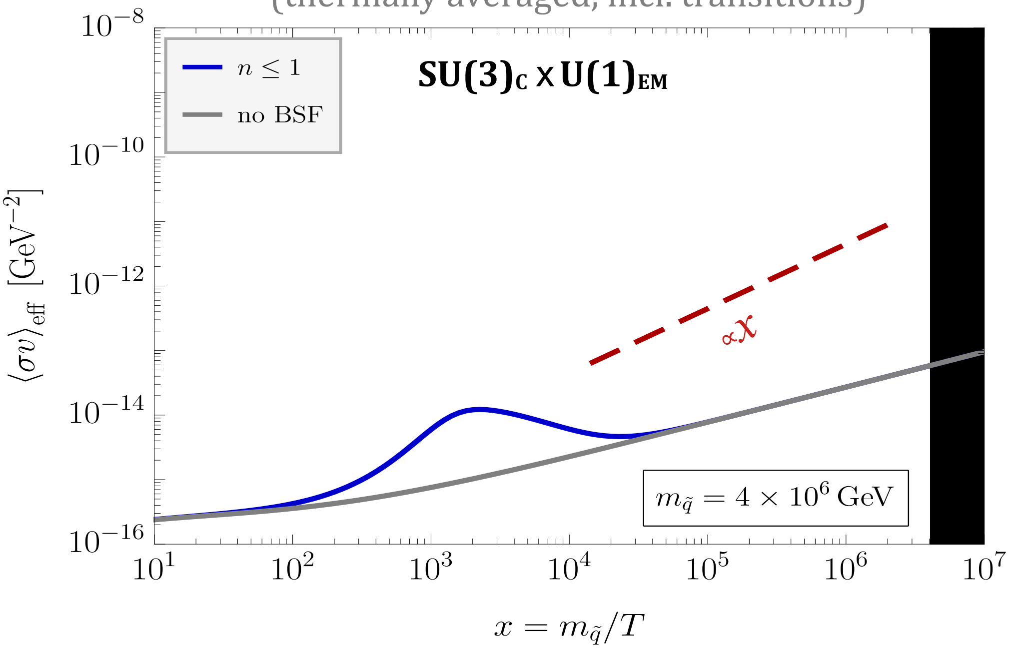


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

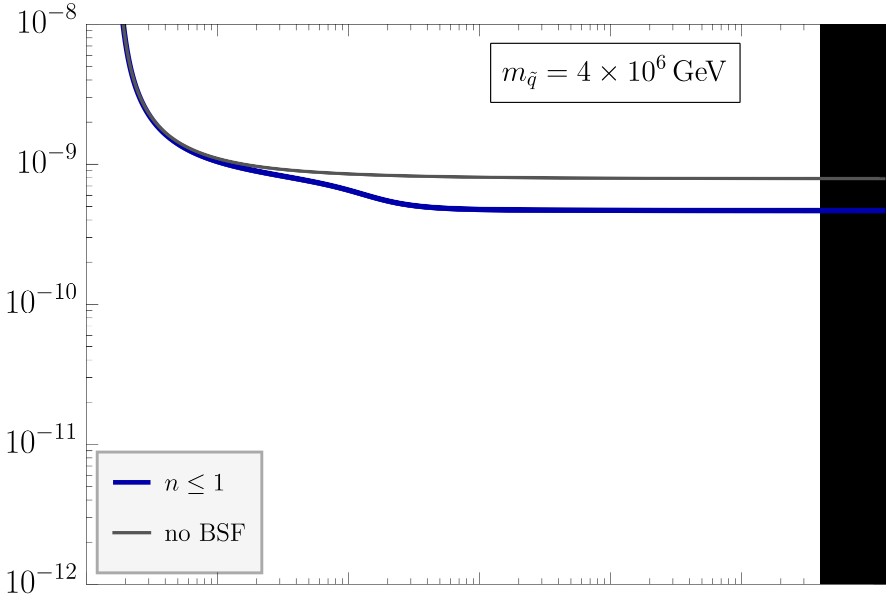
Abundance with an excited state (n=1)

cross section

(thermally averaged, incl. transitions)



mediator Yield

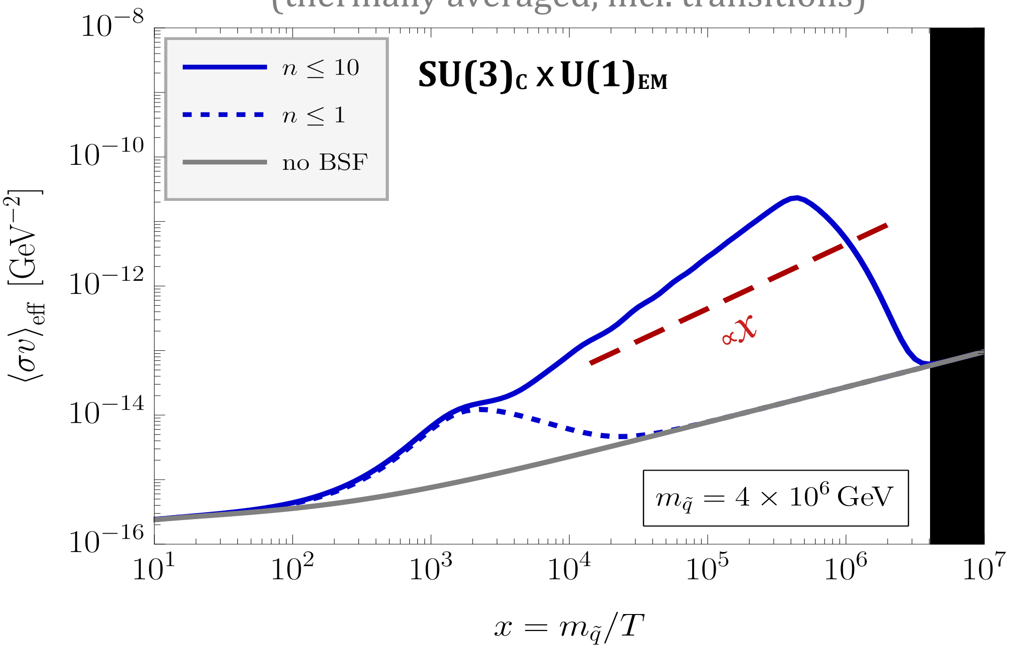


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

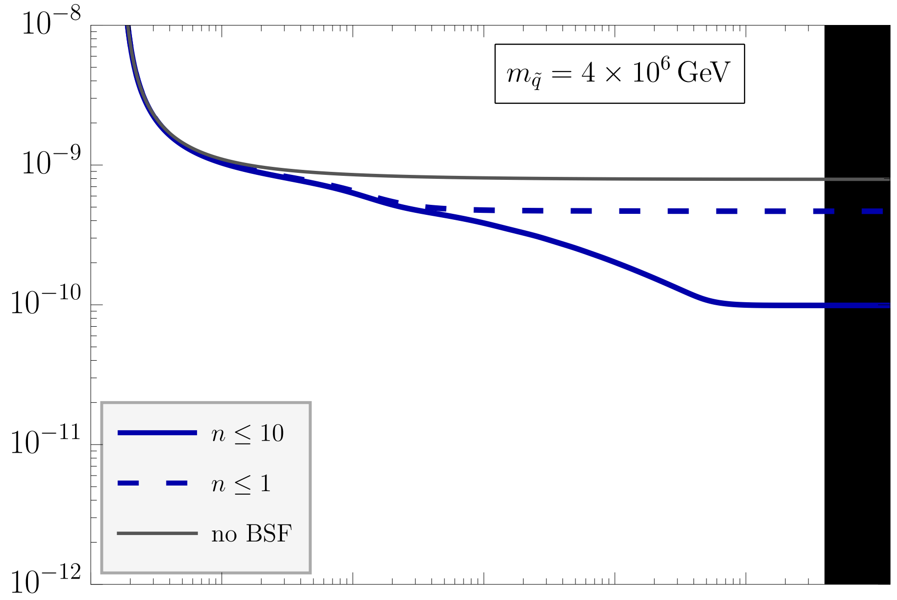
Abundance with some excited states

cross section

(thermally averaged, incl. transitions)



mediator Yield

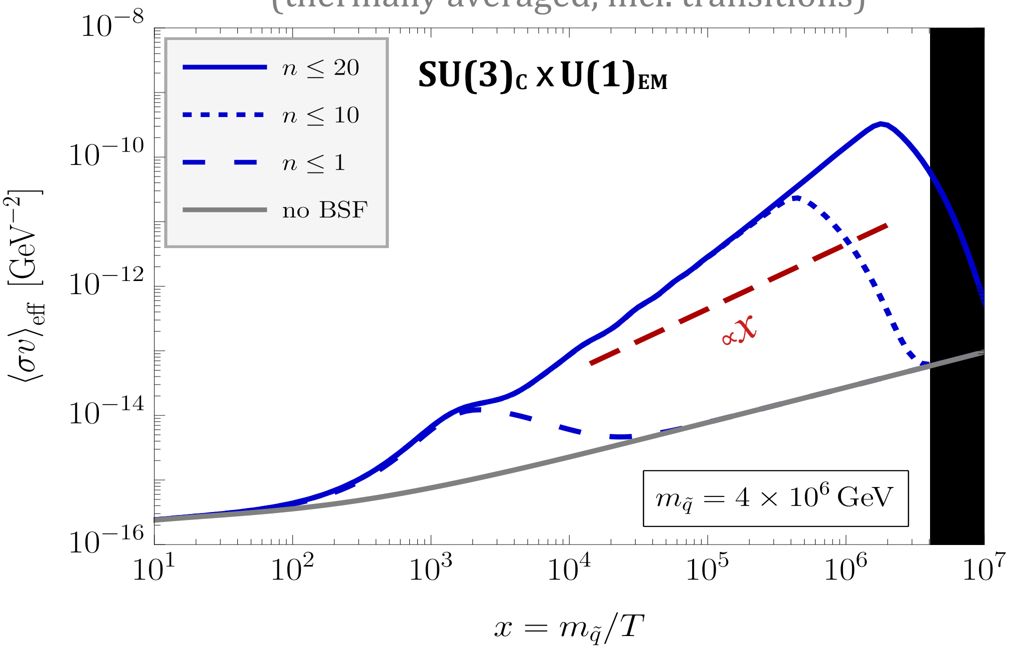


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

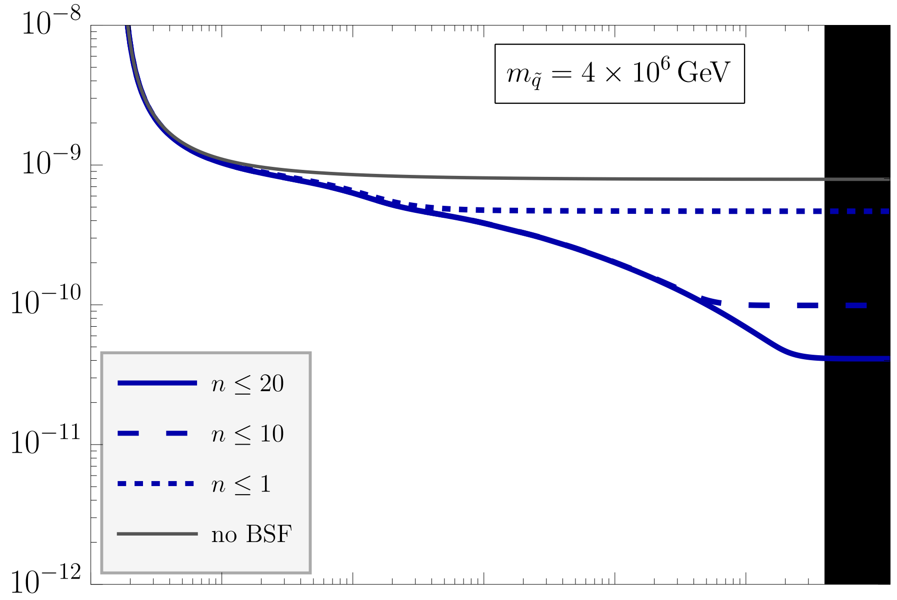
Abundance with many excited states

cross section

(thermally averaged, incl. transitions)



mediator Yield

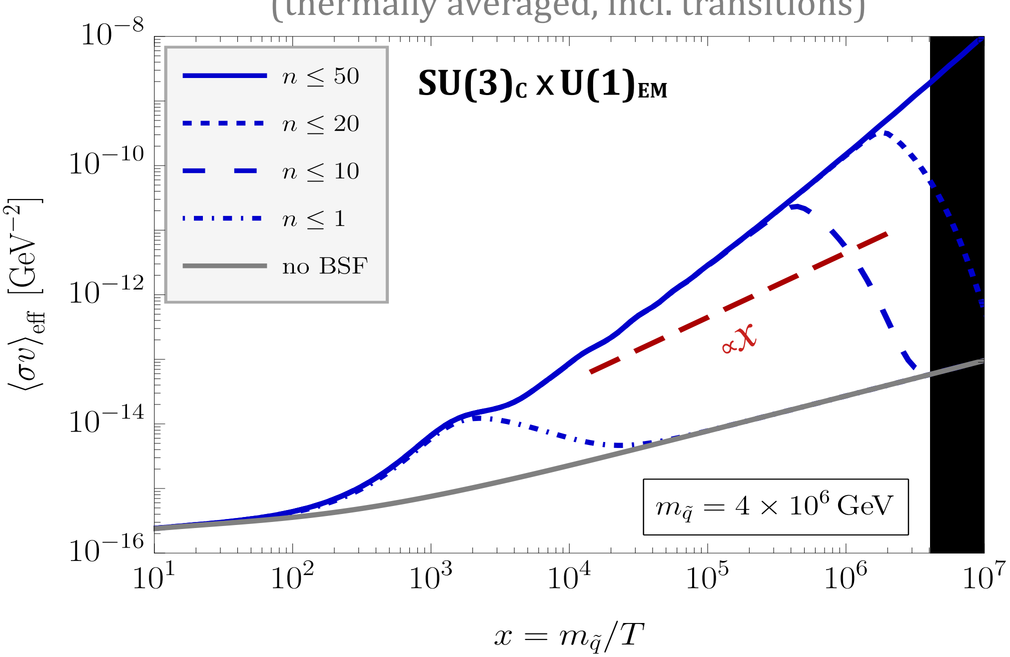


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

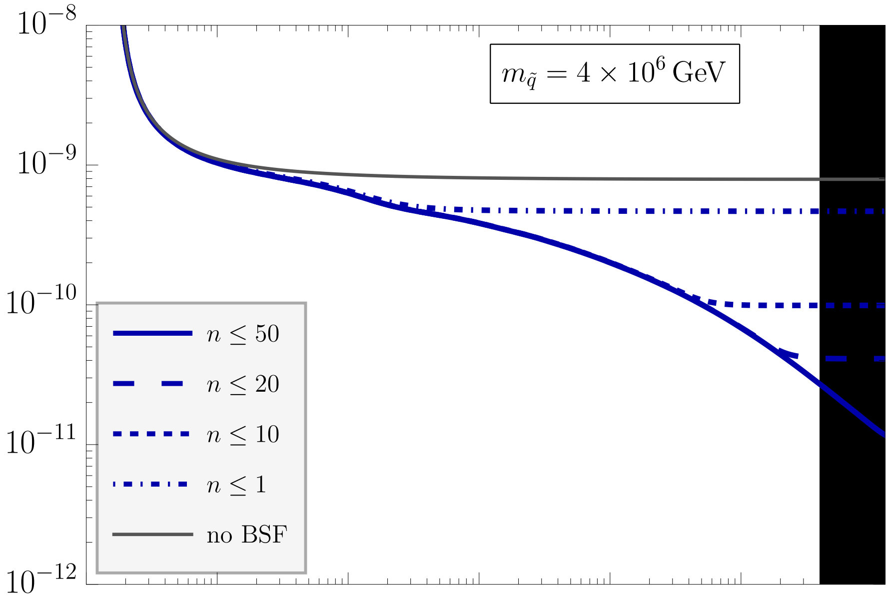
Abundance with enough excited states

effective cross section

(thermally averaged, incl. transitions)



mediator Yield

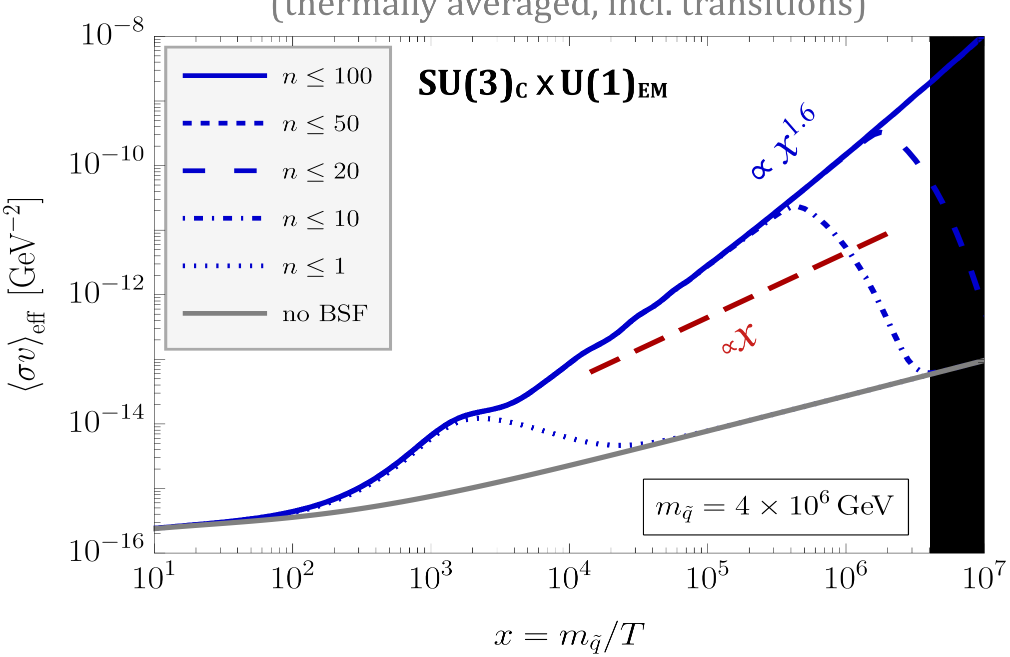


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

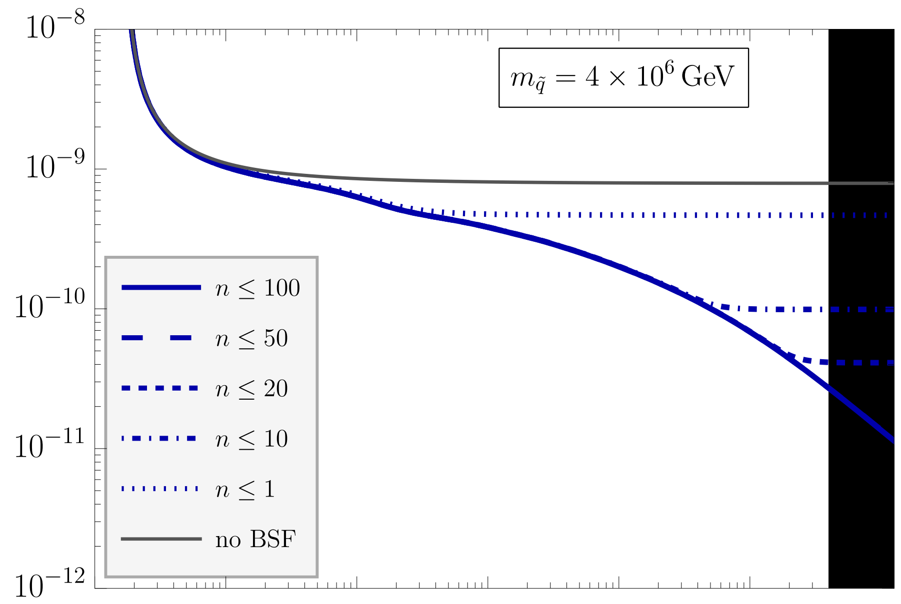
Abundance with too many excited states

- Cross-section converged in the pertubative regime.
- Bound-to-bound transitions give strong enhancement: $x^{1.1} \longrightarrow x^{1.6}$

effective cross section
(thermally averaged, incl. transitions)



mediator Yield



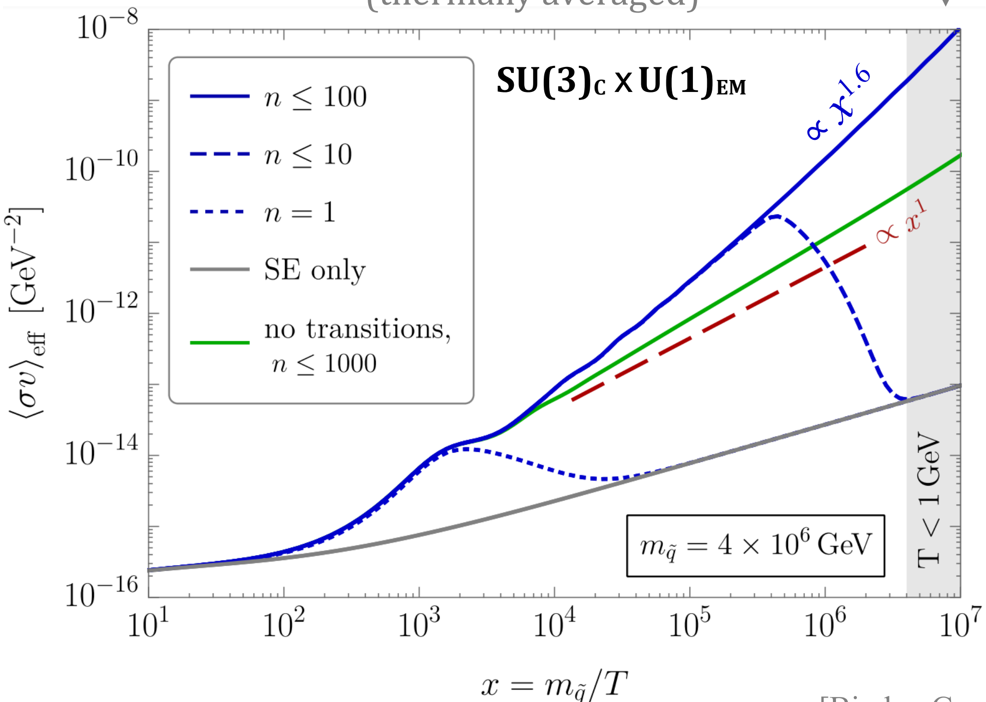
[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

Eternal annihilation – how does it end?

- Respect unitarity bounds ✔
- Avoid non-perturbative regime α ~ 1 ?!

confinement phase

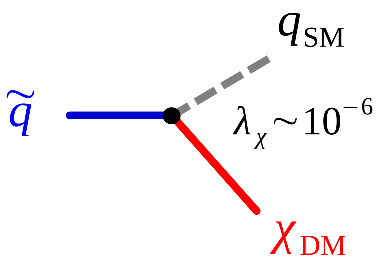
cross section
(thermally averaged)

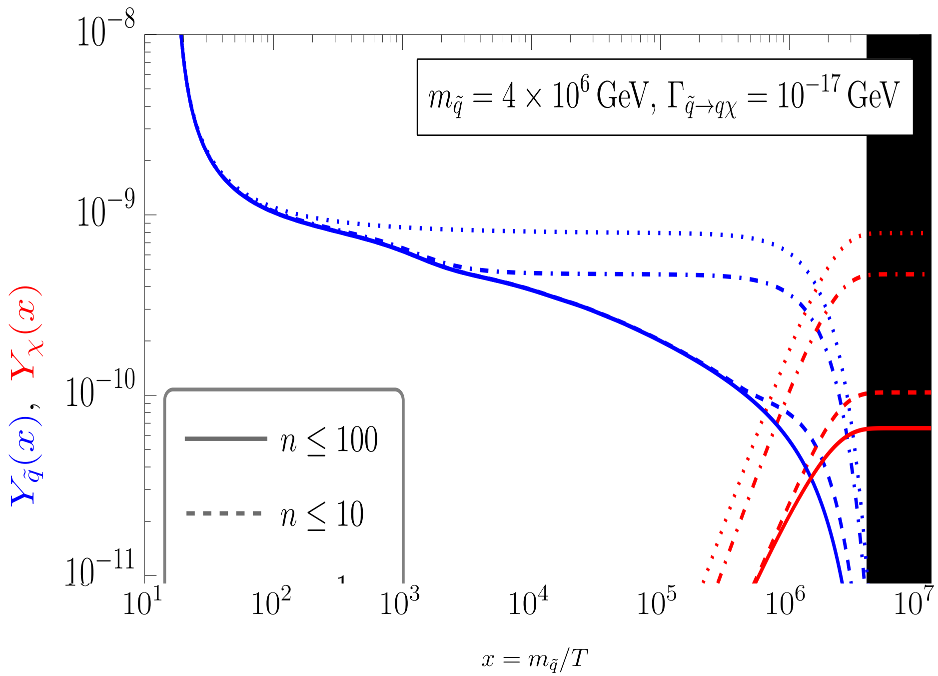


[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

superWIMP production with bound states

Adds a finite life-time: χ = DM,
$\tilde{q}$ = „t-channel mediator“

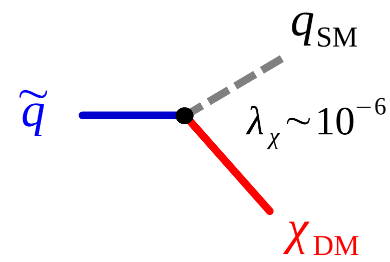


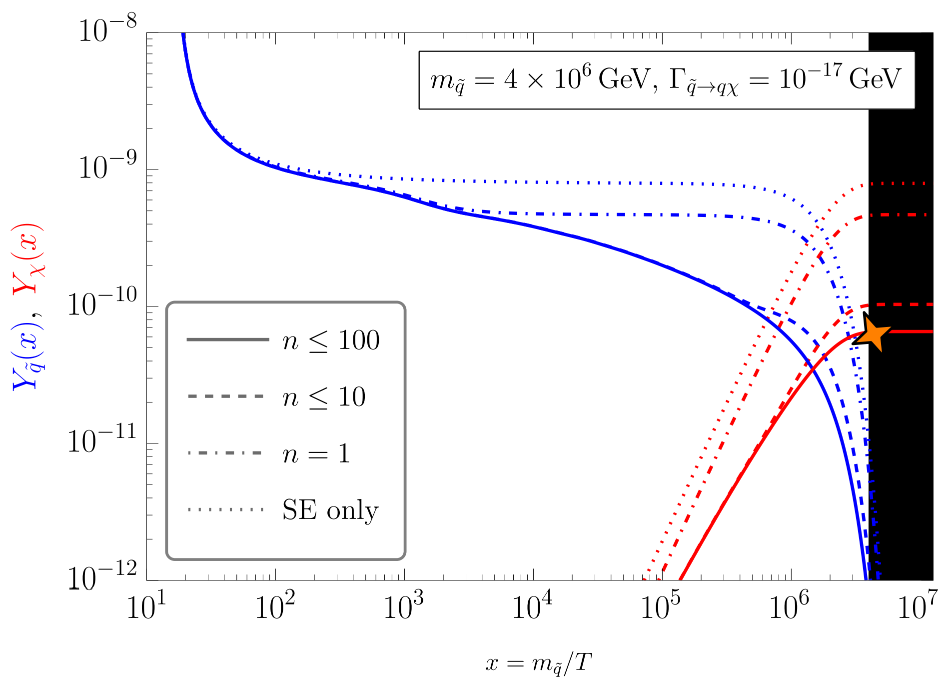


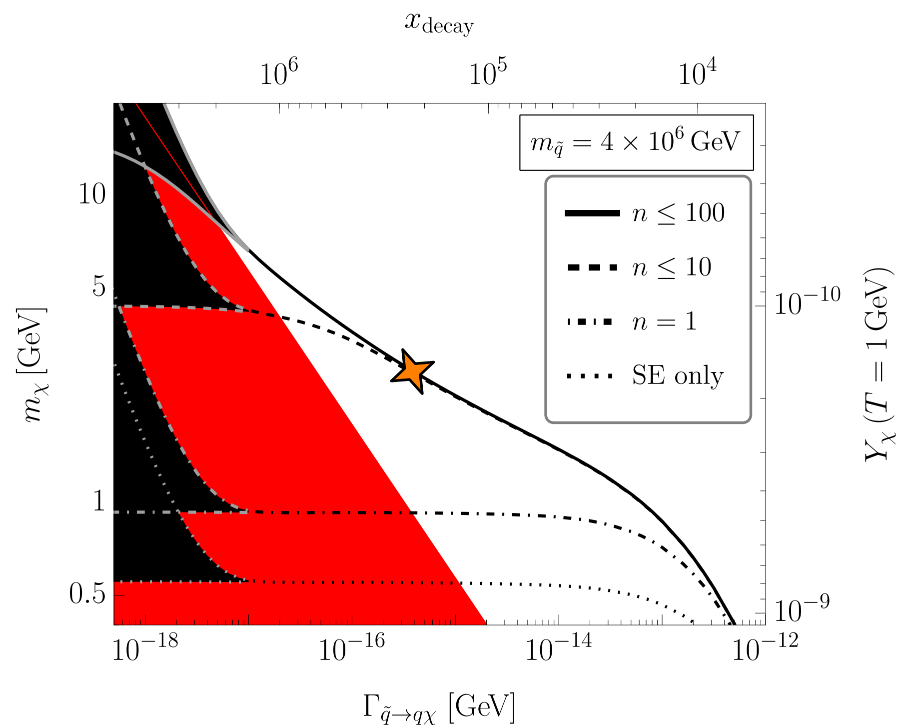
[Garny, Heisig: 2112.01449]
[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

superWIMP production with bound states

Adds a finite life-time: χ = DM,
$\tilde{q}$ = „t-channel mediator“







[Garny, Heisig: 2112.01449]
[Binder, Garny, Heisig, **SL**, Urban: 2308.01336]

Summary

Importance of **bound states** for dark matter:

1. Repulsive potentials show **enhanced** BSF.
2. Unitarity is systematically violated in BSF at leading order.
3. Dominant effect at small temperatures.
4. Non-Abelian excited states can not be neglected.
5. Large effects from transitions between bound states.
6. Bound state formation can source eternal depletion.

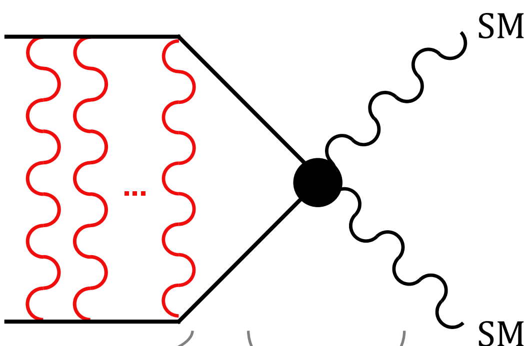
Using intuition from QED is dangerous
& bound states are exciting!

Backup Slides

Sommerfeld enhancement

Light mediators form long-range potentials

$\Rightarrow$ strong enhancement at small v_{rel}



Coulomb wavefunction $|\Psi(0)|^2$

(solve Schrödinger equation)

„Sommerfeld Effect"

Born cross-section

$$(\sigma v) \sim \frac{1}{v_{\text{rel}}} \cdot (\sigma v)_{\text{Born}}$$

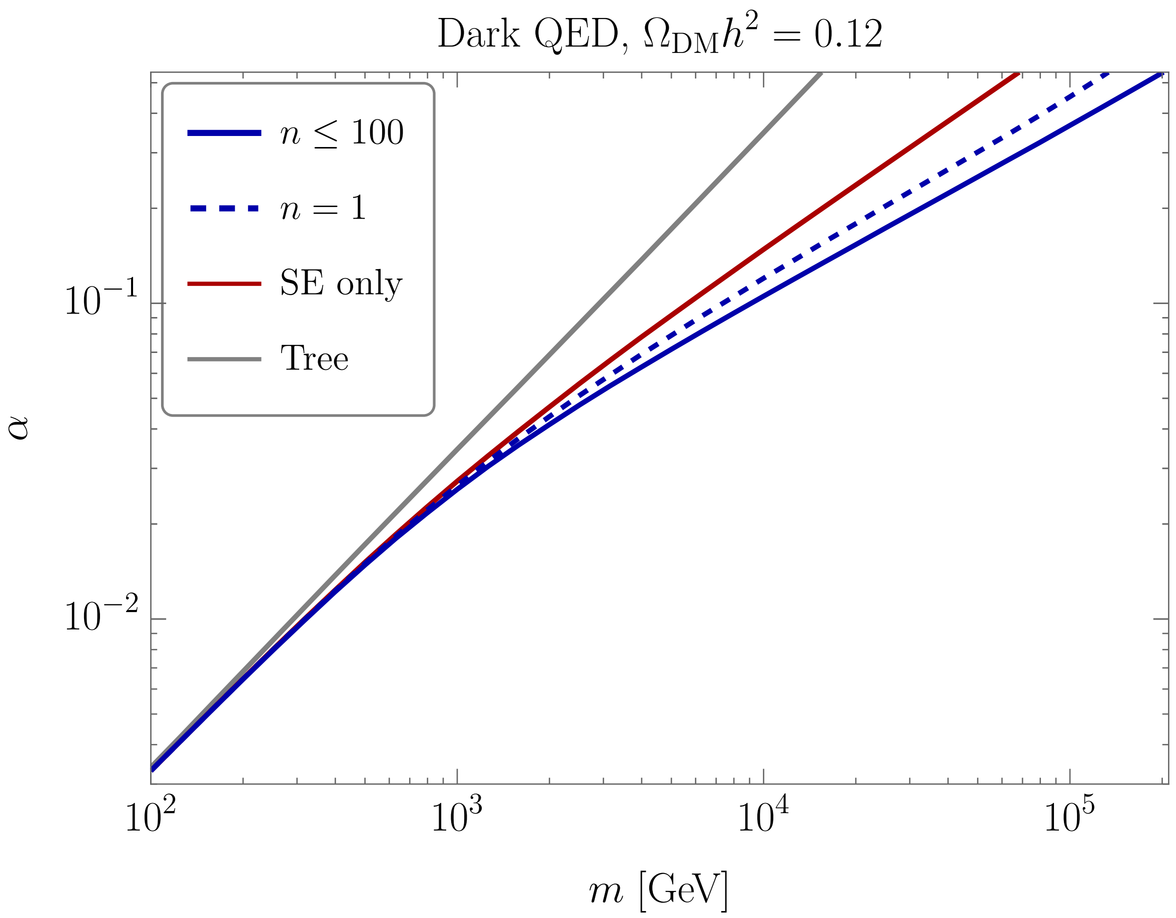
$$\langle \sigma v \rangle \sim T^{-1/2}$$

$$\gamma = \frac{1}{2}$$

✔ freeze out.

dark QED coupling strength

Unitarity bounds on the value of minimal dark QED.



Effective cross-sectioion – definitions

$$\langle \sigma v \rangle_{\text{ann}}^{\text{eff}} = \langle \sigma_{\chi\chi}^{\text{ann}} v \rangle + \sum_i R_i \langle \sigma_{\text{BSF,i}} v \rangle$$

<u>no transitions</u>:
$$R_i = \frac{\Gamma_i^{\text{decay}}}{\Gamma_i^{\text{tot}}} = 1 - \frac{\Gamma_i^{\text{ion}}}{\Gamma_i^{\text{tot}}}$$

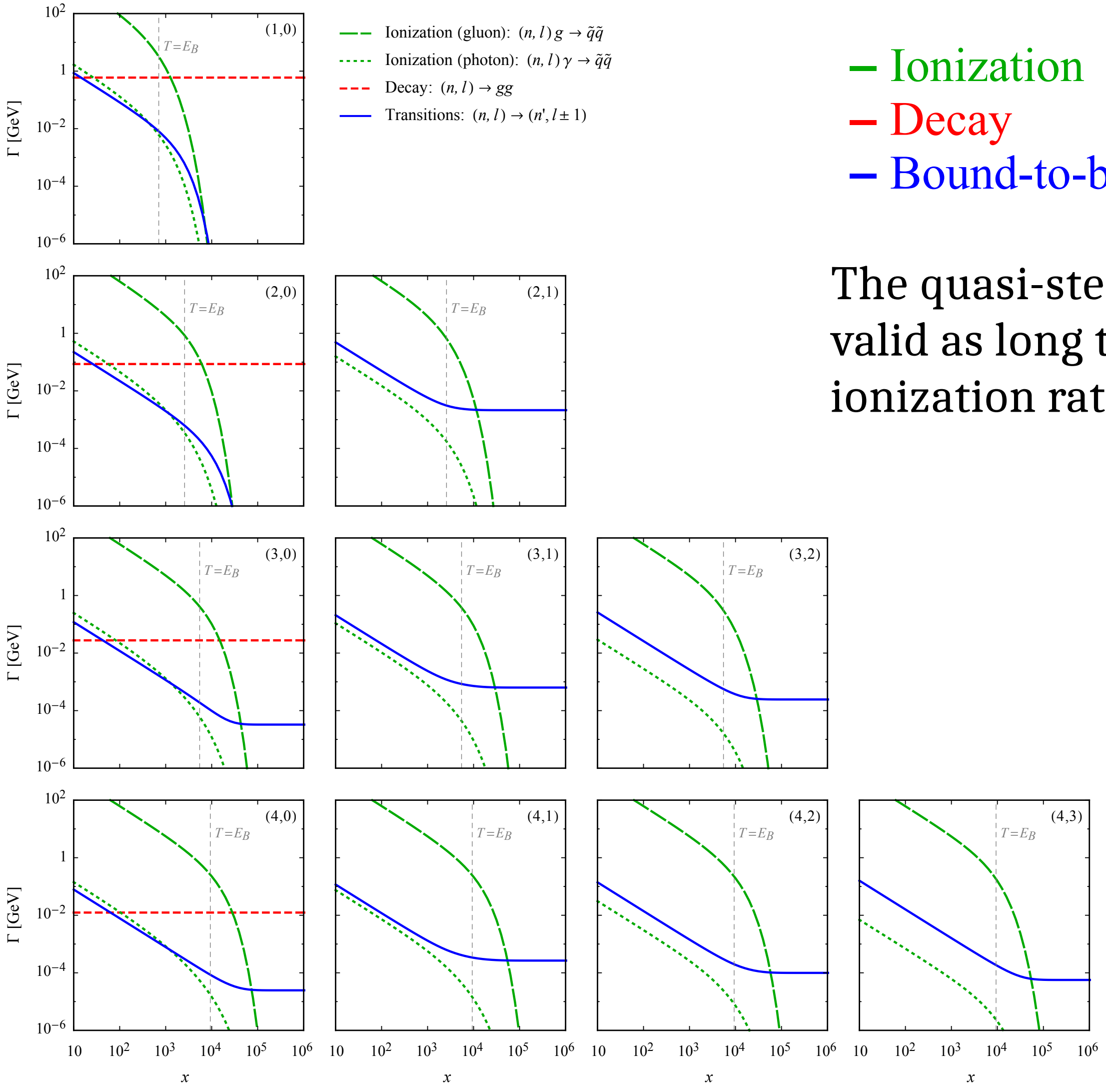
Realized in pure non-abelian interactions. (for fundamental constituents)

<u>with transitions</u>:
$$\vec{R} = 1 - M^{-1} \cdot \frac{\vec{\Gamma}^{\text{ion}}}{\vec{\Gamma}^{\text{tot}}}$$

$$M_{ij} = \delta_{ij} - \frac{\Gamma_{i \to j}^{\text{trans}}}{\Gamma_j^{\text{tot}}}$$ i, j denote bound states {n,l}

M_{ij} has $\sim O(n_{\text{max}})^3$ non-trivial entries.

Various interaction rates of bound states

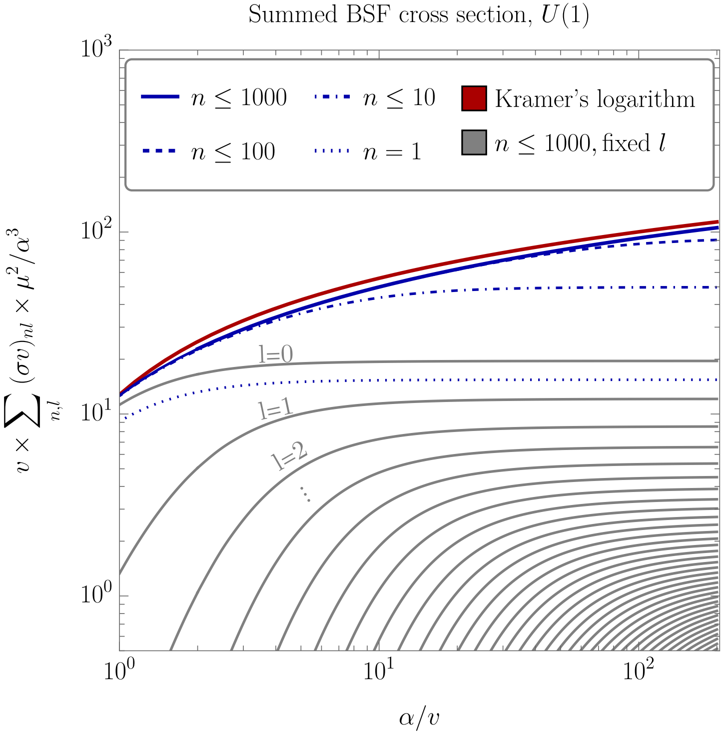


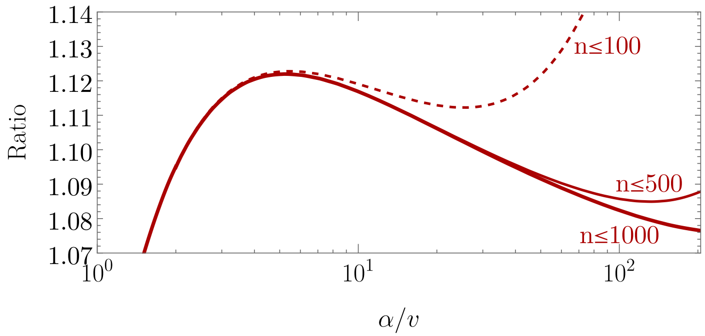
– Ionization
– Decay
– Bound-to-bound transition

The quasi-steady state assumption remains valid as long the decay, transition* or ionization rate of a bound state is large.

*The efficiently transitioning network of bound states must at some point efficiently couple by other means.

Partial wave unitarity in U(1)





Abelian BSF via dipole: $l' \rightarrow (n, l) + \gamma$

Summing all n, l:

scales as log(v)/v

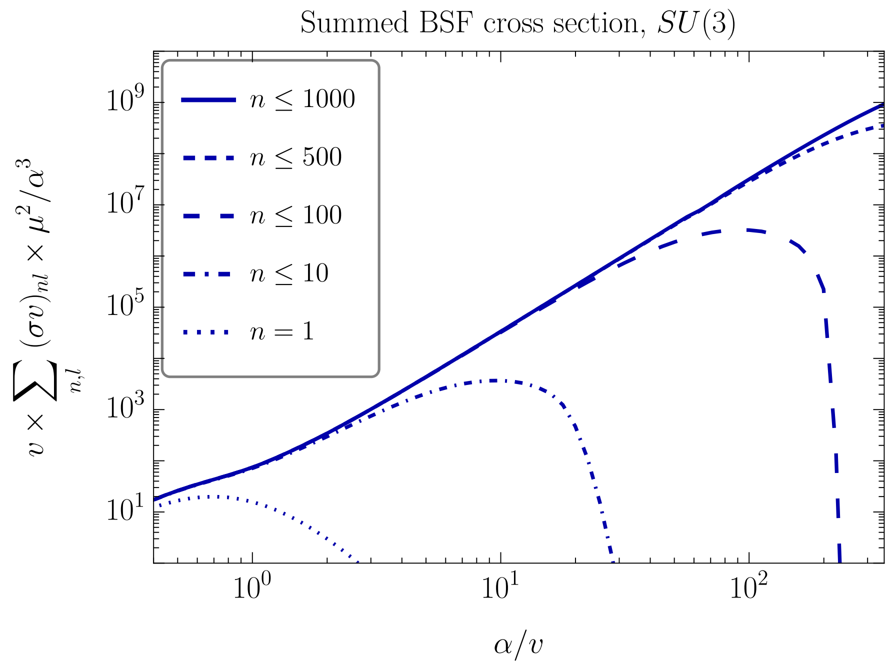
Summing all n with l (l') fixed:

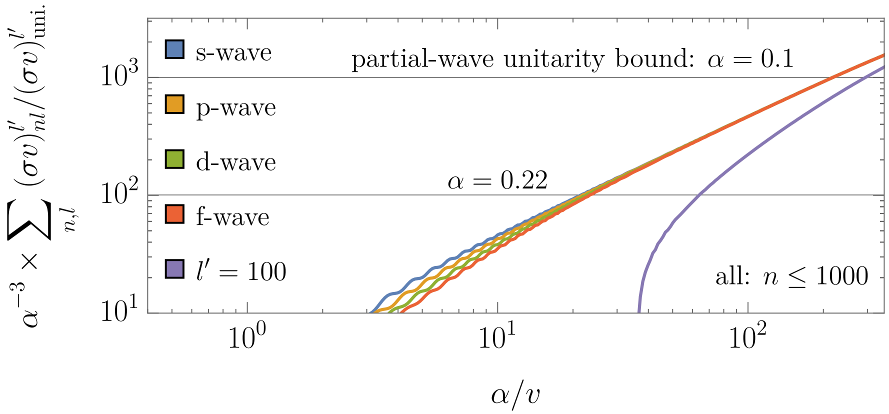
- scales as 1/v
- unitarity respected at small α

Reproduces Kramer's logarithm

(but requires very large n)

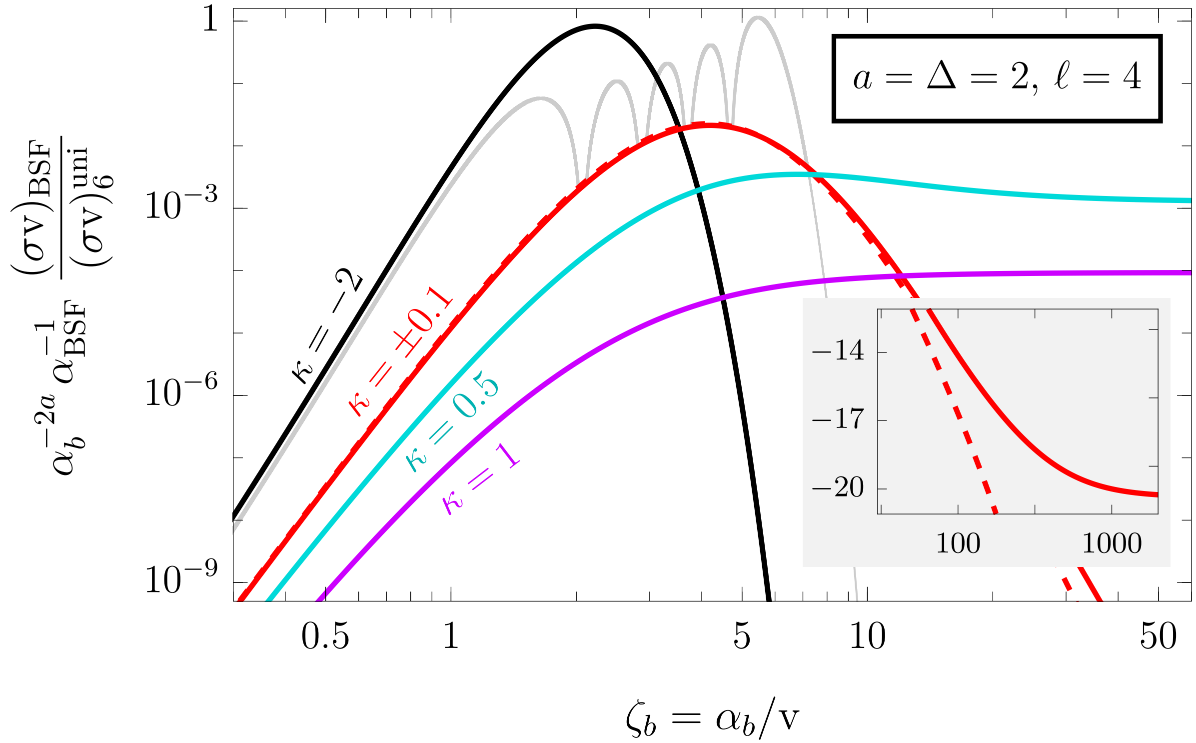
Systematic perturbative UVi in SU(3)





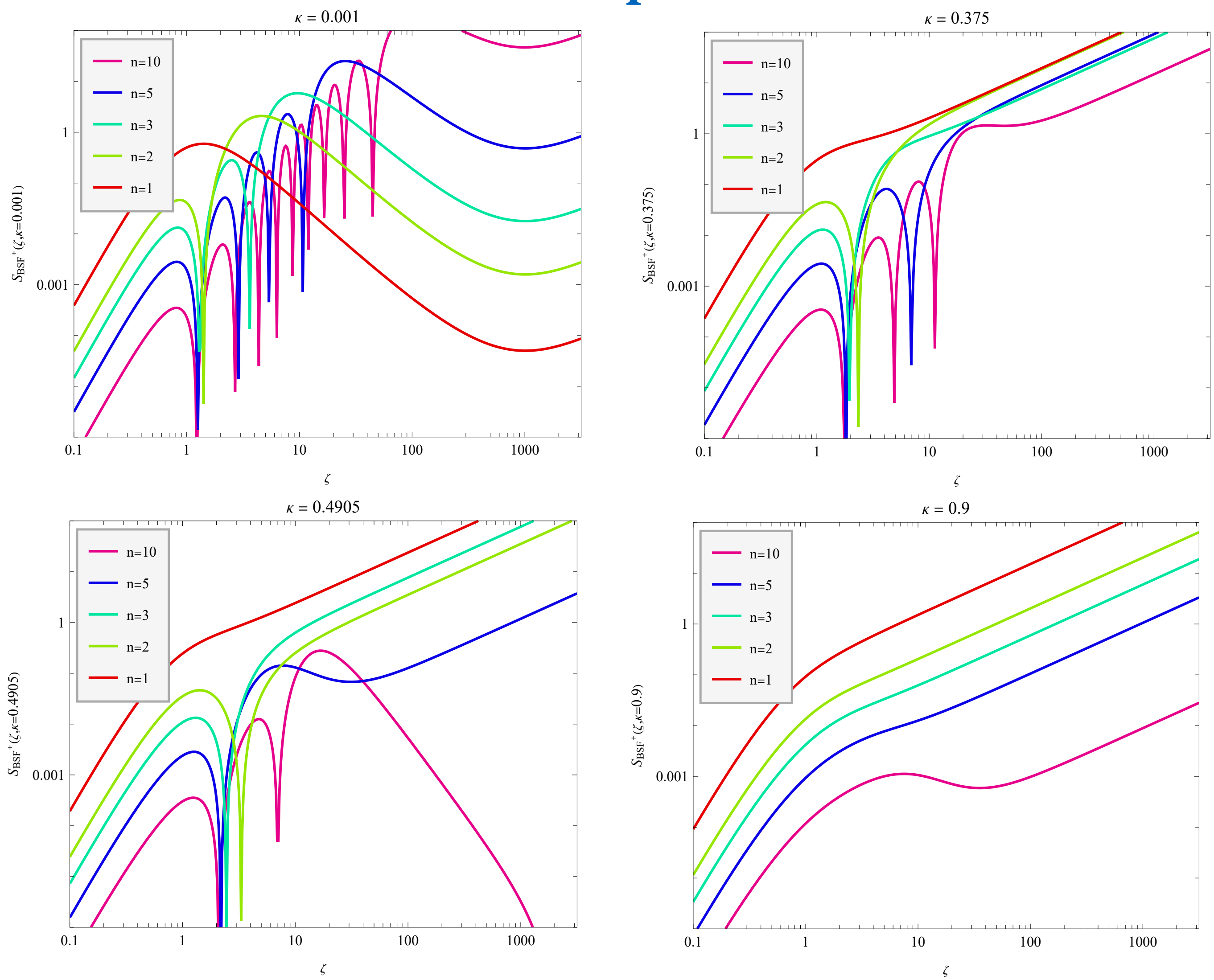
Impact of ϰ on the BSF cross-section

~ BSF cross-section [a.u.]



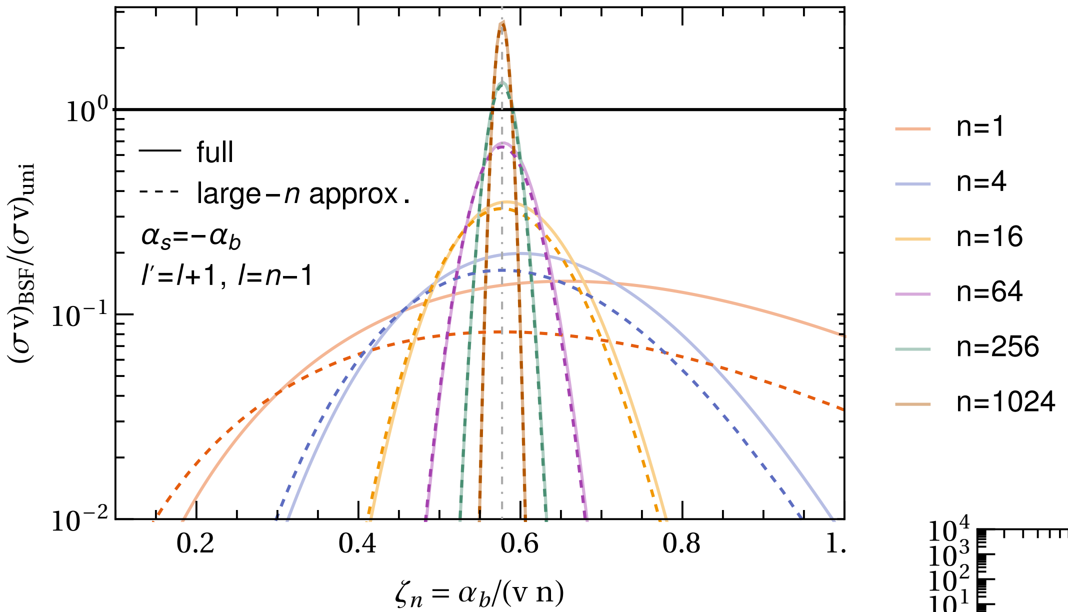
Anti-resonances in the $\varkappa$ dependence

~ BSF cross-section [a.u.]

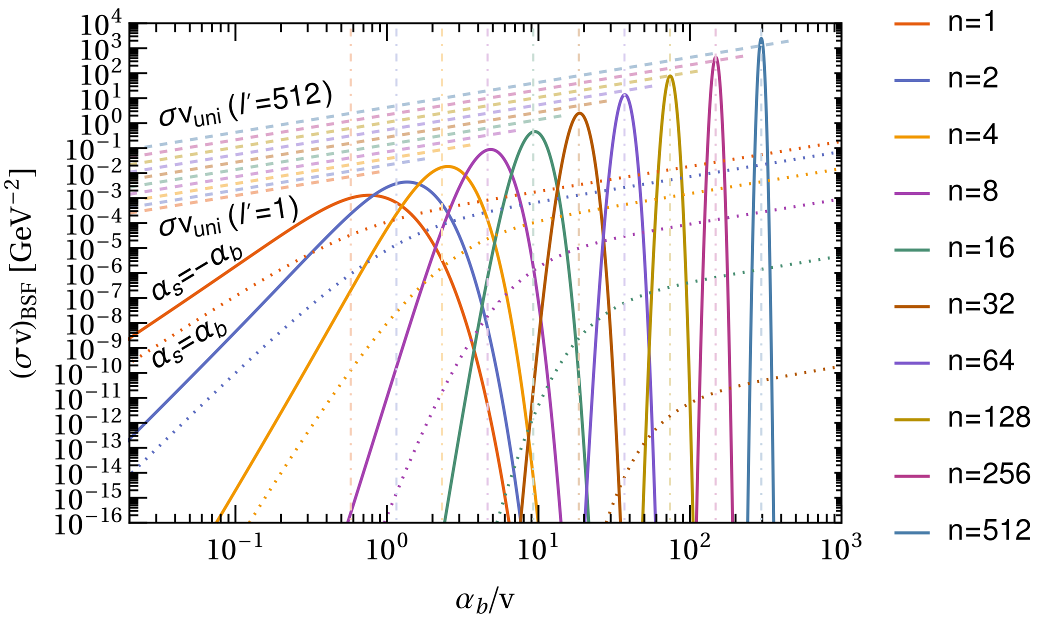


$\zeta = \alpha_b / v$

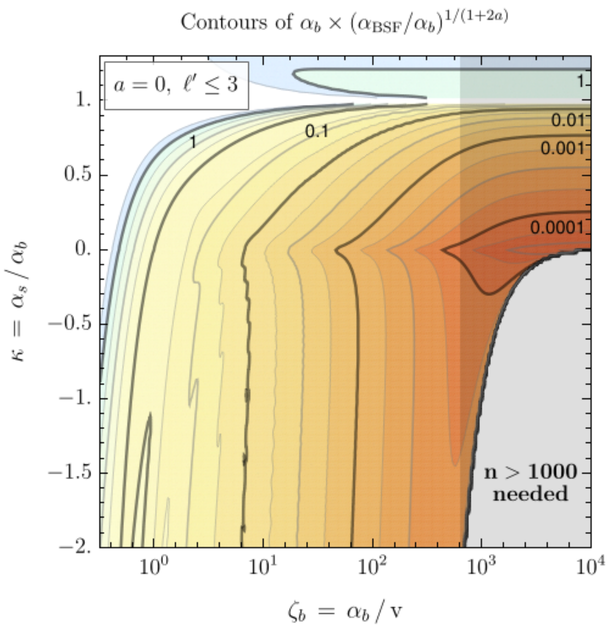
n-dependence of fixed-l BSF cross-sections



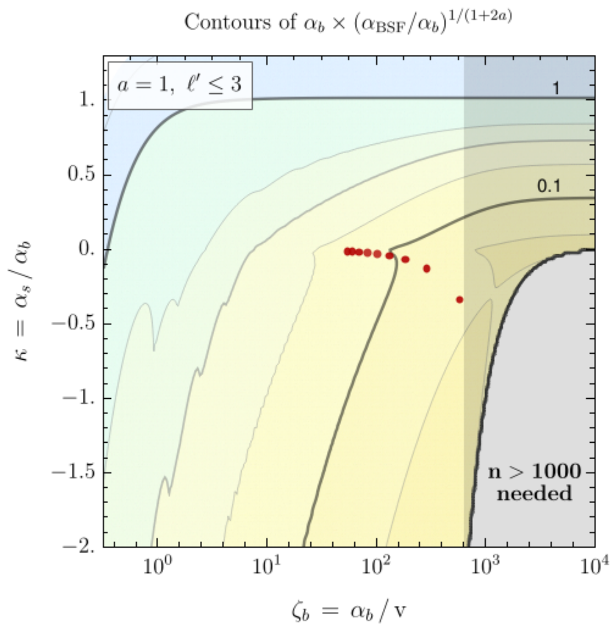
S(l′=l+1) −> B(n, l=n−1) + γ



Unitarity constraints in different multipoles

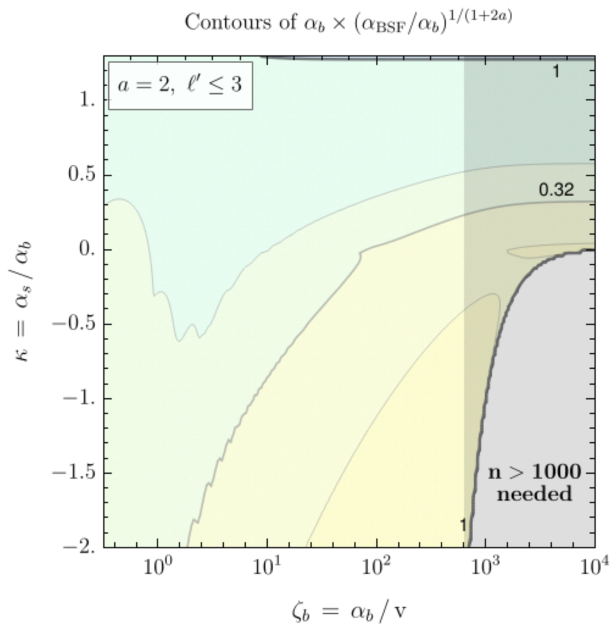


Monopole



Dipole

(+ SU(N) points)



Quadrupole

Terminating eternal depletion

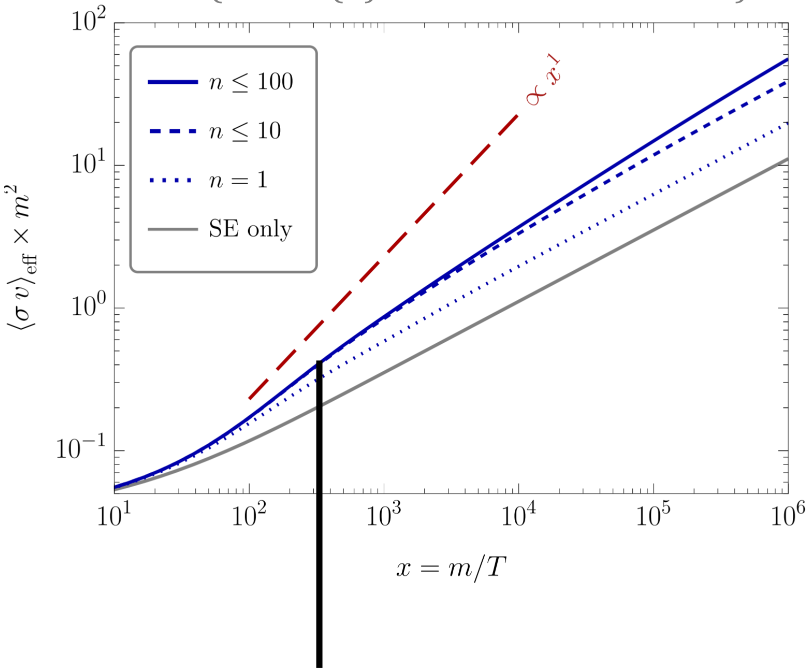
To arrive at a non-zero DM-abundance, the depletion must be stopped!

1. Extended dark sector (here superWIMP)
   the BS-constituents decay into DM eventually (no DM-BSF)
2. IR-Landau pole
   usually included in SU(N)
   breaks perturbative expansions („squeeze-out“, …)
3. finite-T effects
   (plasma interactions)
4. finite BS-width
   eventually the BS are no longer distinguishable „states“ but a continuum
5. phase space distortions, …

Importance of repulsive states & B-B transitions

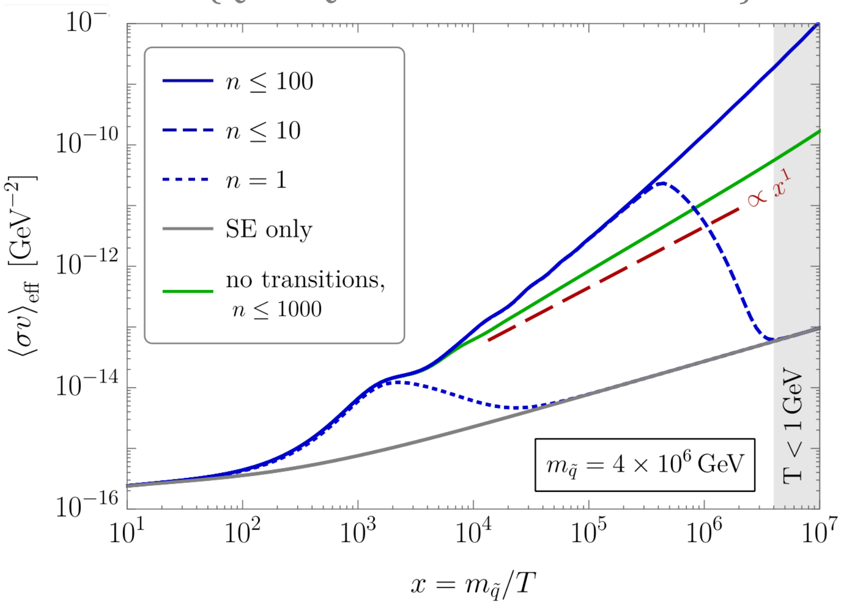
Abelian: identical potentials

(dark U(1) effective cross section)



non-Abelian: opposite potentials

(QCD+QED effective cross section)



Freeze-out almost complete:

⇒ small corrections from excited states

Freeze-out **never** completes:

⇒ excited states prevent decoupling:

"eternal" depletion

t-channel model: parameter space