Nonparametric analysis of CMB power spectrum data and consistency test of cosmological models

Amir Aghamousa

Asia Pacific Center for Theoretical Physics, Pohang, South Korea

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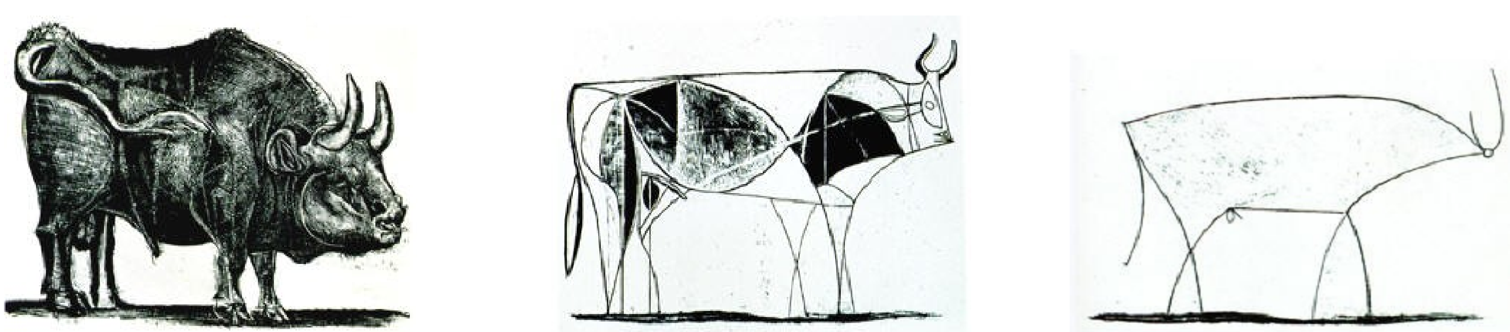


Bull pictures courtesy of “Pablo Picasso”

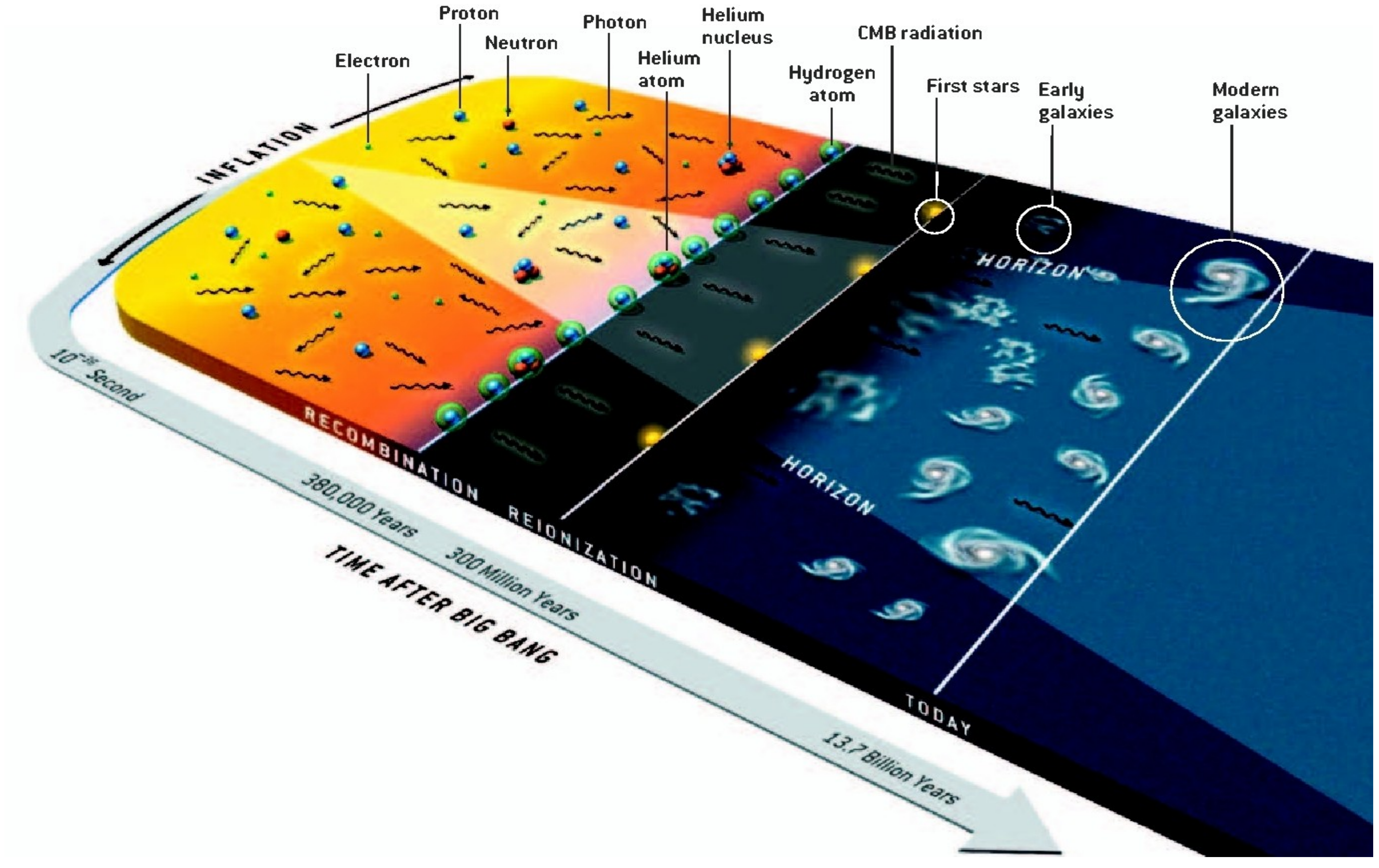
All models are false, some are useful. (George E. P. Box)

I will talk about:

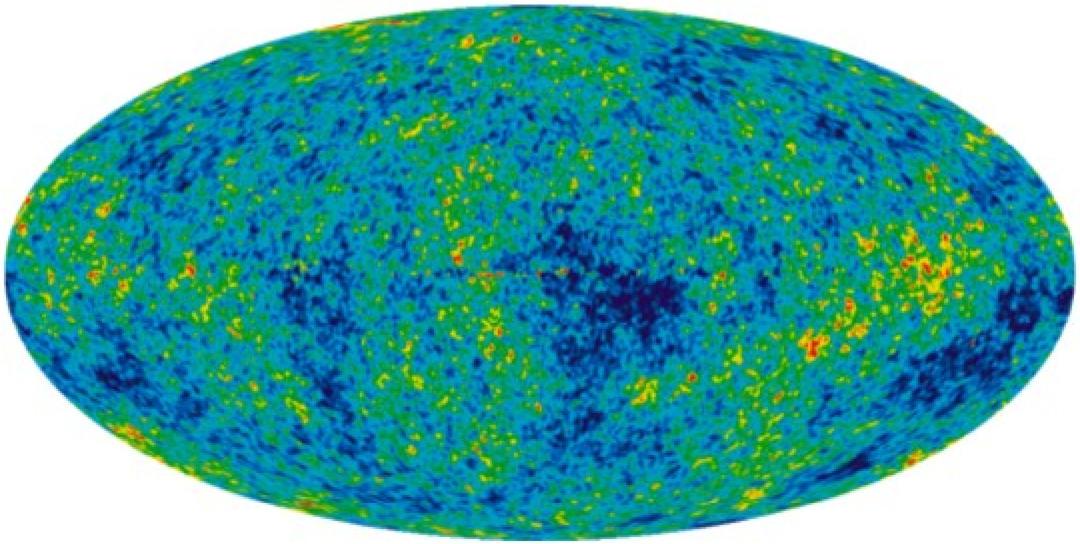
- Model-independent estimation of CMB angular power spectrum (with Arman Shafieloo, Mihir Arjunwadkar, Tarun Souradeep).
- Nonparametric test of consistency between cosmological models and CMB data (with Arman Shafieloo).

Model-independent estimation of CMB angular power spectrum

Timeline of the Universe



CMB Anisotropies and the Power Spectrum



- **Expansion in spherical harmonics**

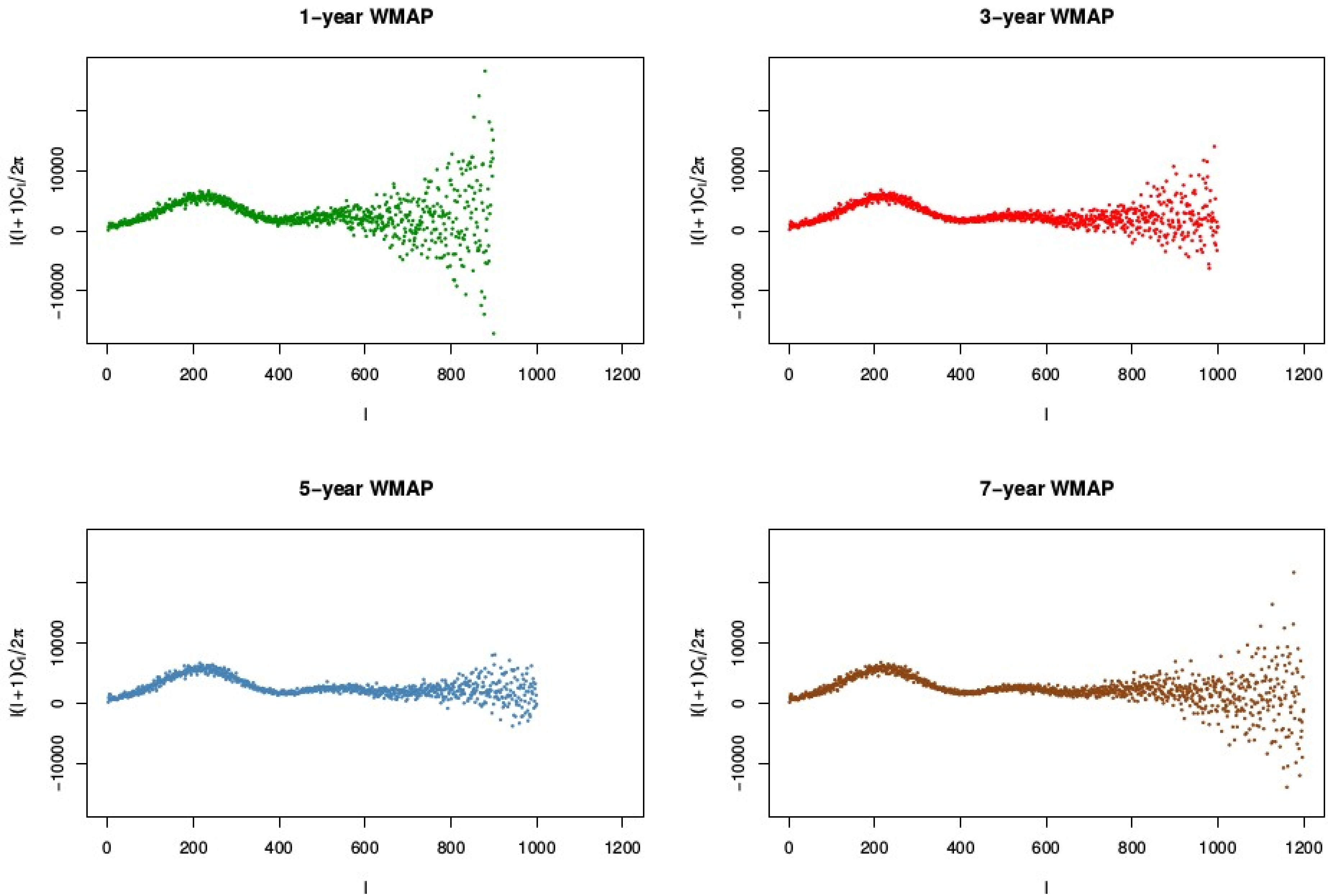
$$\Delta T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{l,m} Y_{l,m}(\theta, \phi),$$

- **$\Delta T(\theta, \phi)$ is a Gaussian random field**

  $\longrightarrow$ $a_{l,m}$ are mean-0 random variables with variance $C_l := E|a_{l,m}|^2$.

$$\tilde{C}_l := \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{l,m}|^2$$

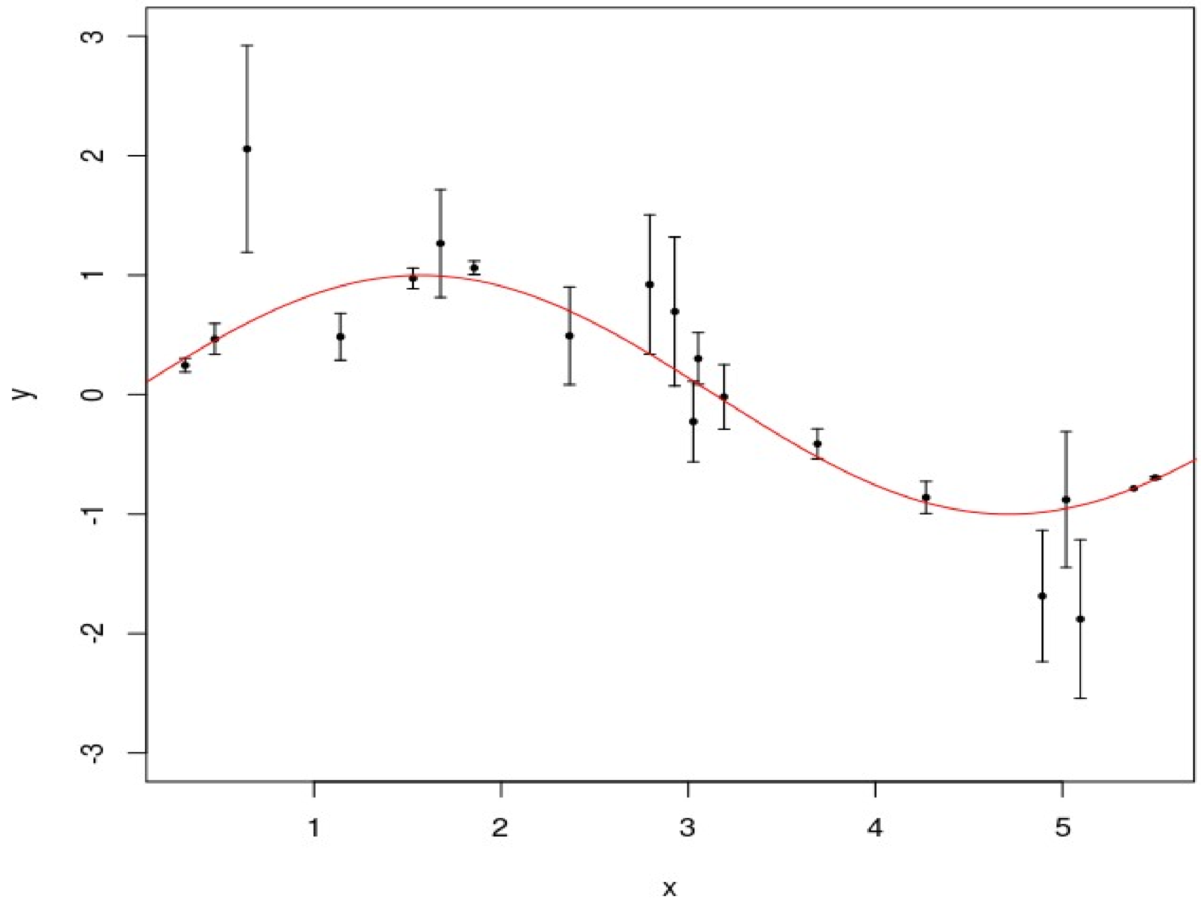
WMAP 1/3/5/7: Power Spectrum Data



Regression Problems

Data: $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

$$Y = f(x) + \epsilon$$



Parametric regression

$Y_i = f(x_i) + \epsilon_i$

- Assume $f(x) = ax + b$.
- Assume noise $\epsilon_i \sim N(0, \sigma^2)$ IID.
- Likelihood function

$$L(a, b|\text{data}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(Y_i - (ax_i + b))^2}{2\sigma^2}\right)$$

- To estimate a, b: Maximize $L(a, b|\text{data})$ w.r.t. a, b.
- This is same as linear least-squares regression, under the assumptions made.

Frequentist vs Bayesian

- **Frequentist**

It defines a probability as the limit of its relative frequency in a large number of trials.

- **Bayesian**

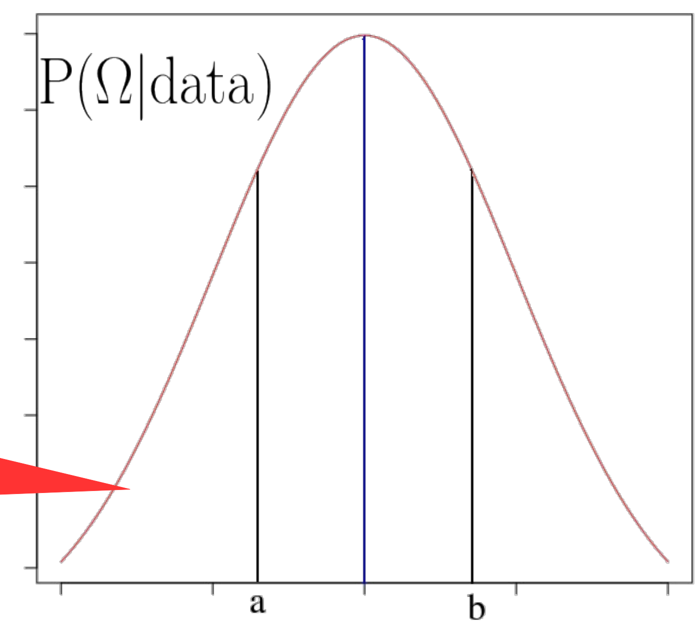
It defines a probability as a degree of belief.

Bayes' theorem:

posterior **likelihood** **prior**

$$P(\Omega|\text{data}) = \frac{P(\text{data}|\Omega)P(\Omega)}{P(\text{data})}$$

How to sample a high dimensional probability distribution?



REACT: nonparametric regression

- $Y_i = f(x_i) + \epsilon_i$, with $\epsilon_i \sim N(0, \sigma^2)$ IID, σ^2 known.
- Assume $f \in L_2(a, b)$ and a complete orthonormal basis $\{\phi_j(x)\}$.

$$f(x) = \sum_{j=0}^{\infty} \beta_j \phi_j(x), \;\; \beta_j = \int_a^b f(x)\phi_j(x)dx$$

- Regression estimator $\widehat{f}(x)$:

$$f(x) = \sum_{j=0}^{n-1} \widehat{\beta}_j \phi_j(x) + \text{(some truncation bias)}$$

$\widehat{\beta}_j := \lambda_j Z_j$ with $1 \geq \lambda_0 \geq \ldots \geq \lambda_{n-1} \geq 0.$ and $Z_j = \sum_{i=1}^{n} Y_i \phi_j(x_i)$

- Inverse-noise-weighted squared loss function

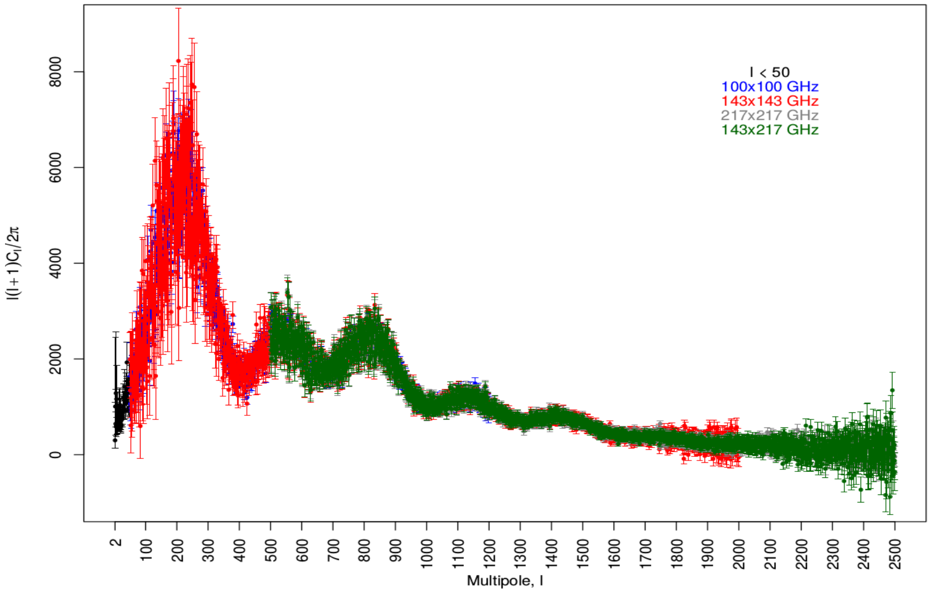
$$L(\widehat{f}, f) = \int \left(\frac{\widehat{f}(x) - f(x)}{\sigma(x)} \right)^2 dx.$$

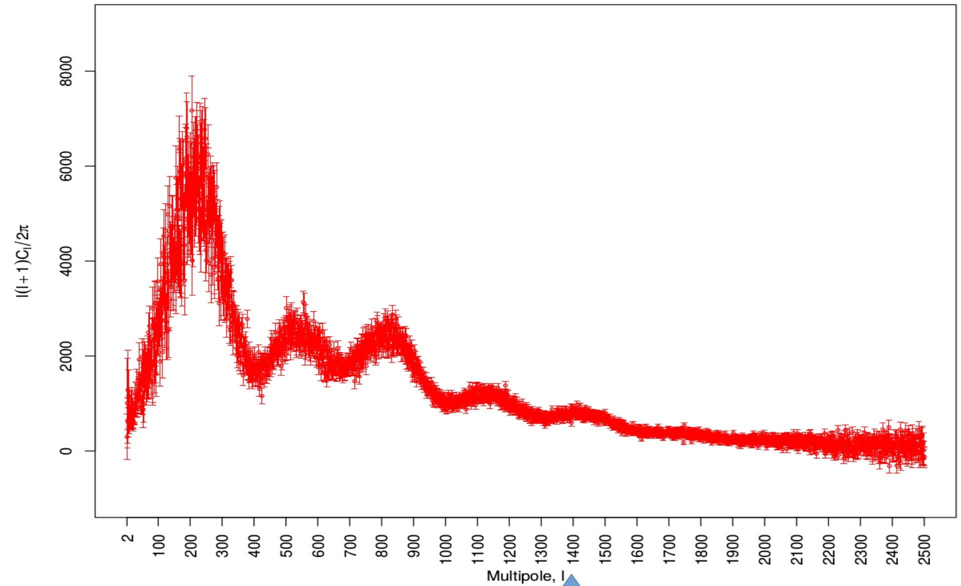
- Risk estimator

$$\widehat{R}(\lambda) = Z^T \bar{D} W \bar{D} Z + \text{tr}(DWDB) - \text{tr}(\bar{D} W \bar{D} B),$$

subject to the constraint $1 \geq \lambda_0 \geq \ldots \geq \lambda_{n-1} \geq 0.$

Planck: Power Spectrum Data



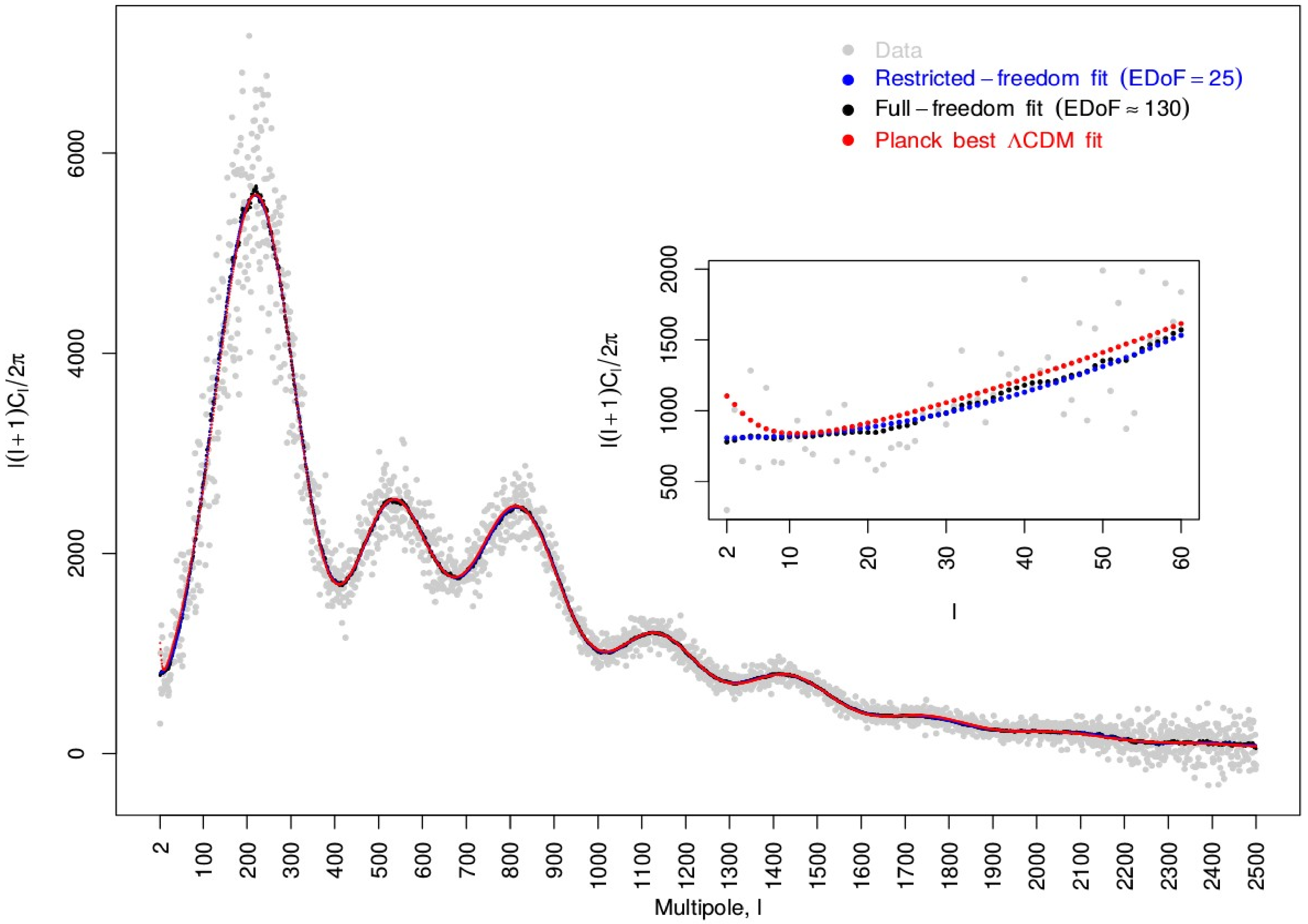


$$\bar{\mathcal{D}}_\ell = \frac{\sum_{ch} w_\ell^{ch} \mathcal{D}_\ell^{ch}}{\sum_{ch} w_\ell^{ch}}, \quad w_\ell^{ch} = (\sigma_\ell^{ch})^{-2}$$

$$\mathrm{Cov}(\bar{\mathcal{D}}_\ell, \bar{\mathcal{D}}_{\ell'}) = \frac{1}{\sum_{ch} w^{ch} \sum_{ch'} w^{ch'}} \sum_{ch,ch'} w_\ell^{ch} w_{\ell'}^{ch'} \mathrm{Cov}(\mathcal{D}_\ell^{ch}, \mathcal{D}_{\ell'}^{ch'})$$

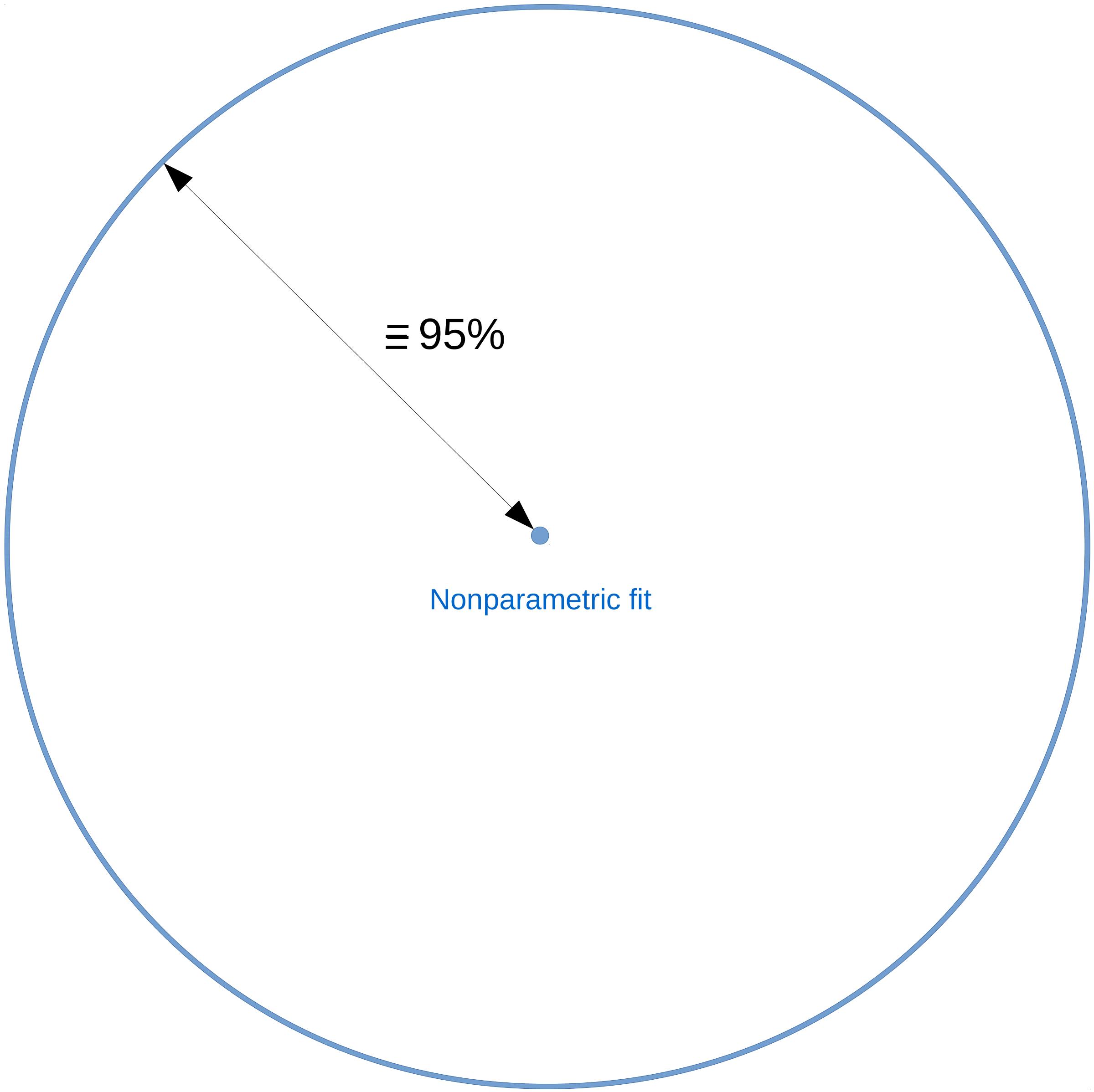
(Aghamousa, Shafieloo, Arjunwadkar and Souradeep, JCAP, 2015)

Planck 2013 power spectrum estimation

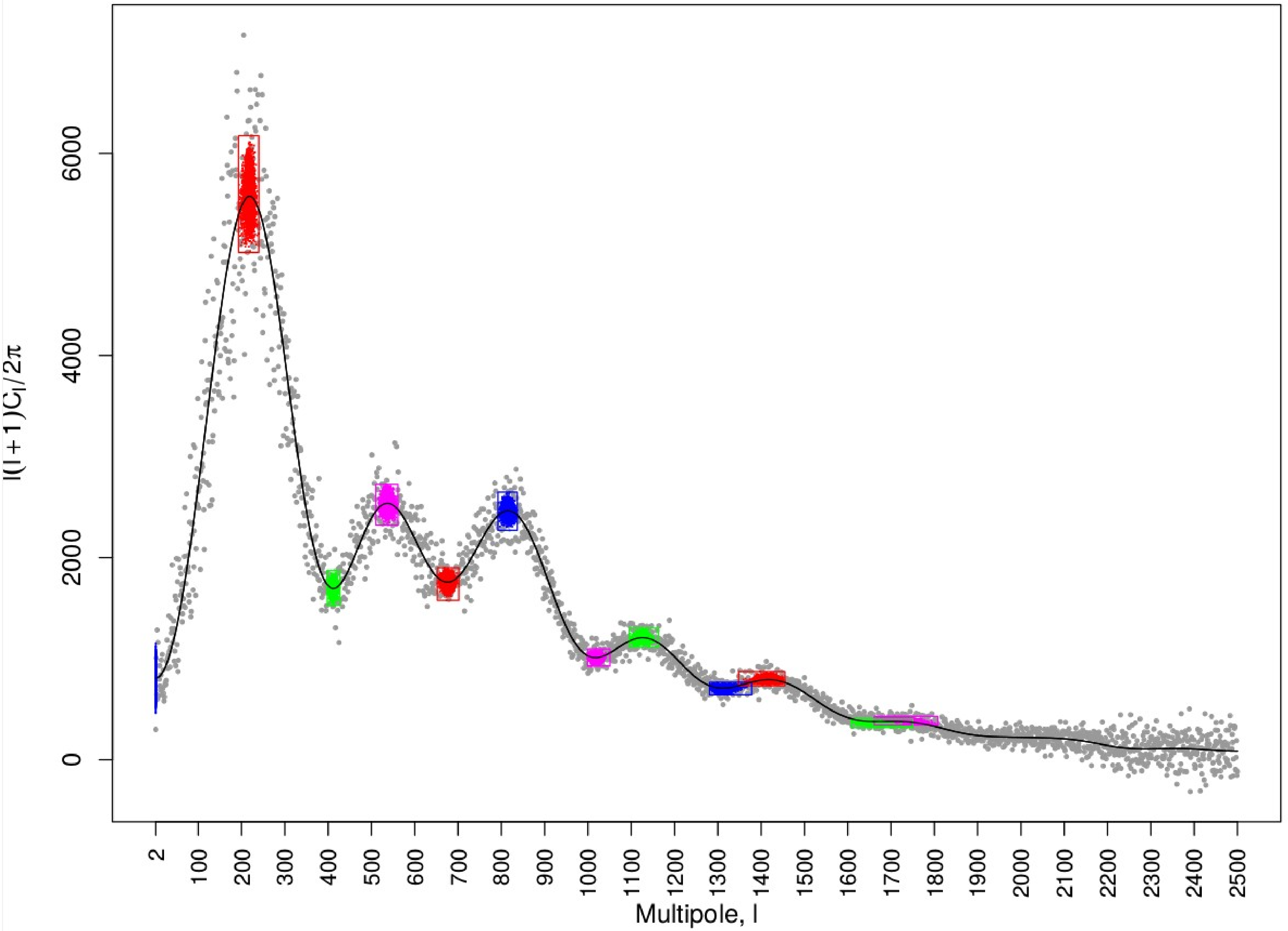


(Aghamousa, Shafieloo, Arjunwadkar and Souradeep, JCAP, 2015)

Confidence set in Function space

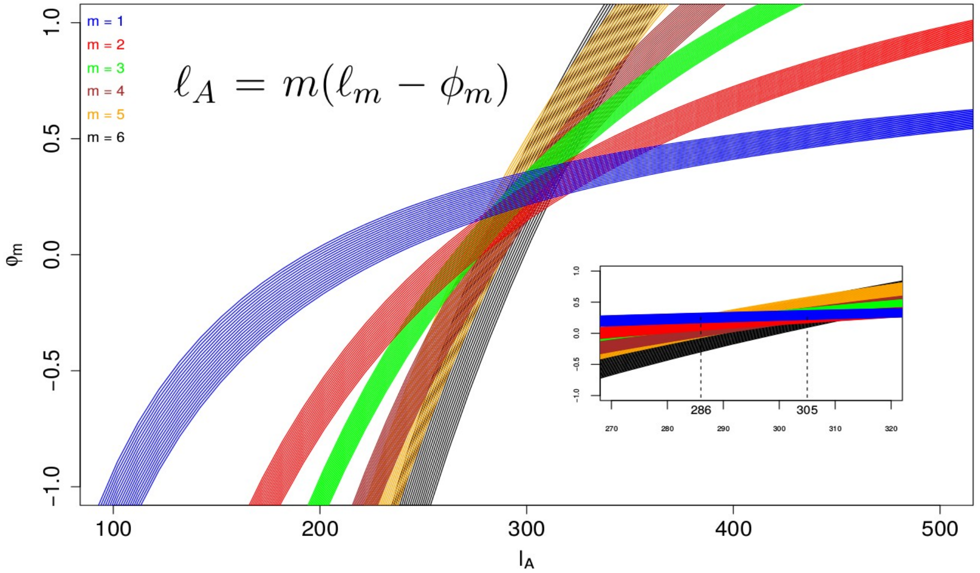


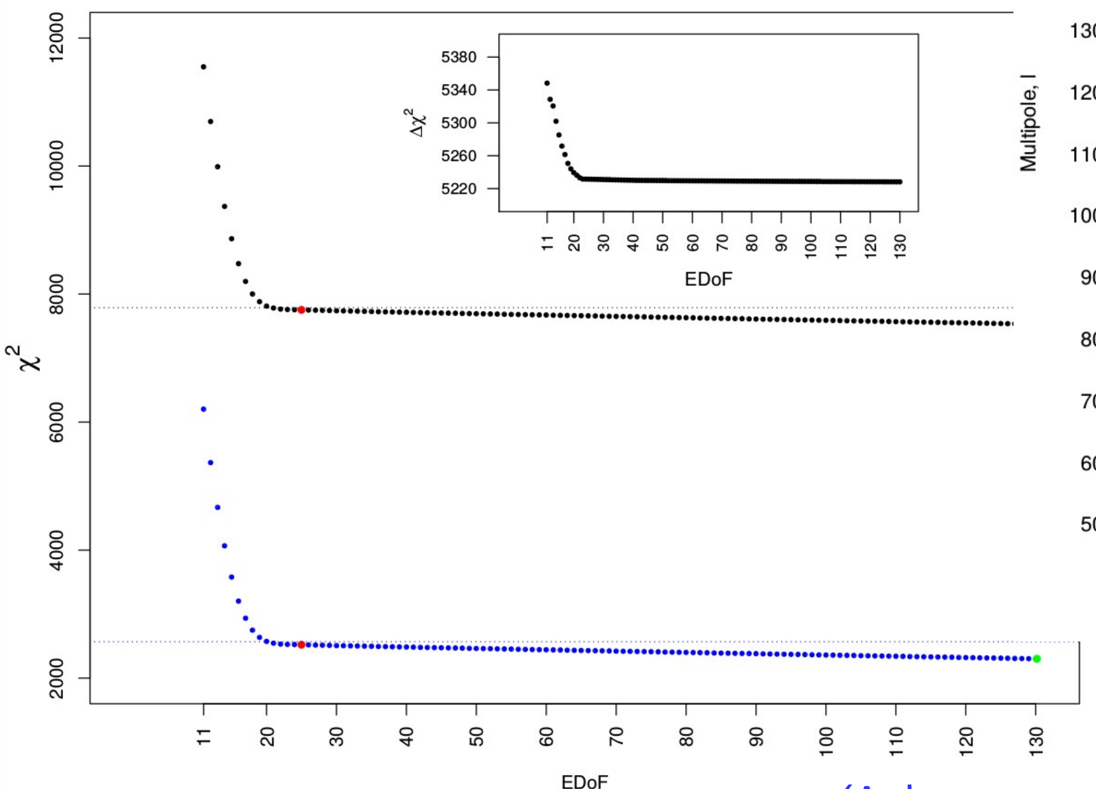
Picks and dips

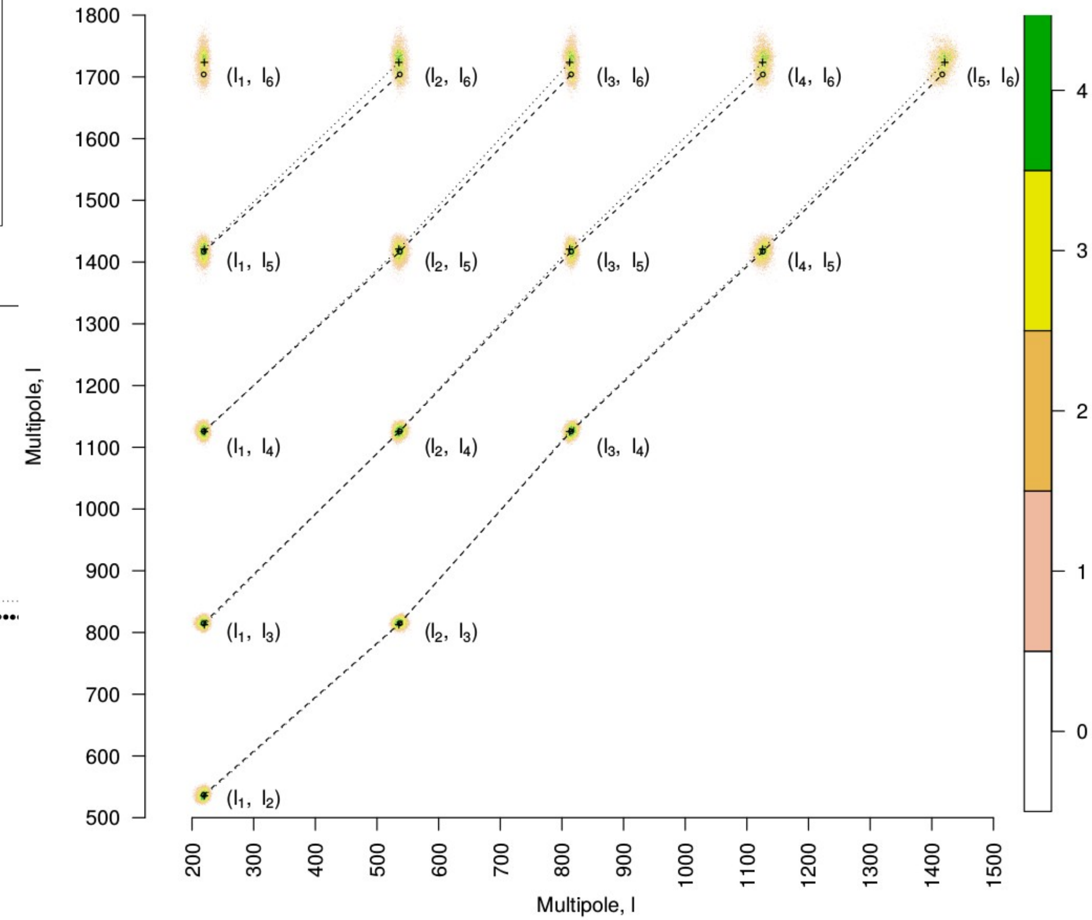


(Aghamousa, Shafieloo, Arjunwadkar and Souradeep, JCAP, 2015)

Other results



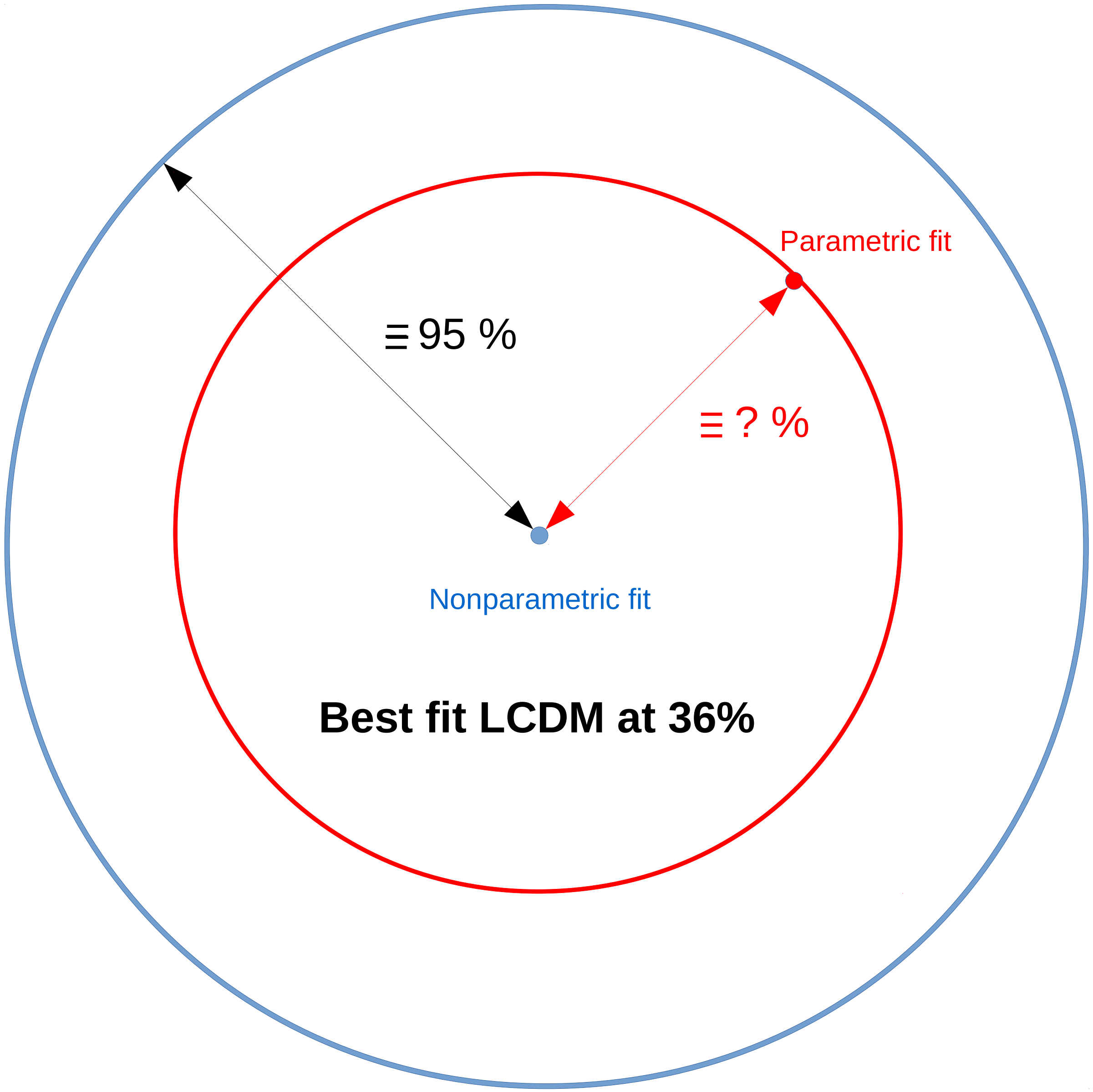




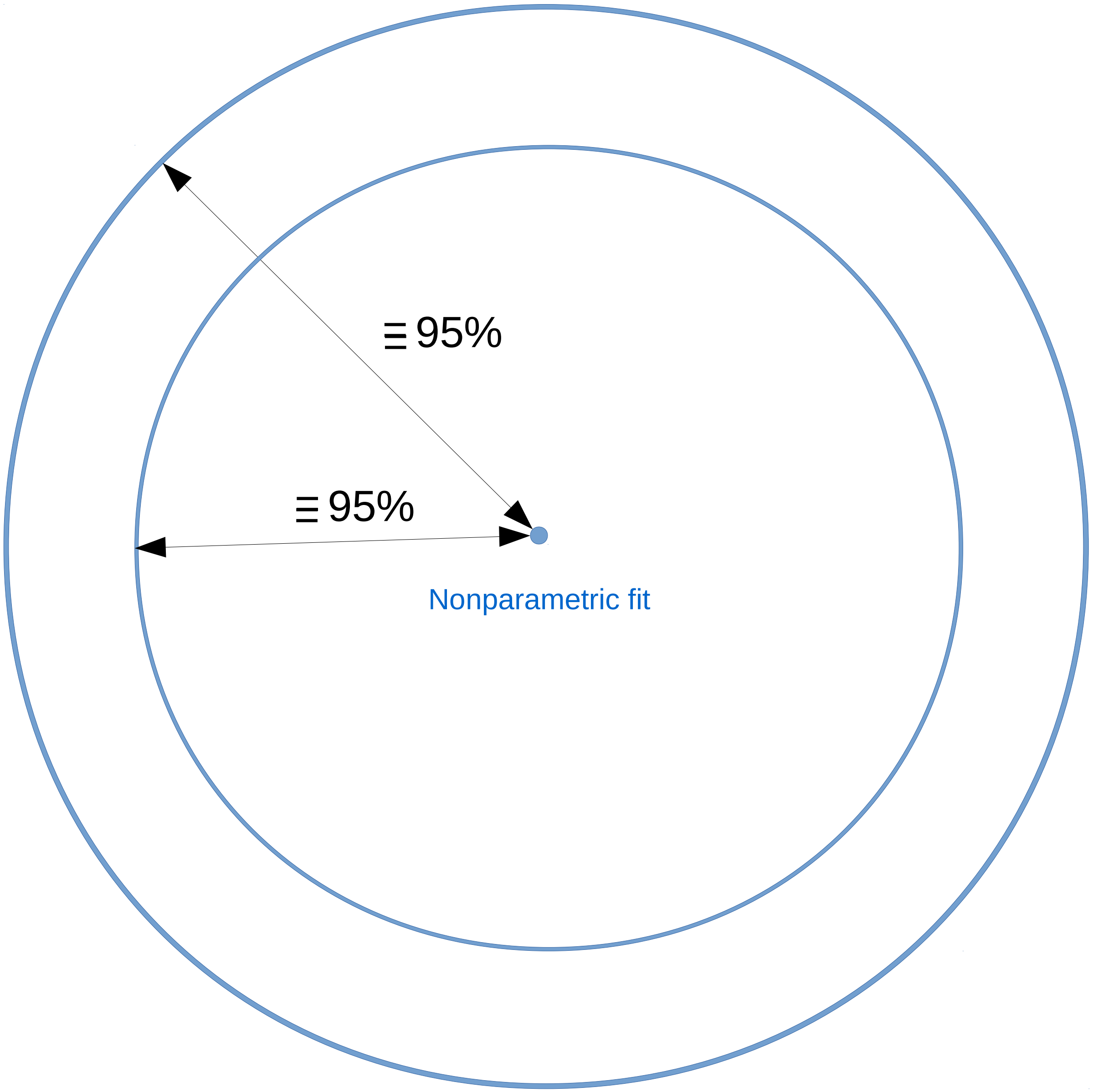
(Aghamousa, Shafieloo, Arjunwadkar and Souradeep, JCAP, 2015)

Nonparametric test of consistency between cosmological models and CMB data

Validating Cosmological Models



Calibrating Confidence distances



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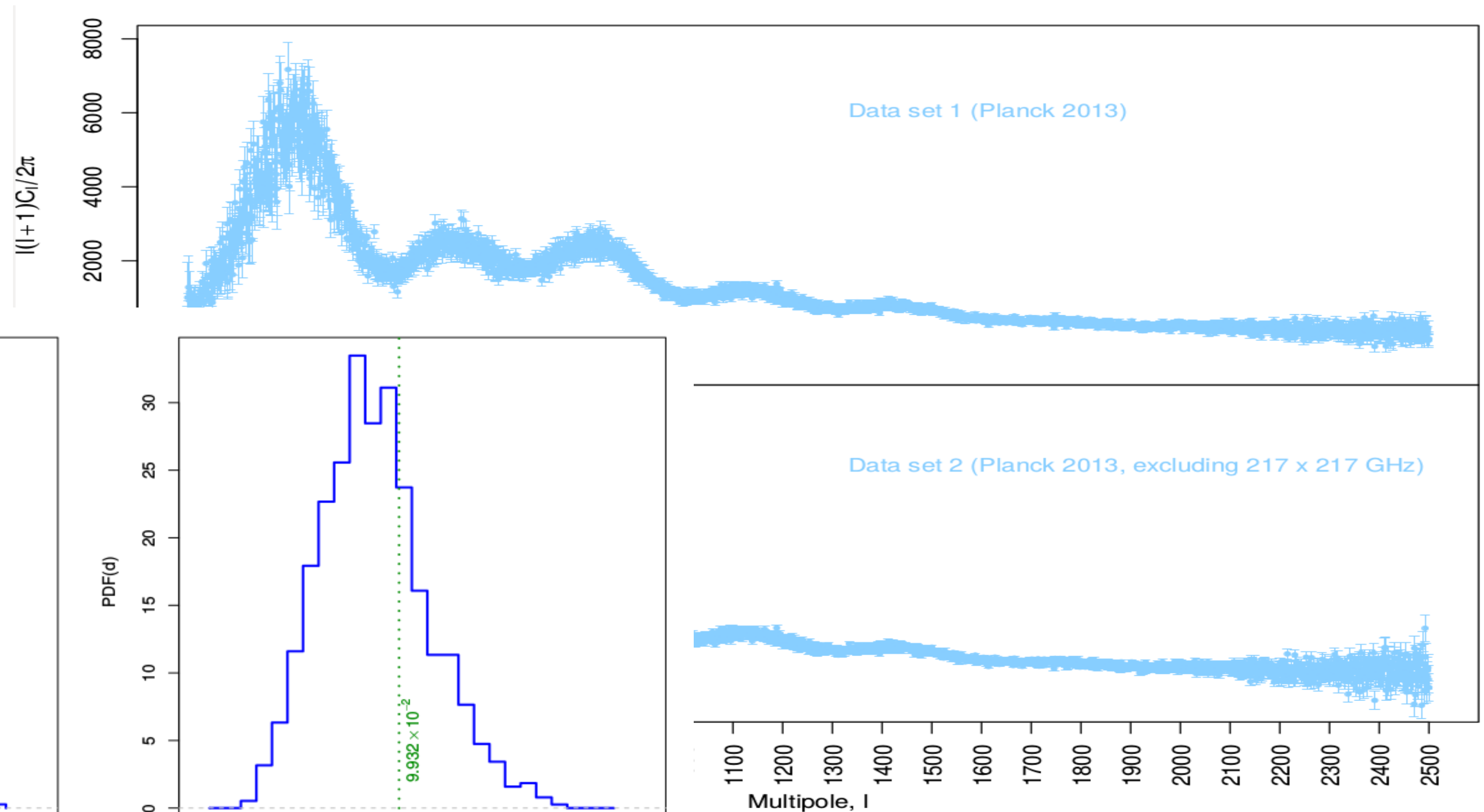
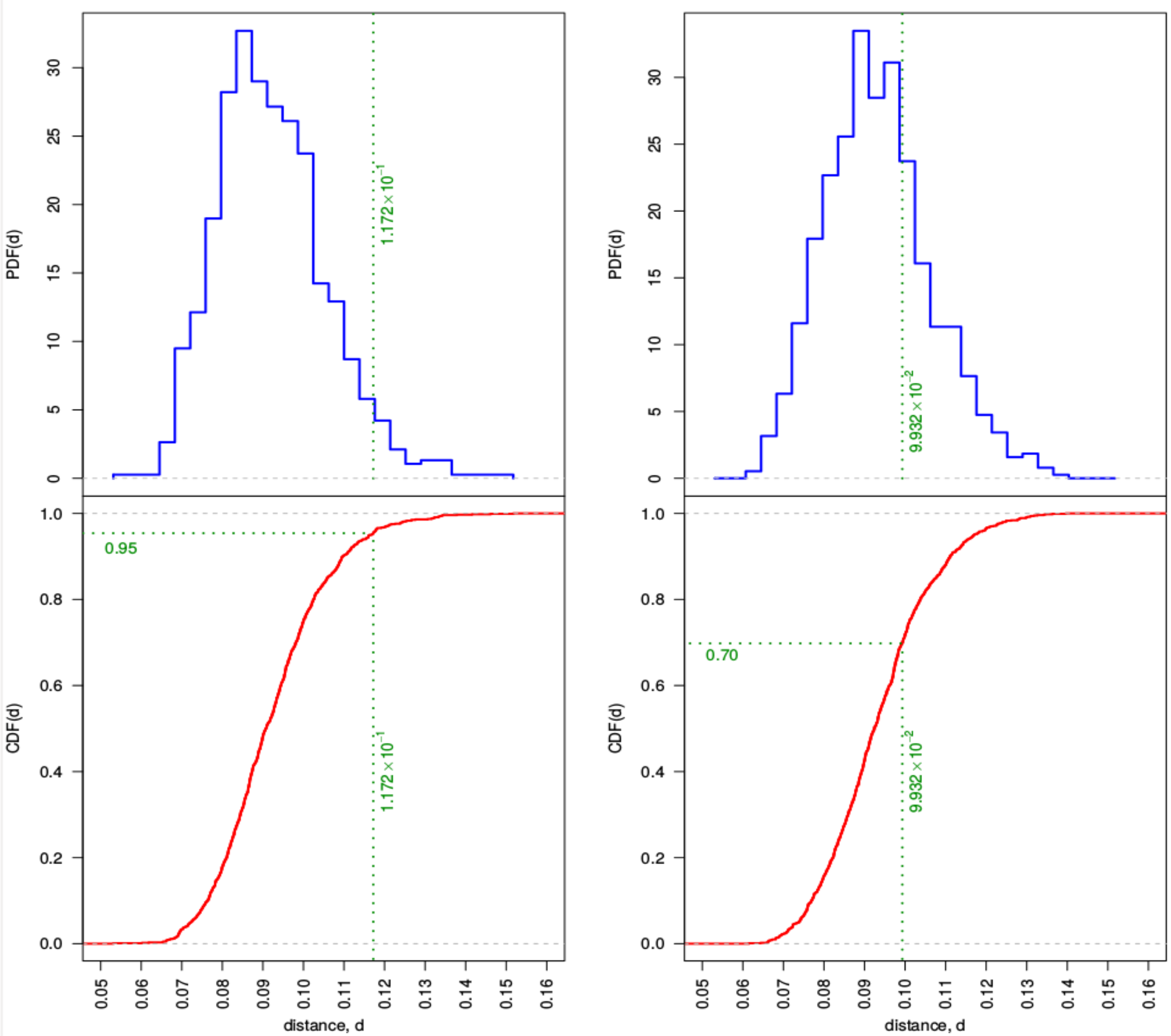
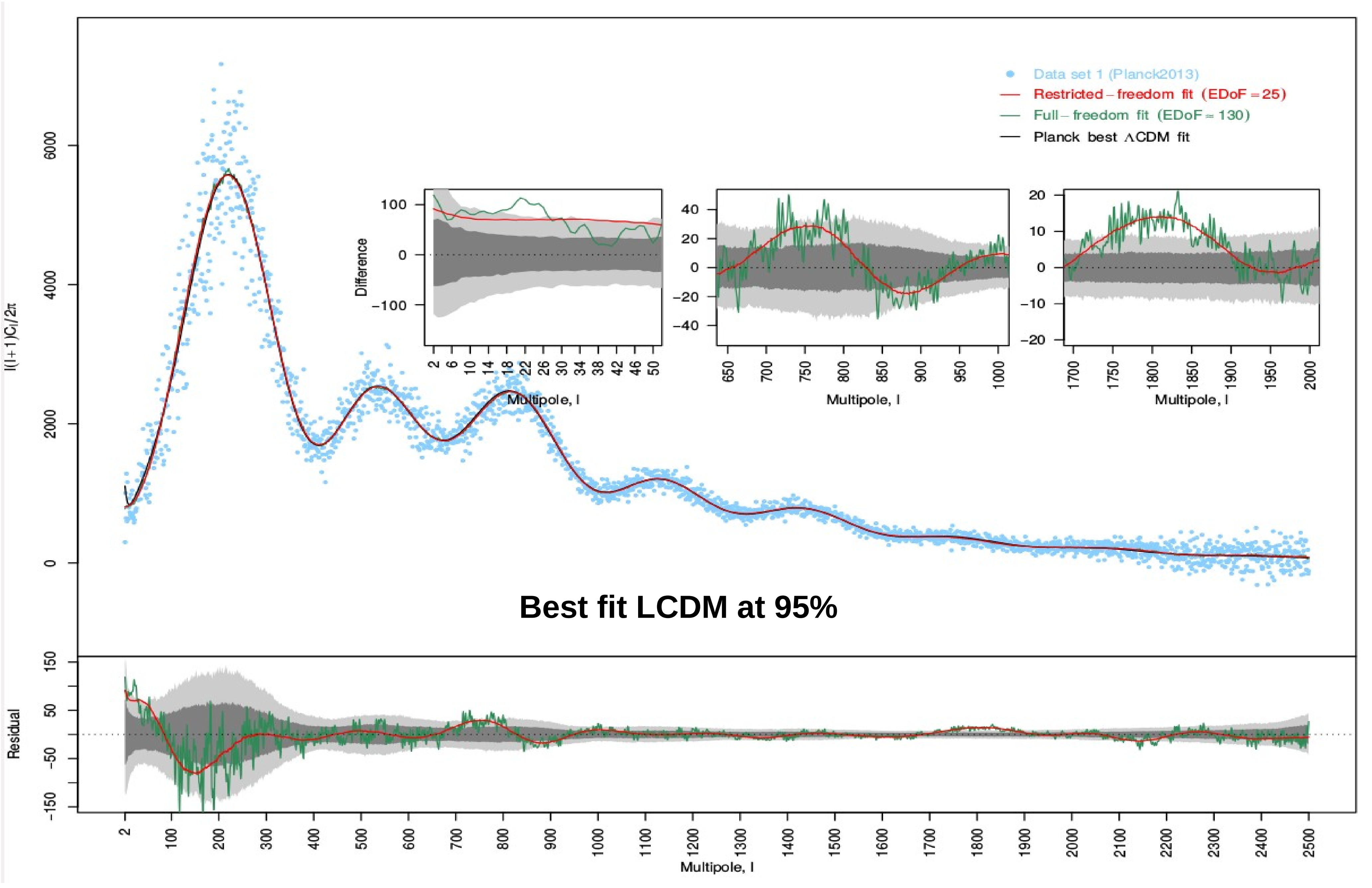




(Aghamousa, Shafieloo, JCAP, 2015)

Bias control



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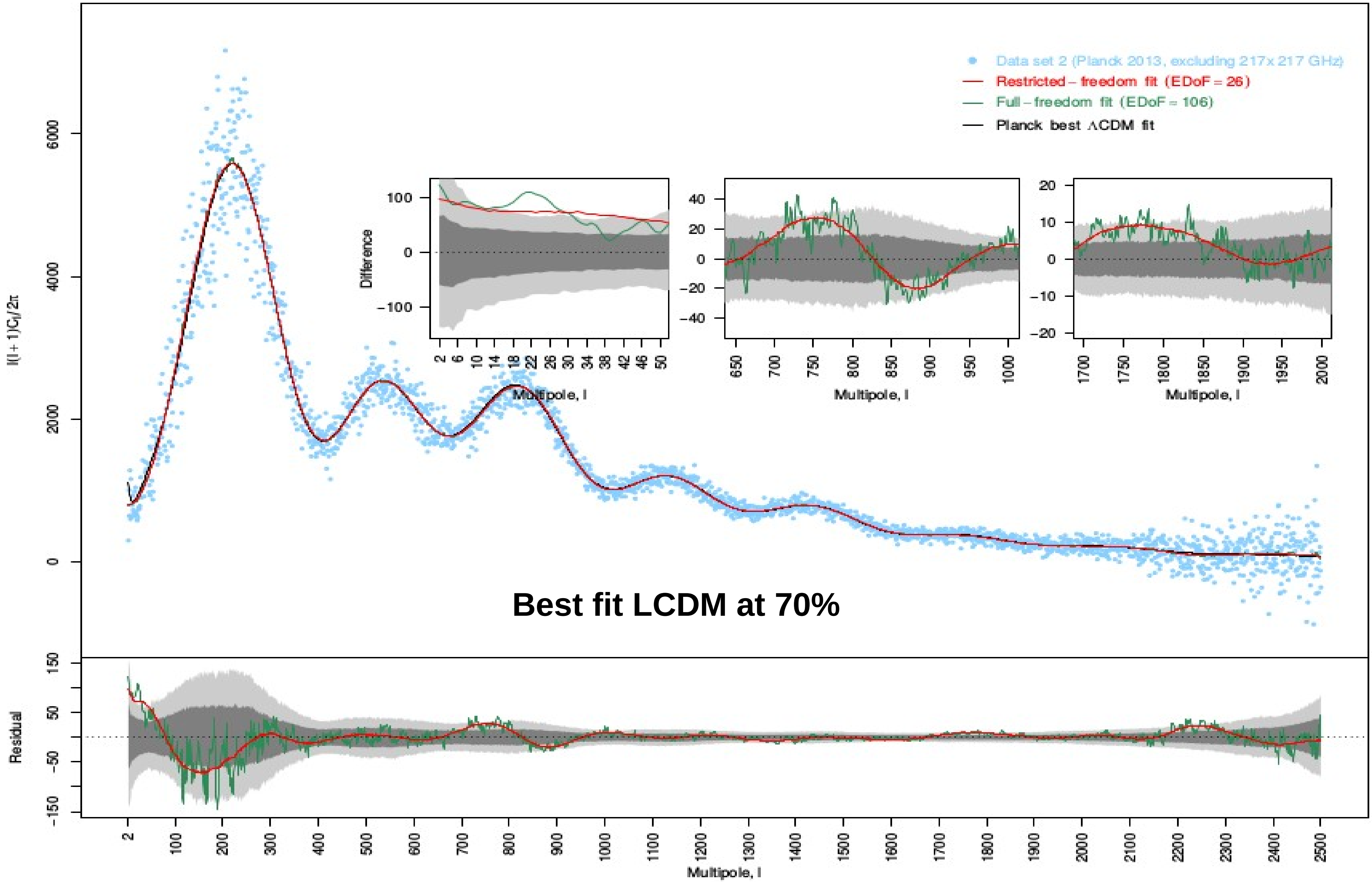
Bias control



(Aghamousa, Shafieloo, JCAP, 2015)

Thank you