Cosmological and Astrophysical Vainshtein mechanism in Bigravity

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Based on
## Abstract

Bigravity provides some interesting cosmological phenomena.

Are massive spin-2 theories restored to GR in massless limit?

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<td>(KA, K. Maeda, R. Namba, arXiv: 1506.04543)</td>
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   KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

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5. Summary and Discussions
**Why Massive?**

What is the graviton?
- It must be spin-2 field.
- How about mass? Massless field or Massive field?

GR describes a massless spin-2 field

Is there a theory with a massive spin-2 field?

If there is, which theory describes our Universe?

Experimental constraint: \( m < 7.1 \times 10^{-23} \text{eV} \)
(from the solar-system experiment)

Dark components hint us that
GR should be modified at large scale.

![Pie Chart](image)
Massive gravity and Bigravity

In order to add a mass to the graviton, we need two metrics
e.g., Fierz-Pauli theory

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \text{background metric + spin-2 field} \]

\[ S_{\text{massive}} = \frac{1}{2\kappa^2} \int d^4x \left[ \mathcal{L}_{\text{EH}} - \frac{m^2}{4} (h_{\mu\nu}h^{\mu\nu} - h^2) \right] \]

Assuming only one metrics is dynamical
→ Massive gravity = massive spin-2

Assuming both metrics are dynamical
→ Bigravity theory = massless spin-2 + massive spin-2
Hassan-Rosen bigravity theory

\[ S = \frac{1}{2\kappa_g^2} \int d^4x \sqrt{-g} R(g) + \frac{1}{2\kappa_f^2} \int d^4x \sqrt{-f} \mathcal{R}(f) \]

\[ - \frac{m^2}{\kappa^2} \int d^4x \sqrt{-g} \sum_{i=0}^{4} b_i \mathcal{U}_i(g, f) + S^{[m]} \]

\[ \kappa^2 = \kappa_g^2 + \kappa_f^2 \]

→ Accelerating expansion?

How about gravity-matter coupling?

Twin matters

\[ S^{[m]} = S^{[m]}_g(g, \phi_g) + S^{[m]}_f(f, \psi_f) \]

Physical matter

Dark matter?

Doubly coupled matter

\[ S^{[m]}(g, f, \psi_{\text{double}}) \]

Reappearance of ghost?

(Y. Yamashita et al. ’14, C. de Rham et al. ’14)
Bigravity theory in low energy scale

✓ Low energy scale
Can the bigravity explain the origin of dark components?
- Mass term produces an effective cosmological constant
- $f$-matter behaves as a dark matter in $g_{\mu \nu}$

\[ m \sim 10^{-33} \text{eV} \sim \text{Gpc}^{-1} \Rightarrow \text{Dark energy} \]
\[ m \geq 10^{-27} \text{eV} \sim \text{kpc}^{-1} \Rightarrow \text{Dark matter} + \text{(DE)} \]

KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

*If we admit fine tuning, bigravity can explain both dark components

Experimental constraint: $m < 7.1 \times 10^{-23} \text{eV}$
(from the solar-system experiment)
Bigravity theory in high energy scale

- High energy scale (= Graviton mass can be ignored)
- The theory should be restored to two GRs

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Homothetic solutions in bigravity

If two metrics are proportional, the equation of motion is exactly same as GR with a cosmological constant.

\[ f_{\mu\nu} = K^2 g_{\mu\nu}, \quad K = \text{const} \Rightarrow \text{GR solution} \]

\[
G_{\mu\nu}(g) + \Lambda_g g_{\mu\nu} = \kappa_g^2 T^{[m]}_{\mu\nu}, \quad \Lambda_g(K) = K^2 \Lambda_f(K), \quad \kappa_f^2 T^{[m]}_{\mu\nu} = \kappa_g^2 T^{[m]}_{\mu\nu}
\]

where

\[
\Lambda_g(K) = m^2 \frac{\kappa_g^2}{\kappa^2} \left( b_0 + 3b_1 K + 3b_2 K^2 + b_3 K^3 \right),
\]

\[
\Lambda_f(K) = m^2 \frac{\kappa_f^2}{\kappa^2} \left( b_4 + 3b_3 K^{-1} + 3b_2 K^{-2} + b_1 K^{-3} \right)
\]

\[ K \] is a root of a quartic equation

\[ \rightarrow \] There are four different vacuum solutions

Even if we assume the existence of Minkowski spacetime, there can be de Sitter solution as a vacuum solution.
Attractor universe

For a particular coupling constants, we find de Sitter and AdS solutions as well as Minkowski solution

\[ \Lambda_g(K_M) = 0, \quad K = K_M \quad : \text{Minkowski solution} \]

\[ \Lambda_g(K_{dS}) > 0, \quad K = K_{dS} \quad : \text{de Sitter solution} \]

\[ \Lambda_g(K_{AdS}) < 0, \quad K = K_{AdS} \quad : \text{AdS solution} \]

We consider the homogenous and isotropic spacetimes

\[ ds_g^2 = -N_g^2(t) dt^2 + a_g^2(t) \gamma_{ij} dx^i dx^j, \]
\[ ds_f^2 = -N_f^2(t) dt^2 + a_f^2(t) \gamma_{ij} dx^i dx^j \]

\[ \Lambda_g > 0 \text{ homothetic and } \Lambda_g = 0 \text{ homothetic solutions are obtained as attractor solutions as the Universe expands.} \]

KA and K. Maeda, PRD 89, 064051 (2014)
Perturbation around homothetic solution

The Universe approaches the homothetic spacetime.

→ In the low energy scale, spacetimes may be described by linear perturbations around the homothetic spacetime.

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{[g]}_{\mu\nu}, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + K^2 h^{[f]}_{\mu\nu} = K^2 \left( \bar{g}_{\mu\nu} + h^{[f]}_{\mu\nu} \right) \]

Linear perturbation can be decomposed into linearized GR + FP

\[ h^{[-]}_{\mu\nu} = h^{[g]}_{\mu\nu} - h^{[f]}_{\mu\nu} \quad \leftrightarrow \quad \text{Massive mode = FP theory} \]

\[ h^{[+]}_{\mu\nu} = \frac{m^2_f}{m^2_{\text{eff}}} h^{[g]}_{\mu\nu} + \frac{m^2_g}{m^2_{\text{eff}}} h^{[f]}_{\mu\nu} \quad \leftrightarrow \quad \text{Massless mode = GR} \]

Massless and massive modes couple to both twin matters.

\[ m^2_{\text{eff}} := m^2_g + m^2_f \]

\[ m^2_g := \frac{m^2_k^g}{\kappa^2} \left( b_1 K + 2b_2 K^2 + b_3 K^3 \right), \quad m^2_f := \frac{m^2_k^f}{K^2 \kappa^2} \left( b_1 K + 2b_2 K^2 + b_3 K^3 \right) \]
Dark matter is f-matter?

$$h_{\mu\nu}^{[-]} = h_{\mu\nu}^{[g]} - h_{\mu\nu}^{[f]} \quad \text{Massive mode} = \text{FP theory}$$

$$h_{\mu\nu}^{[+]} = \frac{m_f^2}{m_{\text{eff}}^2} h_{\mu\nu}^{[g]} + \frac{m_g^2}{m_{\text{eff}}^2} h_{\mu\nu}^{[f]} \quad \text{Massless mode} = \text{GR}$$

Massless and massive modes couple to both twin matters. Our spacetime is given by both massive and massless modes.

$$h_{\mu\nu}^{[g]} = h_{\mu\nu}^{[+]} + \frac{m_g^2}{m_{\text{eff}}^2} h_{\mu\nu}^{[-]} \quad h_{\mu\nu}^{[g]} \approx h_{\mu\nu}^{[+]}$$

Both massive and massless modes survive. The massive mode decays. Only the massless mode survives.

$$m_{\text{eff}}^{-1}$$

scale
Dark matter is f-matter?

The gravitational potential is induced by $f$-matter field as well as $g$-matter field through the interaction terms.

Outside Vainshtein radius

$$\Phi_g = -\frac{G M_g}{r} \left( \frac{m_f^2}{m_{\text{eff}}^2} + \frac{4m_g^2}{3m_{\text{eff}}^2} e^{-m_{\text{eff}} r} \right) v_{\text{DVZ}} \text{ discontinuity}$$

$$- \frac{m_g^2}{m_{\text{eff}}^2} \frac{K^2 G M_f}{r} \left( 1 - \frac{4}{3} e^{-m_{\text{eff}} r} \right)$$

repulsive force in $m_{\text{eff}} r \ll 1$

Screened | Repulsive | Attractive

= dark matter

$$r_V = \left( \frac{|G M_g - K^2 G M_f|}{m_{\text{eff}}^2} \right)^{1/3}$$
Dark energy and Dark matter in bigravity

✓ $\Lambda_g > 0$ homothetic and $\Lambda_g = 0$ homothetic solutions are obtained as attractor solutions as the Universe expands.

→ Accelerating expansion is an attractor solution

✓ $f$-matter can be a dark matter if the graviton mass is large

(rotation curve, structure formation, missing mass of Universe)

$m \sim 10^{-33} \text{eV} \sim \text{Gpc}^{-1} \Rightarrow \text{Dark energy}$

$m \geq 10^{-27} \text{eV} \sim \text{kpc}^{-1} \Rightarrow \text{Dark matter} + (\text{DE})$

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Viable cosmology with massive graviton

Linear perturbations are stable in the late stage \((H \ll m)\)

However, there is an instability in the early stage \((H \gg m)\)

(Comelli et al. ’12, ’14, De Felice et al. ‘14)

The fact implies that the massive graviton has instability in massless limit on a curved background.

Unstable \(\rightarrow\) We must take into account non-linear interactions

Can we find a viable cosmology with non-linear effects?

For simplicity, we assume the background evolution of the Universe is given by GR (not general cosmological background)

\(\iff\) There is no value of massive spin-2 field in the background
Instability of massive graviton on curved spacetime

We consider a linear massive spin-2 field, where the background spacetime is given by GR solution.

→ There is no value of massive spin-2 field in the background.

The action is given by linearized EH action with FP mass term

\[ S_{\text{massive}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \mathcal{L}_{\text{EH}}[h; \Lambda_g] - \frac{m^2}{4} (h_{\mu\nu} h^{\mu\nu} - h^2) \right] \]

We focus on the scalar graviton mode \( \pi \).

To recover gauge symmetry, we introduce Stueckelberg fields as

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\bar{\nabla}_{(\mu} A_{\nu)} + 2\bar{\nabla}_\mu \bar{\nabla}_\nu \pi \]
Instability of massive graviton on curved spacetime

Fierz-Pauli theory on the FLRW background

- Higuchi ghost (Higuchi, 1972, Grisa and Sorbo, 2010)
  de Sitter background (or the accelerating universe)
  with \( m/H \rightarrow 0 \).
  \[ \Rightarrow \text{ Scalar graviton has ghost instability} \]

- Gradient instability (Grisa and Sorbo, 2010)
  the decelerating universe \((-1/3 < w < 1)\)
  with \( m/H \rightarrow 0 \).
  \[ \Rightarrow \text{ Scalar graviton has gradient instability} \]

Massive graviton on the FLRW background is unstable in the massless limit!
Why? Massive field should be massless field in \( m/H \rightarrow 0 \).
Hassan-Rosen Bigravity theory

Unstable $\rightarrow$ We must take into account non-linear interactions

\[
S = \frac{1}{2\kappa_g^2} \int d^4 x \sqrt{-g} R(g) + \frac{1}{2\kappa_f^2} \int d^4 x \sqrt{-f} R(f) \\
- \frac{m^2}{\kappa^2} \int d^4 x \sqrt{-g} \sum_{i=0}^{4} b_i \mathcal{U}_i(g, f) + S^{[m]} \\
\mathcal{U}_n(g, f) = -\frac{1}{n!(4-n)!} \epsilon^{...} \epsilon...(\gamma^\mu{}^\nu)^n
\]

with \( \gamma^\mu{}^\alpha\gamma^\alpha{}^\nu = g^\mu{}^\alpha f^{}_{\alpha\nu} \)

Physical matter

\[
S^{[m]} = S_g^{[m]}(g, \psi_g) + S_f^{[m]}(f, \psi_f)
\]
Stability in the Early Universe in Bigravity

The background spacetimes:

\[ ds_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2) , \]
\[ ds_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2) . \]

We consider spherically symmetric configurations:

\[ ds_g^2 = a^2(\eta) \left[ -e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2d\Omega^2 \right] , \]
\[ ds_f^2 = K^{-2} a^2(\eta_f) \left[ -e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2d\Omega^2 \right] , \]
\[ \eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r), \]

We assume spacetimes are almost homogenous and isotropic \( \rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, we do “not” assume \( A^\eta \ll 1, \quad A^r \ll 1 \)
Stability in the Early Universe in Bigravity

The background spacetimes:

\[ d\tilde{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2), \]

\[ d\tilde{s}_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2). \]

We consider spherically symmetric configurations:

\[ ds^2 = a^2(\eta)(-d\eta^2 + dr^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2) \]

We assume the background is given by GR solution. → realized by homothetic spacetime

\[ f_{\mu\nu} = K^2 g_{\mu\nu}, \quad K = const \Rightarrow \text{GR solution} \]

Evolution of scale factor is given by GR isotropic → \( \Psi_{g/f}, \Psi_{g/f} \ll 1 \)

However, we do “not” assume \( A^\eta \ll 1, \quad A^r \ll 1 \)
Stability in the Early Universe in Bigravity

The background spacetimes:

We are interested in scalar graviton

→ **Spherically symmetric configurations**

For bigravity, there are 6 independent variables

\[ \text{6} = 2 \ (g_{\mu\nu}) + 2 \ (f_{\mu\nu}) + 2 \ (\text{Stueckelberg fields}) \]

We consider spherically symmetric configurations:

\[
\begin{align*}
    ds_g^2 &= a^2(\eta) \left[ -e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2 \right], \\
    ds_f^2 &= K^2 a^2(\eta_f) \left[ -e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2 \right],
\end{align*}
\]

\[ \eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r), \]

We assume spacetimes are almost homogenous and isotropic → \( \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, we do “not” assume \( A^\eta \ll 1, \quad A^r \ll 1 \)
Stability in the Early Universe in Bigravity

The background spacetimes:

\[ ds_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2), \]
\[ ds_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2). \]

We consider spherically symmetric configurations:

\[ ds_g^2 = a^2(\eta) \left[-e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2 \right], \]
\[ ds_f^2 = K^2 a^2(\eta_f) \left[-e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2 \right], \]
\[ \eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r), \]

We assume spacetimes are almost homogenous and isotropic \( \rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, we do “not” assume \( A^\eta \ll 1 \), \( A^r \ll 1 \)
Strategy

\[ ds_g^2 = a^2(\eta) \left[ -e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2 \right], \]

\[ ds_f^2 = K^2 a^2(\eta_f) \left[ -e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2 \right], \]

\[ \eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r), \]

✓ Assume $\Phi_{g/f}, \Psi_{g/f} \ll 1$, but do not assume $A^\eta \ll 1, A^r \ll 1$

✓ Consider only sub-horizon scale.

✓ Decompose all variables into adiabatic modes and oscillation modes.

\[ X = X^{\text{ad}} + X^{\text{osc}} \quad \text{with} \quad |\partial_\eta X^{\text{ad}}| \sim |a H X^{\text{ad}}|, \]

\[ |\partial_\eta X^{\text{osc}}| \sim |\partial_r X^{\text{osc}}|. \]

\[ X^{\text{ad}} = \langle X \rangle, \quad X^{\text{osc}} \ll 1 \]
Stability in pure graviton case

We concentrate on the early stage of the Universe ($m_{\text{eff}} \ll H$)

We solve the equations up to $\epsilon^2$, $\epsilon \sim aLH \ll 1$

If there is no matter perturbation

$$\rightarrow \Phi_{g/f} \sim (arm_{\text{eff}})^2 \approx 0, \quad \Psi_{g/f} \sim (arm_{\text{eff}})^2 \approx 0$$

Pure scalar graviton solution:

oscillation mode

$$\eta_f \approx \eta - \frac{1}{2} Har^2 (2\mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0)r + \delta r$$

adiabatic mode

where $\mu_0 = 0$ or $O(1)$

$$\delta \eta = -\frac{\partial_{\eta} \pi}{a^2} + \frac{\mu_0 arH}{1 + \mu_0} \frac{\partial_{r} \pi}{a^2}, \quad \delta r = \frac{\partial_{r} \pi + \mu_0 arH \partial_{\eta} \pi}{a^2 (1 + \mu_0)}$$
Stability in pure graviton case

Pure scalar graviton solution: \((\mu_0 = 0 \text{ or } \mathcal{O}(1))\)

\[
\eta_f \approx \eta - \frac{1}{2} H a r^2 (2\mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0) r + \delta r
\]

\[
\delta \eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 a r H}{1 + \mu_0} \frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 a r H \partial_\eta \pi}{a^2(1 + \mu_0)}
\]

Quadratic action: \(\pi\) is the scalar graviton mode

\[
S_2 = \frac{m_{\text{eff}}^2}{\kappa_-^2} \int d\Omega \int d\eta dr (arH)^2 \mathcal{K}_S \left[ (\partial_\eta \pi)^2 - c_S^2 (\partial_r \pi)^2 \right],
\]

\[\checkmark \quad \mu_0 = 0 \Rightarrow \text{Ghost or gradient instability appears for } w < 1\]

\[\checkmark \quad \mu_0 \sim 1 \Rightarrow \text{Stability depends on the background dynamics as well as the coupling constants}\]

\[
b_2^2 - b_1 b_3 > 0, \quad b_2 < 0 \Rightarrow \mathcal{K}_S \geq 0, \quad c_S^2 > 0 \quad \text{for any } w \quad (m_{\text{eff}}^2 > 0)
\]
Stability in pure graviton case

As a result, we find a stable cosmological solution as

\[ ds^2_g \simeq a^2(\eta) \left[ -d\eta^2 + dr^2 + r^2 d\Omega^2 \right], \]
\[ ds^2_f \simeq K^2 a^2(\eta_f) \left[ -d\eta_f^2 + dr_f^2 + r_f^2 d\Omega^2 \right], \]
\[ \eta_f \approx \eta - \frac{1}{2} Har^2(2\mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0)r + \delta r \]
\[ \delta \eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 ar H}{1 + \mu_0} \frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 ar H \partial_\eta \pi}{a^2(1 + \mu_0)} \]

Cosmological evolution is same as the homothetic background.

When \( w > 1 \rightarrow \mu_0 = 0 \) is stable (linear Stueckelberg field)
When \( w < 1 \rightarrow \mu_0 \sim 1 \) is stable (non-linear Stueckelberg field)

Although two spacetimes are homogeneous and isotropic, two foliations are related by the non-linear coordinate transformation.
Including matter perturbations

When there are matter perturbations

\[ \Phi_g \sim \Phi_{\text{GR}} + (am_{\text{eff}})^2, \]
\[ \Psi_g \sim \Psi_{\text{GR}} + (am_{\text{eff}})^2 \]
\[ \Phi_{\text{GR}}, \Psi_{\text{GR}} \sim (arH)^2 \times \tilde{\delta}_g \quad \text{for } \mu \sim 1 \]

The fifth force is screened in

\[ \tilde{\delta}_g := \frac{\int 4\pi r^2 \delta_g dr}{\int 4\pi r^2 dr} \gg \frac{m_{\text{eff}}^2}{H^2} \rightarrow 0 \quad \text{in the early Universe} \]

\[ \leftrightarrow r \ll r_V := \left( \frac{G\delta M}{m_{\text{eff}}^2} \right)^{1/3} \]
\[ G\delta M := G \int 4\pi r^2 \delta \rho_g dr \sim H^2 \int 4\pi r^2 \delta_g dr \]

\[ \rightarrow \text{Vainshtein mechanism on a cosmological background} \]
Including matter perturbations

Although the branch is unstable unless \( w > 1 \), there is a linear adiabatic solution

\[
f_{\mu\nu} \simeq K^2 g_{\mu\nu}
\]

\[
\Phi_g \sim \Phi_{GR} + (a r m_{eff})^2 \times \tilde{\delta}_g,
\]

\[
\Psi_g \sim \Psi_{GR} + (a r m_{eff})^2 \times \tilde{\delta}_g
\]

\[
\Phi_{GR}, \Psi_{GR} \sim (a r H)^2 \times \tilde{\delta}_g \quad \text{for} \quad \mu \ll 1
\]

The fifth force is always screened in \( \frac{m^2_{\text{eff}}}{H^2} \ll 1 \)

The non-linear terms are not necessary for the screening.
Cosmological Vainshtein mechanism

The result is a generalization of the Vainshtein mechanism

Conventional Vainshtein mechanism (on Minkowski)
→ Non-linear terms are necessary to screen the fifth force
  in the case with matter perturbation

Cosmological Vainshtein mechanism (on FLRW)
→ The fifth force can be screened even at linear order.
  Non-linear terms are necessary to stabilize the fluctuation
  even in the case without matter perturbation
Why is the scalar mode stabilized?

It may be interpreted by a mechanism like the ghost condensate.

\[
\mathcal{L}_{\text{eff}} = -\frac{3}{4} (\partial \phi)^2 + \frac{c_{\text{NL}}}{\Lambda^3} (\partial \phi)^2 \Box \phi + \cdots \\
+ \frac{\tilde{R}^{\mu \nu}}{2m^2} \partial_\mu \phi \partial_\nu \phi + \tilde{c}_{\text{NL}} \frac{\tilde{R}^{\mu \nu \rho \sigma}}{m^2} \partial_\mu \phi \partial_\rho \phi \partial_\nu \partial_\sigma \phi + \cdots + \kappa_- \phi \delta T
\]

When \( R_0 \gg m^2 \), \( R_0 \sim R_{\mu \nu} \)

\[
\kappa_{\text{eff}} = \frac{m}{\sqrt{R_0}} \kappa_- \ll \kappa_-
\]

Fifth force can be screened even at linear order.

However, third term produces an instability

\[
\text{e.g., } \tilde{R}^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi = + \Lambda_g (\partial \phi)^2 \rightarrow \text{Higuchi ghost}
\]
Why is the scalar mode stabilized?

\[ \mathcal{L}_{\text{eff}} = -\frac{3}{4} (\partial \phi)^2 + \frac{c_{NL}}{\Lambda^3} (\partial \phi)^2 \Box \phi + \cdots \]

\[ + \frac{\tilde{R}^{\mu \nu}}{2m^2} \partial_\mu \phi \partial_\nu \phi + \frac{\tilde{c}_{NL}}{\Lambda^3} \frac{\tilde{R}^{\mu \nu \rho \sigma}}{m^2} \partial_\mu \phi \partial_\rho \phi \partial_\nu \partial_\sigma \phi + \cdots + \kappa_- \phi \delta T \]

When \( R_0 \gg m^2 \), \( R_0 \sim R_{\mu \nu} \Rightarrow \kappa_{\text{eff}} = \frac{m}{\sqrt{R_0}} \kappa_- \ll \kappa_- \)

Non-zero expectation value \( \langle \pi_0' \rangle \) (= spatial derivative)

\[ \phi = \pi_0 + \pi \]

oscillation mode

adiabatic mode

Although the scalar mode has an inhomogeneity, the spacetime is homogenous due to the screening mechanism.

\[ \text{c.f. Non-zero} \ \langle \dot{\pi}_0 \rangle \text{ can stabilize in the ghost condensation} \]

(Arkani-Hamed, et al., 2004)
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   KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

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Appearance of singularity in strong gravitational field in bigravity

Screening mechanism in bigravity
✓ For weak gravity (Volkov, ’12, Babichev and Crisostomi ’13)
✓ On cosmological background
  (KA, K. Maeda, R. Namba arXiv: 1506.04543)

How about strong gravity effect?

Black holes? (Volkov, ’12, Babichev et al., ’13, Brito et al., ’13, …)

Relativistic stars?
We find a critical value of the gravitational field strength, beyond which the solution turns to a wormhole.
(A singularity appears beyond the critical value)
(KA, K. Maeda, M. Tanabe, in preparation)
Static spherically symmetric spacetimes

\[ ds_g^2 = -N^2_g dt^2 + \frac{dr^2_g}{F^2_g} + r_g^2 d\Omega^2, \]

\[ ds_f^2 = -N^2_f dt^2 + \frac{dr^2_f}{F^2_f} + r_f^2 d\Omega^2, \quad r_f(r_g) = r_g(1 + \mu(r_g)) \]

We find two branches for uniform density \( g \)-star

Two branches can be connected in strong gravity region unless introducing negative c.c. with Vainshtein mechanism

Asymptotically flat

shell singularity

Two branches can be connected in strong gravity region
Static spherically symmetric spacetimes

\[ ds_g^2 = -N_g^2 dt^2 + \frac{dr_g^2}{F_g^2} + r_g^2 d\Omega^2 , \]

\[ ds_f^2 = -N_f^2 dt^2 + \frac{dr_f^2}{F_f^2} + r_f^2 d\Omega^2 , \quad r_f(r_g) = r_g (1 + \mu(r_g)) \]

We find two branches for uniform density stars.

At wormhole throat

\[ \frac{d\mu}{dr_g} \to \infty \iff \frac{dr_f}{dr_g} \to \infty \]

\[ r_f(r_g) = r_g (1 + \mu(r_g)) \]

Asymptotically flat with Vainshtein mechanism

Two branches can be connected in strong gravity region
Wormhole solution?

\[ ds_g^2 = -N_g^2 dt^2 + \frac{dr_g^2}{F_g^2} + r_g^2 d\Omega^2 , \]

\[ ds_f^2 = -N_f^2 dt^2 + \frac{dr_f^2}{F_f^2} + r_f^2 d\Omega^2 , \quad r_f(r_g) = r_g(1 + \mu(r_g)) \]

\[ \frac{dr_f}{dr_g} = 0 \text{ or } \infty \rightarrow \text{Coordinate transformation is singular} \]

\[ \text{not spacetime singularity} \quad R \ldots R \ldots = \text{finite} \]

However, there is a physical degree of freedom in \( r_f(r_g) \)

→ Singularity of Stueckelberg field

Wormhole type solution exists even in vacuum.

→ Singularity appears by strong gravity effect?
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Summary and Discussions

✓ Bigravity theory can explain the origin of dark components.

✓ We show that Higuchi ghost and the gradient instability can be resolved by the nonlinear interactions of the scalar graviton for a cosmological background.

✓ We find an example of the appearance of singularity in the strong gravitational field.

✓ Transition from GR to bigravity (FP theory)?

✓ Bigravity is low energy effective theory? We need high energy physics? Need modification of bigravity?
**Attractor Universe in bigravity**

\[ A = \frac{N_f}{N_g}, \quad B = \frac{a_f}{a_g} \]

Assuming dust dominant

\[ \kappa_g^2 \rho_g = \frac{c_{g,m}}{a_g^3}, \quad \kappa_f^2 \rho_f = \frac{c_{f,m}}{a_f^3} \]

\[ r_m = \frac{c_{g,m}}{c_{f,m}} \]

\( g_{\mu\nu} \) is singular at \( A = \infty \)

\( f_{\mu\nu} \) is singular at \( A = 0 \)

\( \Lambda_g > 0 \) and \( \Lambda_g = 0 \) homothetic spacetime are attractors

*Fixed coupling constants*
Equations of motion

“Graviton” energy-momentum tensors

\[
G^{\mu \nu} = \kappa^2_g \left( T^{[\gamma]}_{\mu \nu} \right) + \left( T^{[m]}_{\mu \nu} \right),
\]

\[
G_{\mu \nu} = \kappa^2_f \left( T^{[\gamma]}_{\mu \nu} \right) + \left( T^{[m]}_{\mu \nu} \right),
\]

Matter energy-momentum tensors

Matter conservation laws

\[
\nabla_{\mu} T^{[m]}_{\mu \nu} = 0, \quad \nabla_{\mu} T^{[m]}_{\mu \nu} = 0,
\]

“Graviton” conservation laws

\[
\nabla_{\mu} T^{[\gamma]}_{\mu \nu} = 0, \quad \nabla_{\mu} T^{[\gamma]}_{\mu \nu} = 0
\]