Cosmological and Astrophysical Vainshtein mechanism in Bigravity

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Based on
KA, K. Maeda, and R. Namba, arXiv: 1506.04543.
KA, K. Maeda, and M. Tanabe, in preparation.

Abstract

Bigravity provides some interesting cosmological phenomena.

Are massive spin-2 theories restored to GR in massless limit?

	Linear theory	Non-linear theory
Weak gravity	Discontinuity	Vainshtein mechanism
Cosmological background	Instability	Stabilization mechanism (KA, K. Maeda, R. Namba arXiv: 1506.04543)
Strong gravity	Not valid	Breaking screening? (KA, K. Maeda, M. Tanabe, in preparation)

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   KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

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   KA, K. Maeda, and R. Namba, arXiv: 1506.04543.

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   KA, K. Maeda, and M. Tanabe, in preparation.

5. Summary and Discussions

Why Massive?

What is the graviton?

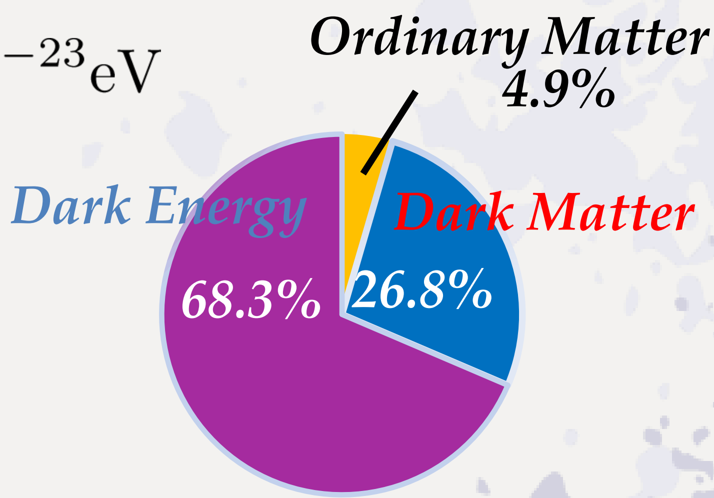
- It must be spin-2 field.
- How about mass? Massless field or Massive field?

GR describes a massless spin-2 field

Is there a theory with a massive spin-2 field?

If there is, which theory describes our Universe?

Experimental constraint: $m < 7.1 \times 10^{-23}\mathrm{eV}$
(from the solar-system experiment)



Dark components hint us that
GR should be modified at large scale.

Massive gravity and Bigravity

In order to add a mass to the graviton, we need two metrics

e.g., Fierz-Pauli theory

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \text{background metric} + \text{spin-2 field}$$

$$S_{\text{massive}} = \frac{1}{2\kappa^2}\int d^4x \left[\mathcal{L}_{\text{EH}} - \frac{m^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^2)\right]$$

Assuming only one metrics is dynamical

$\rightarrow$ Massive gravity = massive spin-2

Assuming both metrics are dynamical

$\rightarrow$ Bigravity theory = massless spin-2 + massive spin-2

Hassan-Rosen bigravity theory

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$

$$- \frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} \boxed{b_i\mathscr{U}_i(g,f)} + S^{[\mathrm{m}]}$$

$\rightarrow$ Accelerating expansion?

$$\kappa^2 = \kappa_g^2 + \kappa_f^2$$

How about gravity-matter coupling?

Twin matters — Doubly coupled matter

$$S^{[\mathrm{m}]} = \boxed{S_g^{[\mathrm{m}]}(g,\phi_g) + S_f^{[\mathrm{m}]}(f,\psi_f)} + S^{[\mathrm{m}]}(g,f,\psi_{\mathbf{double}})$$

Physical matter — Dark matter? — Reappearance of ghost?

(Y. Yamashita et al. '14,
C. de Rham et al. '14)

Bigravity theory in low energy scale

✓ Low energy scale

Can the bigravity explain the origin of dark components?

- Mass term produces an effective cosmological constant
- f-matter behaves as a dark matter in $g_{\mu\nu}$

$$m \sim 10^{-33}\text{eV} \sim \text{Gpc}^{-1} \Rightarrow \text{Dark energy}$$

$$m \gtrsim 10^{-27}\text{eV} \sim \text{kpc}^{-1} \Rightarrow \text{Dark matter + (DE)}$$

KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

*If we admit fine tuning, bigravity can explain both dark components

Experimental constraint: $m < 7.1 \times 10^{-23}\text{eV}$
(from the solar-system experiment)

Bigravity theory in high energy scale

✓ High energy scale (= Graviton mass can be ignored)

The theory should be restored to two GRs

	Linear theory	Non-linear theory
Weak gravity	Discontinuity	Vainshtein mechanism
Cosmological background	Instability	Stabilization mechanism (KA, K. Maeda, R. Namba arXiv: 1506.04543) → Sec. 3
Strong gravity	Not valid	Breaking screening? (KA, K. Maeda, M. Tanabe, in preparation) → Sec. 4

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5. Summary and Discussions

Homothetic solutions in bigravity

If two metrics are proportional, the equation of motion is exactly same as GR with a cosmological constant.

$$f_{\mu\nu} = K^2 g_{\mu\nu},\ K = const \Rightarrow \text{GR solution}$$

$$G_{\mu\nu}(g) + \Lambda_g\, g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}{}_{\mu\nu}\,,$$
$$\mathcal{G}_{\mu\nu}(f) + \Lambda_f\, f_{\mu\nu} = \kappa_f^2 \mathcal{T}^{[\mathrm{m}]}{}_{\mu\nu}$$

with

$$\Lambda_g(K) = K^2 \Lambda_f(K)\,,$$
$$\kappa_f^2 \mathcal{T}^{[\mathrm{m}]}{}_{\mu\nu} = \kappa_g^2\, T^{[\mathrm{m}]}{}_{\mu\nu}$$

where

$$\Lambda_g(K) = m^2 \frac{\kappa_g^2}{\kappa^2}\left(b_0 + 3b_1 K + 3b_2 K^2 + b_3 K^3\right)\,,$$
$$\Lambda_f(K) = m^2 \frac{\kappa_f^2}{\kappa^2}\left(b_4 + 3b_3 K^{-1} + 3b_2 K^{-2} + b_1 K^{-3}\right)$$

K is a root of a quartic equation

$\rightarrow$ There are four different vacuum solutions

Even if we assume the existence of Minkowski spacetime, there can be de Sitter solution as a vacuum solution.

Attractor universe

For a particular coupling constants, we find
de Sitter and AdS solutions as well as Minkowski solution

$\Lambda_g(K_{\rm M}) = 0\,, \quad K = K_{\rm M}$: Minkowski solution

$\Lambda_g(K_{\rm dS}) > 0\,, \quad K = K_{\rm dS}$: de Sitter solution

$\Lambda_g(K_{\rm AdS}) < 0\,, \quad K = K_{\rm AdS}$: AdS solution

We consider the homogenous and isotropic spacetimes

$$ds_g^2 = -N_g^2(t)dt^2 + a_g^2(t)\gamma_{ij}dx^i dx^j\,,$$
$$ds_f^2 = -N_f^2(t)dt^2 + a_f^2(t)\gamma_{ij}dx^i dx^j$$

$\Lambda_g > 0$ homothetic and $\Lambda_g = 0$ homothetic solutions are obtained as attractor solutions as the Universe expands.

KA and K. Maeda, PRD 89, 064051 (2014)

Perturbation around homothetic solution

The Universe approaches the homothetic spacetime.

→ In the low energy scale, spacetimes may be described by linear perturbations around the homothetic spacetime.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{[g]}_{\mu\nu}, \quad f_{\mu\nu} \quad = \bar{f}_{\mu\nu} + K^2 h^{[f]}_{\mu\nu} = K^2 \left(\bar{g}_{\mu\nu} + h^{[f]}_{\mu\nu} \right)$$

Linear perturbation can be decomposed into linearized GR + FP

$$h^{[-]}_{\mu\nu} = h^{[g]}_{\mu\nu} - h^{[f]}_{\mu\nu}$$ ⇐ Massive mode = FP theory

$$h^{[+]}_{\mu\nu} = \frac{m_f^2}{m_{\rm eff}^2} h^{[g]}_{\mu\nu} + \frac{m_g^2}{m_{\rm eff}^2} h^{[f]}_{\mu\nu}$$ ⇐ Massless mode = GR

Massless and massive modes couple to both twin matters.

$$m_{\rm eff}^2 := m_g^2 + m_f^2$$

$$m_g^2 := \frac{m^2 \kappa_g^2}{\kappa^2}(b_1 K + 2b_2 K^2 + b_3 K^3), \; m_f^2 := \frac{m^2 \kappa_f^2}{K^2 \kappa^2}(b_1 K + 2b_2 K^2 + b_3 K^3)$$

Dark matter is f-matter?

$$h^{[-]}_{\mu\nu} = h^{[g]}_{\mu\nu} - h^{[f]}_{\mu\nu}$$ ⇐ Massive mode = FP theory

$$h^{[+]}_{\mu\nu} = \frac{m_f^2}{m_{\text{eff}}^2} h^{[g]}_{\mu\nu} + \frac{m_g^2}{m_{\text{eff}}^2} h^{[f]}_{\mu\nu}$$ ⇐ Massless mode = GR

Massless and massive modes couple to both twin matters.

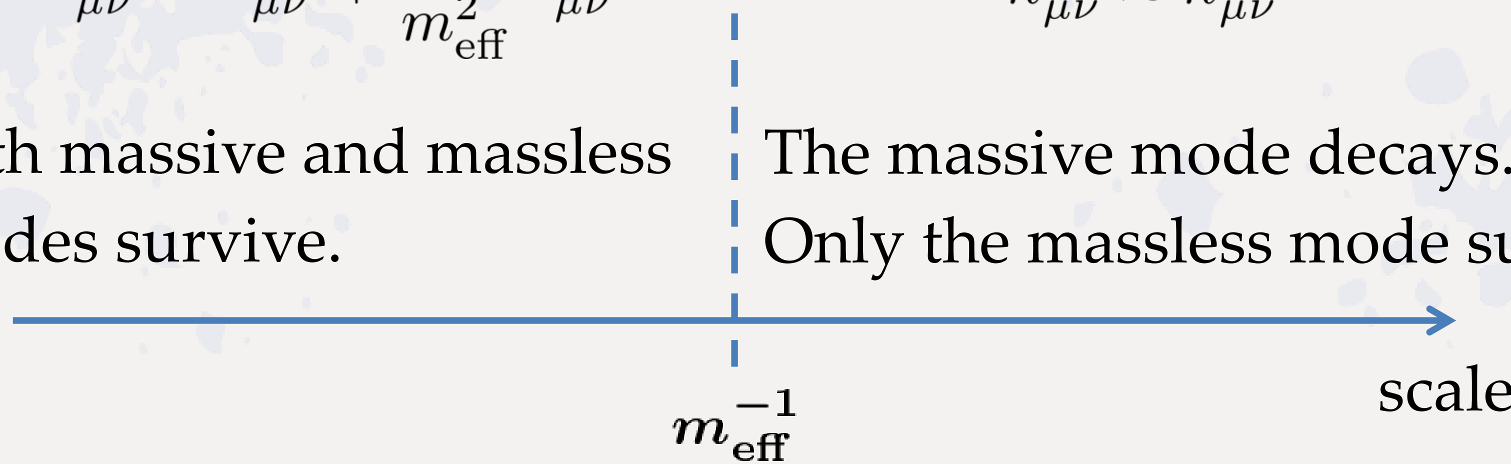
Our spacetime is given by both massive and massless modes.

$$h^{[g]}_{\mu\nu} = h^{[+]}_{\mu\nu} + \frac{m_g^2}{m_{\text{eff}}^2} h^{[-]}_{\mu\nu}$$

$$h^{[g]}_{\mu\nu} \approx h^{[+]}_{\mu\nu}$$

Both massive and massless modes survive.

The massive mode decays.
Only the massless mode survives.



Dark matter is f-matter?

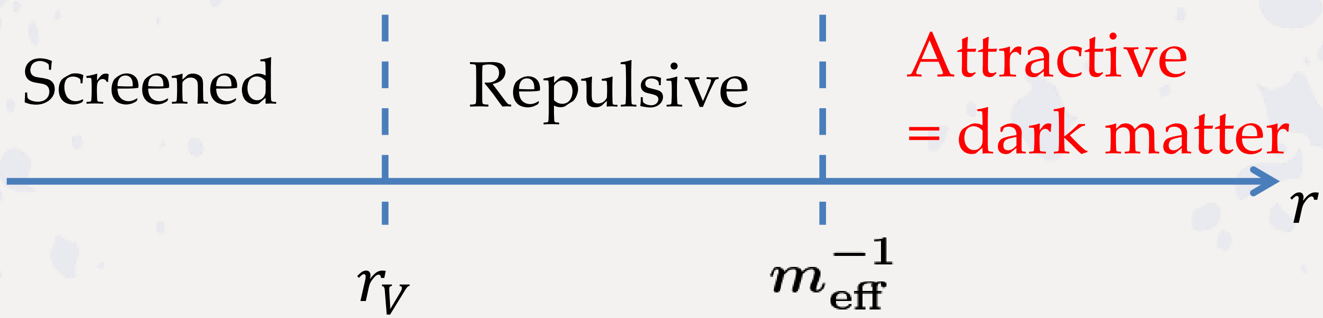
The gravitational potential is induced by f-matter field as well as g-matter field through the interaction terms.

✓ Outside Vainshtein radius

$$\Phi_g = -\frac{GM_g}{r}\left(\frac{m_f^2}{m_{\rm eff}^2} + \frac{4m_g^2}{3m_{\rm eff}^2}e^{-m_{\rm eff}r}\right) - \frac{m_g^2}{m_{\rm eff}^2}\frac{K^2\mathcal{G}\mathcal{M}_f}{r}\left(1 - \frac{4}{3}e^{-m_{\rm eff}r}\right)$$

vDVZ discontinuity

repulsive force in $m_{\rm eff}r \ll 1$



$$r_V := \left(\frac{|GM_g - K^2\mathcal{G}\mathcal{M}_f|}{m_{\rm eff}^2}\right)^{1/3}$$

Dark energy and Dark matter in bigravity

✓ $\Lambda_g > 0$ homothetic and $\Lambda_g = 0$ homothetic solutions are obtained as attractor solutions as the Universe expands.
→ Accelerating expansion is an attractor solution

✓ f-matter can be a dark matter if the graviton mass is large (rotation curve, structure formation, missing mass of Universe)

$m \sim 10^{-33}\text{eV} \sim \text{Gpc}^{-1} \Rightarrow$ Dark energy

$m \gtrsim 10^{-27}\text{eV} \sim \text{kpc}^{-1} \Rightarrow$ Dark matter + (DE)

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5. Summary and Discussions

Viable cosmology with massive graviton

Linear perturbations are stable in the late stage $(H \ll m)$

However, there is an instability in the early stage $(H \gg m)$

(Comelli et al. '12, '14, De Felice et al. '14)

The fact implies that the massive graviton has instability in massless limit on a curved background.

Unstable → We must take into account non-linear interactions

Can we find a viable cosmology with non-linear effects?

For simplicity, we assume the background evolution of the Universe is given by GR (not general cosmological background)

⇔ There is no value of massive spin-2 field in the background

Instability of massive graviton on curved spacetime

We consider a linear massive spin-2 field,
where the background spacetime is give by GR solution.

→ There is no value of massive spin-2 field in the background.

The action is given by linearized EH action with FP mass term

$$S_{\text{massive}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\bar{g}} \left[\mathcal{L}_{\text{EH}}[h; \Lambda_g] - \frac{m^2}{4} (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

We focus on the scalar graviton mode π.

To recover gauge symmetry, we introduce Stueckelberg fields as

$$h_{\mu\nu} \to h_{\mu\nu} + 2\bar{\nabla}_{(\mu} A_{\nu)} + 2\bar{\nabla}_\mu \bar{\nabla}_\nu \pi$$

Instability of massive graviton on curved spacetime

Fierz-Pauli theory on the FLRW background

✓ Higuchi ghost (Higuchi, 1972, Grisa and Sorbo, 2010)

de Sitter background (or the accelerating universe) with $m/H \to 0$.

⇨ Scalar graviton has ghost instability

✓ Gradient instability (Grisa and Sorbo, 2010)

the decelerating universe $(-1/3 < w < 1)$ with $m/H \to 0$.

⇨ Scalar graviton has gradient instability

Massive graviton on the FLRW background is unstable in the massless limit!

Why? Massive field should be massless field in $m/H \to 0$.

Hassan-Rosen Bigravity theory

Unstable → We must take into account non-linear interactions

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f) + S^{[\mathrm{m}]}$$

$$\kappa^2 = \kappa_g^2 + \kappa_f^2$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdots}\epsilon_{\cdots\cdots}(\gamma^\mu{}_\nu)^n$$

with $\gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu}$

Physical matter

$$S^{[\mathrm{m}]} = S_g^{[\mathrm{m}]}(g,\psi_g) + S_f^{[\mathrm{m}]}(f,\psi_f)$$

Stability in the Early Universe in Bigravity

The background spacetimes:

$$d\bar{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,,$$
$$d\bar{s}_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,.$$

We consider spherically symmetric configurations:

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2\right],$$
$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2\right],$$
$$\eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r),$$

We assume spacetimes are almost homogenous and isotropic $\rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1$

However, we do "not" assume $A^\eta \ll 1\,, \quad A^r \ll 1$

Stability in the Early Universe in Bigravity

The background spacetimes:

$$d\bar{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,,$$
$$d\bar{s}_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,.$$

We conside[r sph]erically symmetric configurations:

$$ds_g^2 = a \ldots d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2]$$

We assume the background is given by GR solution.
→ realized by homothetic spacetime

$$f_{\mu\nu} = K^2 g_{\mu\nu},\ K = const \Rightarrow \text{GR solution}$$

Evolution of scale factor is given by GR

We a…
isotropic → $\Psi_{g/f}, \Psi_{g/f} \ll 1$

However, we do "not" assume $A^\eta \ll 1\,, \quad A^r \ll 1$

Stability in the Early Universe in Bigravity

The

We are interested in scalar graviton
$\rightarrow$ Spherically symmetric configurations

For bigravity, there are 6 independent variables
$$6 = 2\,(g_{\mu\nu}) + 2\,(f_{\mu\nu}) + 2\,(\text{Stueckelberg fields})$$

We consider spherically symmetric configurations:

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g}d\eta^2 + e^{2\Psi_g}dr^2 + r^2 d\Omega^2\right],$$
$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f}d\eta_f^2 + e^{2\Psi_f}dr_f^2 + r_f^2 d\Omega^2\right],$$
$$\eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r),$$

We assume spacetimes are almost homogenous and isotropic $\rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1$

However, we do "not" assume $A^\eta \ll 1\,, \quad A^r \ll 1$

Stability in the Early Universe in Bigravity

The background spacetimes:

$$d\bar{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,,$$
$$d\bar{s}_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,.$$

We consider spherically symmetric configurations:

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2\right],$$
$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2\right],$$
$$\eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r),$$

We assume spacetimes are almost homogenous and isotropic $\rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1$

However, we do "not" assume $A^\eta \ll 1\,, \quad A^r \ll 1$

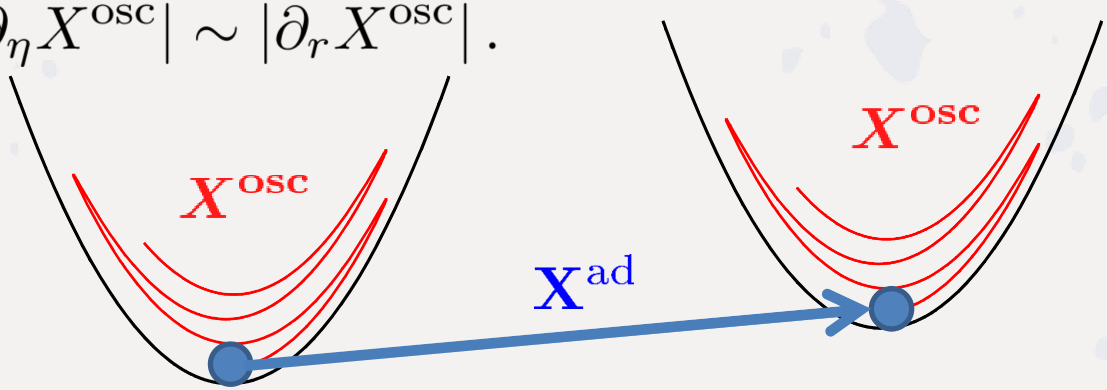
Strategy

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g}d\eta^2 + e^{2\Psi_g}dr^2 + r^2 d\Omega^2\right],$$
$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f}d\eta_f^2 + e^{2\Psi_f}dr_f^2 + r_f^2 d\Omega^2\right],$$
$$\eta_f = \eta + A^\eta(\eta, r), \quad r_f = r + A^r(\eta, r),$$

- ✓ Assume $\Phi_{g/f}, \Psi_{g/f} \ll 1$, but do not assume $A^\eta \ll 1$, $A^r \ll 1$
- ✓ Consider only sub-horizon scale.
- ✓ Decompose all variables into adiabatic modes and oscillation modes.

$$X = X^{\rm ad} + X^{\rm osc} \qquad X^{\rm ad} = \langle X \rangle$$

with
$$|\partial_\eta X^{\rm ad}| \sim |aHX^{\rm ad}|, \qquad X^{\rm osc} \ll 1$$
$$|\partial_\eta X^{\rm osc}| \sim |\partial_r X^{\rm osc}|.$$



Stability in pure graviton case

We concentrate on the early stage of the Universe ($m_{\rm eff} \ll H$)

We solve the equations up to ϵ^2. $\epsilon \sim aLH \ll 1$

If there is no matter perturbation

$$\rightarrow \quad \Phi_{g/f} \sim (arm_{\rm eff})^2 \approx 0, \quad \Psi_{g/f} \sim (arm_{\rm eff})^2 \approx 0$$

Pure scalar graviton solution: oscillation mode

$$\eta_f \approx \eta - \frac{1}{2} Har^2(2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0)r + \delta r$$

adiabatic mode

where $\mu_0 = 0$ or $\mathcal{O}(1)$

$$\delta\eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 arH}{1+\mu_0}\frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 arH \partial_\eta \pi}{a^2(1+\mu_0)}$$

Stability in pure graviton case

Pure scalar graviton solution: $(\mu_0 = 0 \text{ or } \mathcal{O}(1))$

$$\eta_f \approx \eta - \frac{1}{2}Har^2(2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0)r + \delta r$$

$$\delta\eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 arH}{1+\mu_0}\frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 arH \partial_\eta \pi}{a^2(1+\mu_0)}$$

Quadratic action: π is the scalar graviton mode

$$S_2 = \frac{m_{\rm eff}^2}{\kappa_-^2}\int d\Omega \int d\eta dr (arH)^2 \mathcal{K}_S \left[(\partial_\eta \pi)^2 - c_S^2 (\partial_r \pi)^2\right],$$

✓ $\mu_0 = 0 \;\Rightarrow$ Ghost or gradient instability appears for $w < 1$

✓ $\mu_0 \sim 1 \;\Rightarrow$ Stability depends on the background dynamics as well as the coupling constants

$b_2^2 - b_1 b_3 > 0, b_2 < 0 \Rightarrow \mathcal{K}_S \geq 0, c_S^2 > 0$ for any w $(m_{\rm eff}^2 > 0)$

Stability in pure graviton case

As a result, we find a stable cosmological solution as

$$ds_g^2 \simeq a^2(\eta)\left[-d\eta^2 + dr^2 + r^2 d\Omega^2\right],$$

$$ds_f^2 \simeq K^2 a^2(\eta_f)\left[-d\eta_f^2 + dr_f^2 + r_f^2 d\Omega^2\right],$$

$$\eta_f \approx \eta - \frac{1}{2} H a r^2 (2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0) r + \delta r$$

$$\delta\eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 a r H}{1+\mu_0}\frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 a r H \partial_\eta \pi}{a^2(1+\mu_0)}$$

Cosmological evolution is same as the homothetic background.

When $w > 1 \rightarrow \mu_0 = 0$ is stable (linear Stueckelberg field)

When $w < 1 \rightarrow \mu_0 \sim 1$ is stable (non-linear Stueckelberg field)

Although two spacetimes are homogeneous and isotropic, two foliations are related by the non-linear coordinate transformation.

Including matter perturbations

When there are matter perturbations

$$\rightarrow \quad \Phi_g \sim \Phi_{\rm GR} + (arm_{\rm eff})^2,$$
$$\Psi_g \sim \Psi_{\rm GR} + (arm_{\rm eff})^2$$
$$\Phi_{\rm GR}, \Psi_{\rm GR} \sim (arH)^2 \times \tilde{\delta}_g \qquad \text{for } \mu \sim 1$$

The fifth force is screened in

$$\tilde{\delta}_g := \frac{\int 4\pi r^2 \delta_g dr}{\int 4\pi r^2 dr} \gg \frac{m_{\rm eff}^2}{H^2} \to 0 \quad \text{in the early Universe}$$

$$\Leftrightarrow r \ll r_{\rm V} := \left(\frac{G\delta M}{m_{\rm eff}^2}\right)^{1/3}$$

$$G\delta M := G \int 4\pi r^2 \delta\rho_g dr$$
$$\sim H^2 \int 4\pi r^2 \delta_g dr$$

$\rightarrow$ Vainshtein mechanism on a cosmological background

Including matter perturbations

Although the branch is unstable unless $w > 1$,
there is a linear adiabatic solution $\quad f_{\mu\nu} \simeq K^2 g_{\mu\nu}$

$$\rightarrow \quad \Phi_g \sim \Phi_{\rm GR} + (arm_{\rm eff})^2 \times \tilde{\delta}_g,$$

$$\Psi_g \sim \Psi_{\rm GR} + (arm_{\rm eff})^2 \times \tilde{\delta}_g$$

$$\Phi_{\rm GR}, \Psi_{\rm GR} \sim (arH)^2 \times \tilde{\delta}_g \qquad \text{for } \mu \ll 1$$

The fifth force is always screened in $\frac{m_{\rm eff}^2}{H^2} \ll 1$

The non-linear terms are not necessary for the screening.

Cosmological Vainshtein mechanism

The result is a generalization of the Vainshtein mechanism

Conventional Vainshtein mechanism (on Minkowski)

→ Non-linear terms are necessary to screen the fifth force in the case with matter perturbation

Cosmological Vainshtein mechanism (on FLRW)

→ The fifth force can be screened even at linear order. Non-linear terms are necessary to stabilize the fluctuation even in the case without matter perturbation

Why is the scalar mode stabilized?

It may be interpreted by a mechanism like the ghost condensate.

$$\mathcal{L}_{\rm eff} = -\frac{3}{4}(\partial\phi)^2 + \frac{c_{\rm NL}}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots$$
$$+ \frac{\bar{R}^{\mu\nu}}{2m^2}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{c}_{\rm NL}}{\Lambda^3}\frac{\bar{R}^{\mu\nu\rho\sigma}}{m^2}\partial_\mu\phi\partial_\rho\phi\,\partial_\nu\partial_\sigma\phi + \cdots + \kappa_-\phi\delta T$$

When $R_0 \gg m^2$, $\quad R_0 \sim R_{\mu\nu}$

$$\kappa_{\rm eff} = \frac{m}{\sqrt{R_0}}\kappa_- \ll \kappa_-$$

Fifth force can be screened even at linear order.

However, third term produces an instability

e.g., $\bar{R}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = +\Lambda_g(\partial\phi)^2 \rightarrow$ Higuchi ghost

Why is the scalar mode stabilized?

$$\mathcal{L}_{\rm eff} = -\frac{3}{4}(\partial\phi)^2 + \frac{c_{\rm NL}}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots$$
$$+ \frac{\bar{R}^{\mu\nu}}{2m^2}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{c}_{\rm NL}}{\Lambda^3}\frac{\bar{R}^{\mu\nu\rho\sigma}}{m^2}\partial_\mu\phi\partial_\rho\phi\,\partial_\nu\partial_\sigma\phi + \cdots + \kappa_-\phi\delta T$$

When $R_0 \gg m^2$, $R_0 \sim R_{\mu\nu}$ $\Rightarrow \kappa_{\rm eff} = \frac{m}{\sqrt{R_0}}\kappa_- \ll \kappa_-$

Non-zero expectation value $\langle\pi_0'\rangle$ (= spatial derivative) can stabilize the fluctuation.

$\phi = \pi_0 + \pi$ ← oscillation mode

↖ adiabatic mode

Although the scalar mode has an inhomogeneity, the spacetime is homogenous due to the screening mechanism.

c.f. Non-zero $\langle\dot{\pi}_0\rangle$ can stabilize in the ghost condensation (Arkani-Hamed, et al., 2004)

Contents

1. ~~Introduction~~

2. ~~The late Universe in bigravity~~

    KA and K. Maeda, PRD 89, 064051 (2014); PRD 90, 124089 (2014)

3. ~~The early Universe in bigravity~~

    KA, K. Maeda, and R. Namba, arXiv: 1506.04543.

4. Strong gravity effects

    KA, K. Maeda, and M. Tanabe, in preparation.

5. Summary and Discussions

Appearance of singularity in strong gravitational field in bigravity

Screening mechanism in bigravity

- ✓ For weak gravity (Volkov, '12, Babichev and Crisostomi '13)
- ✓ On cosmological background

(KA, K. Maeda, R. Namba arXiv: 1506.04543)

How about strong gravity effect?

Black holes? (Volkov, '12, Babichev et al., '13, Brito et al., '13, …)

Relativistic stars?

We find a critical value of the gravitational field strength, beyond which the solution turns to a wormhole.
(A singularity appears beyond the critical value)

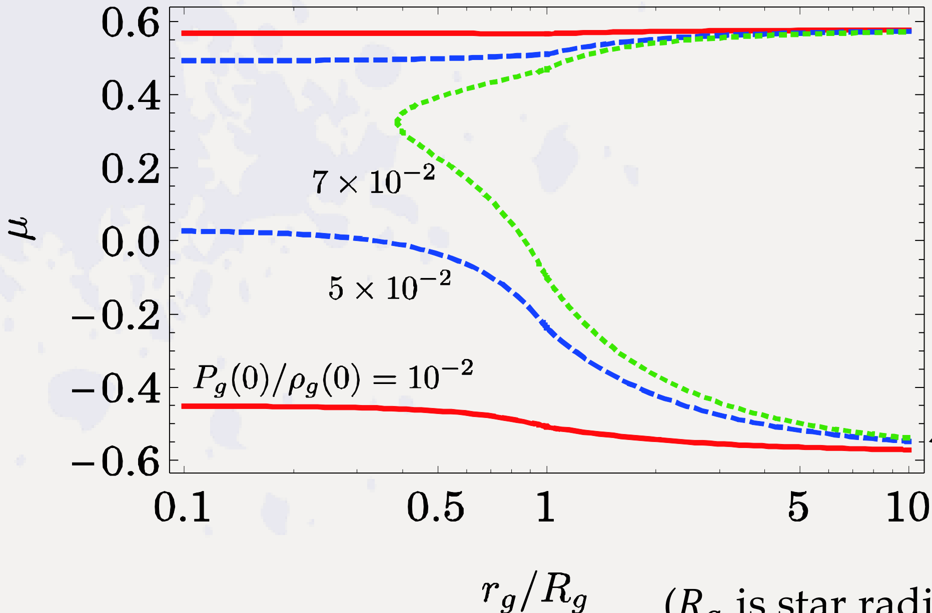
(KA, K. Maeda, M. Tanabe, in preparation)

Static spherically symmetric spacetimes

$$ds_g^2 = -N_g^2 dt^2 + \frac{dr_g^2}{F_g^2} + r_g^2 d\Omega^2 \,,$$

$$ds_f^2 = -N_f^2 dt^2 + \frac{dr_f^2}{F_f^2} + r_f^2 d\Omega^2 \,, \quad r_f(r_g) = r_g(1 + \mu(r_g))$$

We find two branches for uniform density g-star



(R_g is star radius)

shell singularity
unless introducing negative c.c.

Asymptotically flat
with Vainshtein mechanism

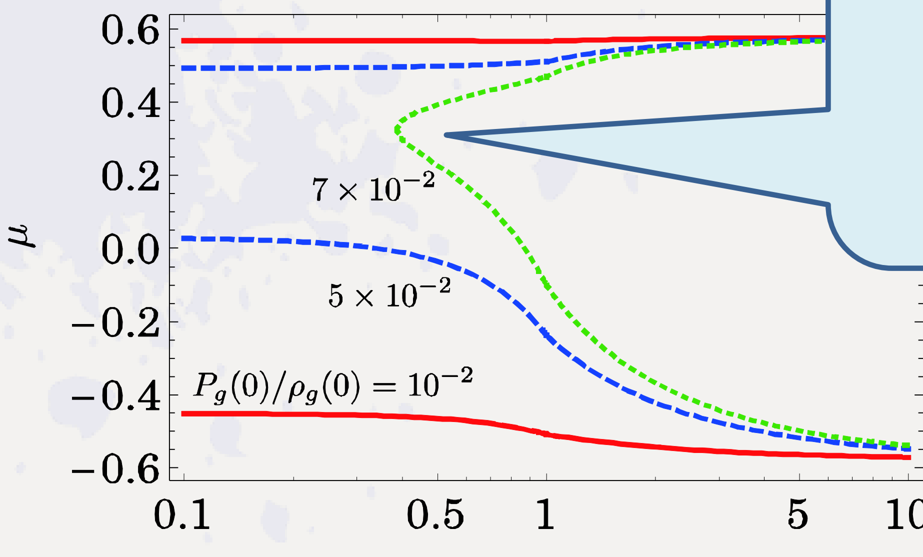
Two branches can be connected in strong gravity region

Static spherically symmetric spacetimes

$$ds_g^2 = -N_g^2 dt^2 + \frac{dr_g^2}{F_g^2} + r_g^2 d\Omega^2 \,,$$

$$ds_f^2 = -N_f^2 dt^2 + \frac{dr_f^2}{F_f^2} + r_f^2 d\Omega^2 \,, \quad r_f(r_g) = r_g(1+\mu(r_g))$$

We find two branches for unifo



r_g/R_g (R_g is star radius)

At wormhole throat

$$\frac{d\mu}{dr_g} \to \infty \Leftrightarrow \frac{dr_f}{dr_g} \to \infty$$

$$r_f(r_g) = r_g(1+\mu(r_g))$$

Asymptotically flat

with Vainshtein mechanism

Two branches can be connected in strong gravity region

Wormhole solution?

$$ds_g^2 = -N_g^2 dt^2 + \frac{dr_g^2}{F_g^2} + r_g^2 d\Omega^2 \,,$$

$$ds_f^2 = -N_f^2 dt^2 + \frac{dr_f^2}{F_f^2} + r_f^2 d\Omega^2 \,, \quad r_f(r_g) = r_g(1+\mu(r_g))$$

$\frac{dr_f}{dr_g} = 0$ or ∞ $\rightarrow$ Coordinate transformation is singular

not spacetime singularity $R_{\cdots\cdot}R^{\cdots\cdot} = \text{finite}$

However, there is a physical degree of freedom in $r_f(r_g)$

$\rightarrow$ Singularity of Stueckelberg field

Wormhole type solution exists even in vacuum.

$\rightarrow$ Singularity appears by strong gravity effect?

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5. Summary and Discussions

Summary and Discussions

- ✓ Bigravity theory can explain the origin of dark components.
- ✓ We show that Higuchi ghost and the gradient instability can be resolved by the nonlinear interactions of the scalar graviton for a cosmological background.
- ✓ We find an example of the appearance of singularity in the strong gravitational field.
- ✓ Transition from GR to bigravity (FP theory)?
- ✓ Bigravity is low energy effective theory?
  We need high energy physics? Need modification of bigravity?

Attractor Universe in bigravity

$A = N_f/N_g, B = a_f/a_g$

Assuming dust dominant

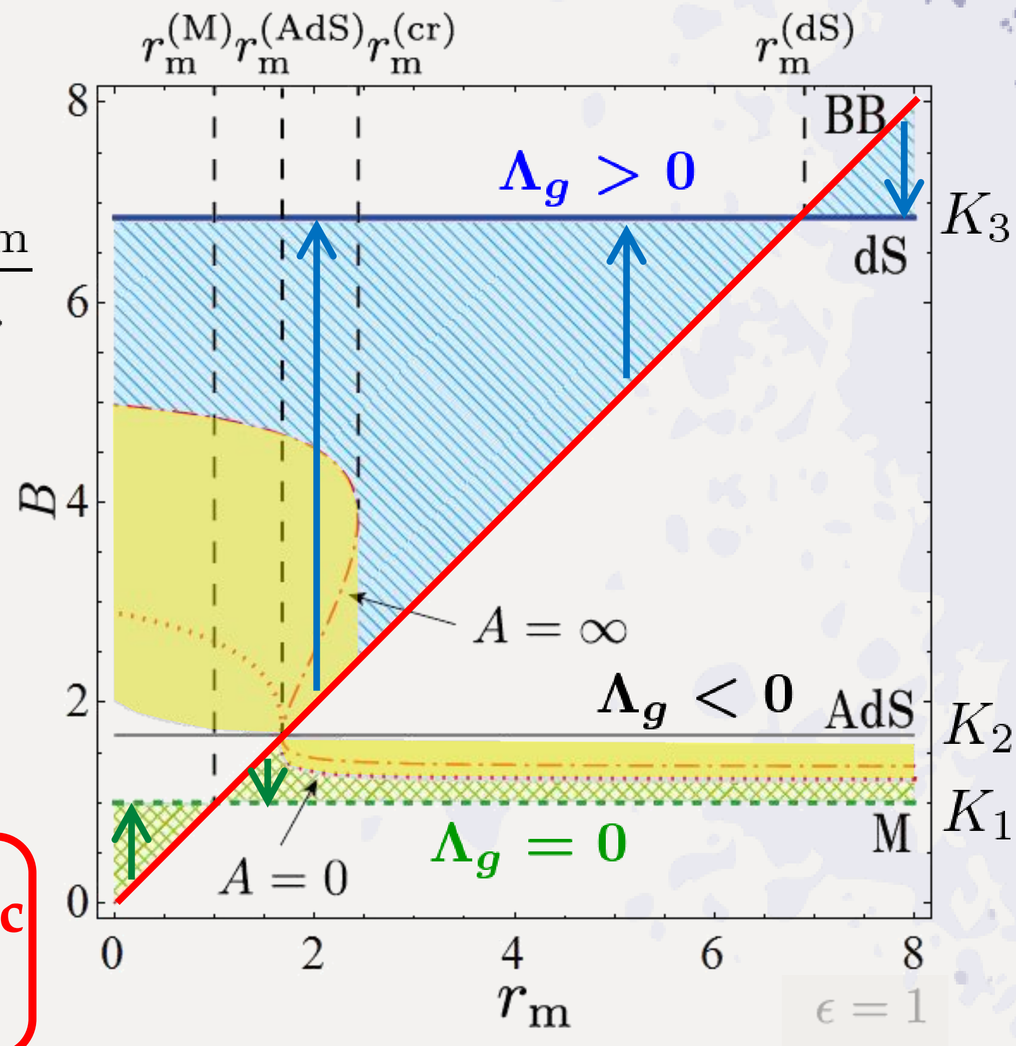
$$\kappa_g^2 \rho_g = \frac{c_{g,\mathrm{m}}}{a_g^3}, \quad \kappa_f^2 \rho_f = \frac{c_{f,\mathrm{m}}}{a_f^3}$$

$$r_\mathrm{m} = c_{g,\mathrm{m}}/c_{f,\mathrm{m}}$$

$g_{\mu\nu}$ is singular at $A = \infty$

$f_{\mu\nu}$ is singular at $A = 0$

$\Lambda_g > 0$ and $\Lambda_g = 0$ homothetic spacetime are attractors



*Fixed coupling constants

Equations of motion

"Graviton" energy-momentum tensors

$$G^{\mu}{}_{\nu} = \kappa_g^2 (T^{[\gamma]\mu}{}_{\nu} + T^{[\mathrm{m}]\mu}{}_{\nu}),$$
$$\mathcal{G}^{\mu}{}_{\nu} = \kappa_f^2 (\mathcal{T}^{[\gamma]\mu}{}_{\nu} + \mathcal{T}^{[\mathrm{m}]\mu}{}_{\nu}),$$

Matter energy-momentum tensors

Matter conservation laws

$$\overset{(g)}{\nabla}_{\mu} T^{[\mathrm{m}]\mu}{}_{\nu} = 0\,, \quad \overset{(f)}{\nabla}_{\mu} \mathcal{T}^{[\mathrm{m}]\mu}{}_{\nu} = 0\,,$$

"Graviton" conservation laws

$$\overset{(g)}{\nabla}_{\mu} T^{[\gamma]\mu}{}_{\nu} = 0\,, \quad \overset{(f)}{\nabla}_{\mu} \mathcal{T}^{[\gamma]\mu}{}_{\nu} = 0$$