Reconstructing the Inflaton Potential from n_s

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Introduction

- recent Planck data $n_s \sim 0.96$ favors

  $n_s = 1 - 2/N$ (for $N \sim 50 \sim 60$)

← quadratic chaotic inflation ϕ^2

Starobinsky model/Higgs inflation $V_0\left(1 - e^{-\sqrt{2/3}\phi}\right)$

or T-model (alpha –attractor) $\tanh^2(\phi/\sqrt{6\alpha})$

what else? consider "→": Given $n_s(N)$, reconstruct $V(\phi)$

1. $V(\phi)$ from $n_s(N)$
2. example: $n_s - 1 = -2/N$
3. reheating temperature

$V(\phi)$ from ns(N)

Reconstructing V(ϕ) from ns(N)

Program:

- Step 1: Reconstruct V(N) via slow-roll parameters from ns(N)

$$n_s - 1 = -6\varepsilon + 2\eta \qquad \varepsilon = \frac{V'^2}{2V^2},\ \eta = \frac{V''}{V}$$

- Step 2: Rewrite N as a function of ϕ via

$$N = \int^{\phi} \frac{V}{V'} d\phi \quad \text{or} \quad dN = \frac{V}{V'} d\phi$$

In terms of N

$$V' = \frac{V}{V'}\frac{dV}{dN} \equiv \frac{V}{V'}V_{,N} \quad \text{from} \quad dN = \frac{V}{V'}d\phi$$

assuming V'>o we have $V' = \sqrt{VV_{,N}}$

similarly, $V'' = \dfrac{V_{,N}{}^2 + VV_{,NN}}{2V_{,N}}$

Therefore, $n_s - 1 = -2\dfrac{V_{,N}}{V} + \dfrac{V_{,NN}}{V_{,N}} = \left(\ln\dfrac{V_{,N}}{V^2}\right)_{,N}$

Moreover,

$$\frac{V'}{V}dN = d\phi \quad \text{becomes} \quad \sqrt{\frac{V_{,N}}{V}}dN = d\phi$$

example: n_s-1=-2/N

$$n_s - 1 = -\frac{2}{N} = \left(\ln\frac{V_{,N}}{V^2}\right)_{,N}$$

- integrated once

$$\frac{\alpha}{N^2} = \frac{V_{,N}}{V^2} = -\left(\frac{1}{V}\right)_{,N} \qquad \alpha > 0 \; (V_{,N} > 0)$$

- second integration

$$V(N) = \frac{1}{\frac{\alpha}{N} + \beta}$$

(α>0, β: integration constants)

example: n_s-1=-2/N

- slow-roll condition :

$$\varepsilon = \frac{V_{,N}}{2V} = \frac{1}{2\left(N + \frac{\beta}{\alpha} N^2\right)} << 1$$

$$N >> 1 \ (\beta \geq 0) \qquad N << -\frac{\alpha}{\beta} \ (\beta < 0)$$

N(ϕ)

- given V(N), N can be written as a function of ϕ :

$$\int \sqrt{\frac{\alpha}{N(\alpha+\beta N)}}dN = \int d\phi \quad from \quad \sqrt{\frac{V_{,N}}{V}}dN = d\phi$$

$$N(\phi) = \frac{\alpha}{\beta}\sinh^2\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha}}(\phi-\phi_0)\right) \qquad for \quad \beta > 0$$

$$N(\phi) = -\frac{\alpha}{\beta}\sin^2\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha}}(\phi-\phi_0)\right) \cong \frac{1}{4}(\phi-\phi_0)^2 \quad for \quad \beta < 0$$

$$N(\phi) = \frac{1}{4}(\phi-\phi_0)^2 \qquad for \quad \beta = 0$$

V(ϕ)

$$V=\frac{1}{\frac{\alpha}{N}+\beta}=\begin{cases}\frac{1}{\beta}\tanh^2\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha}}(\phi-\phi_0)\right)\cong\frac{1}{\beta}\left(1-4e^{-\sqrt{\frac{\beta}{\alpha}}(\phi-\phi_0)}\right) & for\quad \beta>0\\ -\frac{1}{\beta}\tan^2\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha}}(\phi-\phi_0)\right)\cong\frac{1}{4\alpha}(\phi-\phi_0)^2 & for\quad \beta<0\\ \frac{1}{4\alpha}(\phi-\phi_0)^2 & for\quad \beta=0\end{cases}$$

- Reduces to T-model or quadratic chaotic model

r and running

- tensor-scalar ratio:

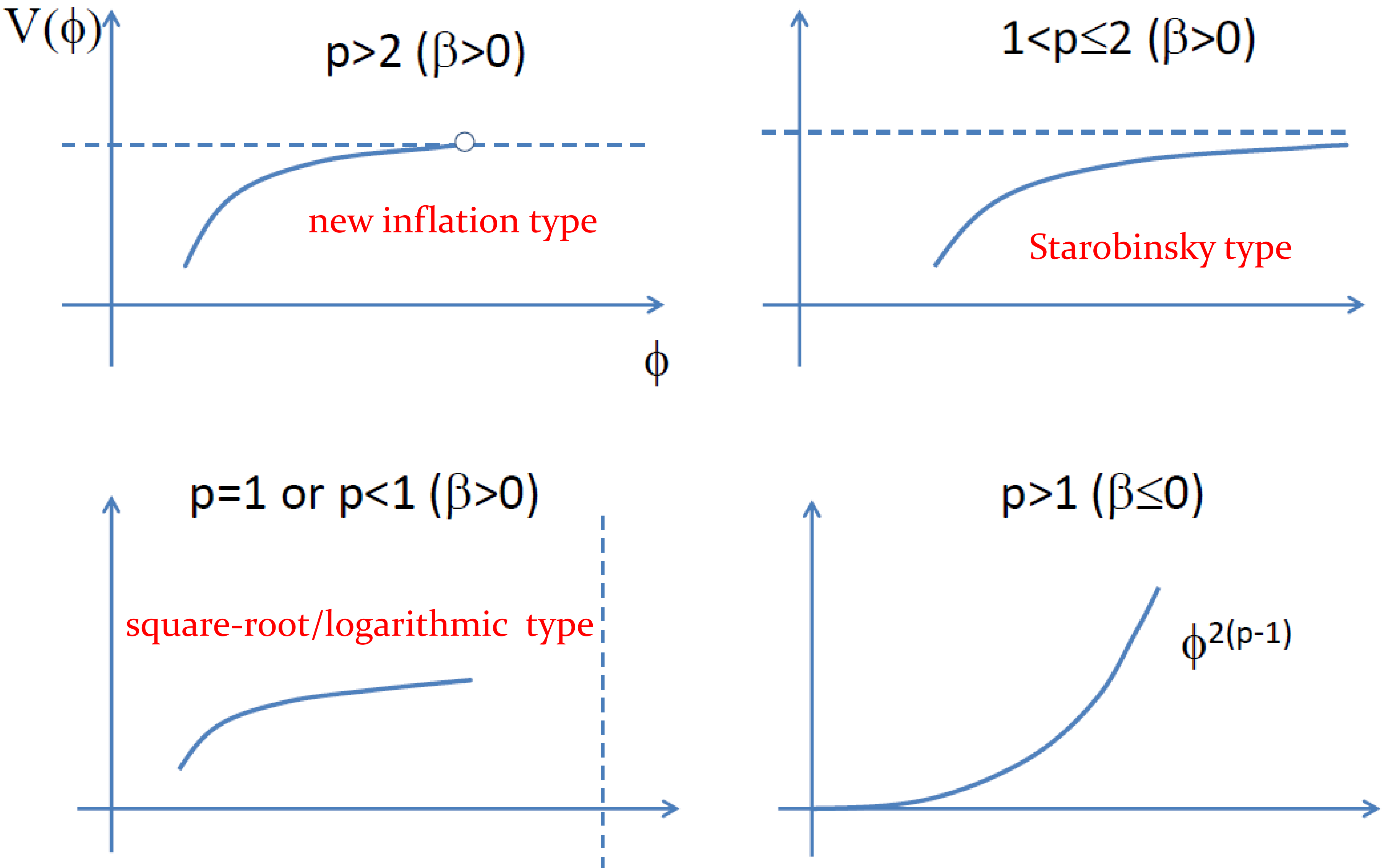
$$r = 16\varepsilon = 8\frac{V_{,N}}{V} = \begin{cases} \dfrac{8}{N + \dfrac{\beta}{\alpha}N^2} \cong 3\times 10^{-3}\dfrac{\alpha}{\beta}\left(\dfrac{50}{N}\right)^2 & for \quad \beta > 0 \\ \dfrac{8}{N} & for \quad \beta \le 0 \end{cases}$$

- running of the spectral index:

$$\frac{dn_s}{d\ln k} = -\frac{dn_s}{dN} = -\frac{2}{N^2} \approx -8\times 10^{-4}\left(\frac{50}{N}\right)^2 < 0$$

$n_s-1=-p/N$

$$V(N)=\frac{1}{\alpha N^{1-p}/(p-1)+\beta}$$



Summary (so far)

- Recipe for $V(\phi)$ from $n_s(N)$

- For $n_s - 1 = -2/N$:

$$V(\phi) \propto \begin{cases} \tanh^2\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha}}\phi\right) & \text{(Starobinsky type/T-model)} \\ \phi^2 \end{cases}$$

- For $n_s - 1 = -p/N$:
  $V(\phi)$: new inflation type ($p>2$) /
  Starobinsky type ($2 \geq p > 1$) /
  logarithmic ($p=1$) / square-root ($p<1$)/
  power law $\phi^{2(p-1)}$ ($p>1$)
- the running of the spectral index is definitely negative
  $\sim -10^{-3}$ ← consistency check

reheating temperature

reheating temperature and r

- given V(N), we can relate reheating temperature T_{RH} to N assuming V(N) is still valid for small N

- for the mode k=aH

$$\frac{k}{a_0 H_0} = \frac{a}{a_{end}} \frac{a_{end}}{a_{RH}} \frac{a_{RH}}{a_0} \frac{H}{H_0}$$

$$e^{-N}$$

$$\rho_{RH} / \rho_{end} = (a_{end} / a_{RH})^3$$

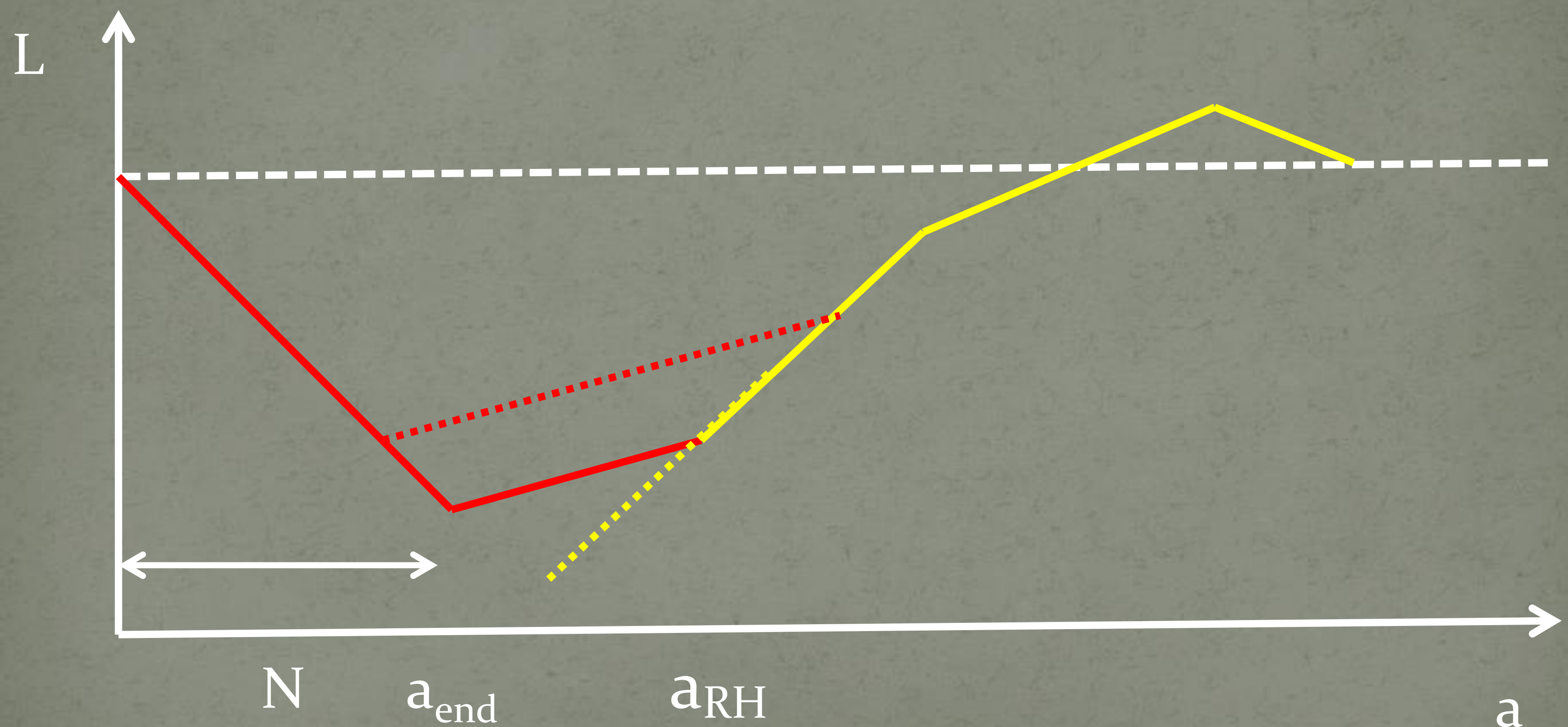
$$g_{S,RH} a_{RH}^3 T_{RH}^3 = \frac{43}{11} g_{S,RH} a_0^3 T_{RH}^3$$

$$H^2 = V/3 = \frac{\pi^2}{2} r A_s$$

$$N = 56.9 - \ln\frac{k}{a_0 H_0} - \ln\frac{h}{0.67} - \frac{1}{3}\ln\frac{\rho_{end}}{V(N)} + \frac{1}{3}\ln\frac{T_{RH}}{10^9\,\mathbf{GeV}} + \frac{1}{6}\ln r(N)$$

- ns-1=-2/N case
- k=0.05/Mpc

$$1/k = 1/aH \propto \begin{cases} a^{-1} & for \;\; w = -1 \; (inflation) \\ a^{1/2} & for \;\; w = 0 \; (reheating, matter) \\ a & for \;\; w = 1/3 \; (radiation) \end{cases}$$

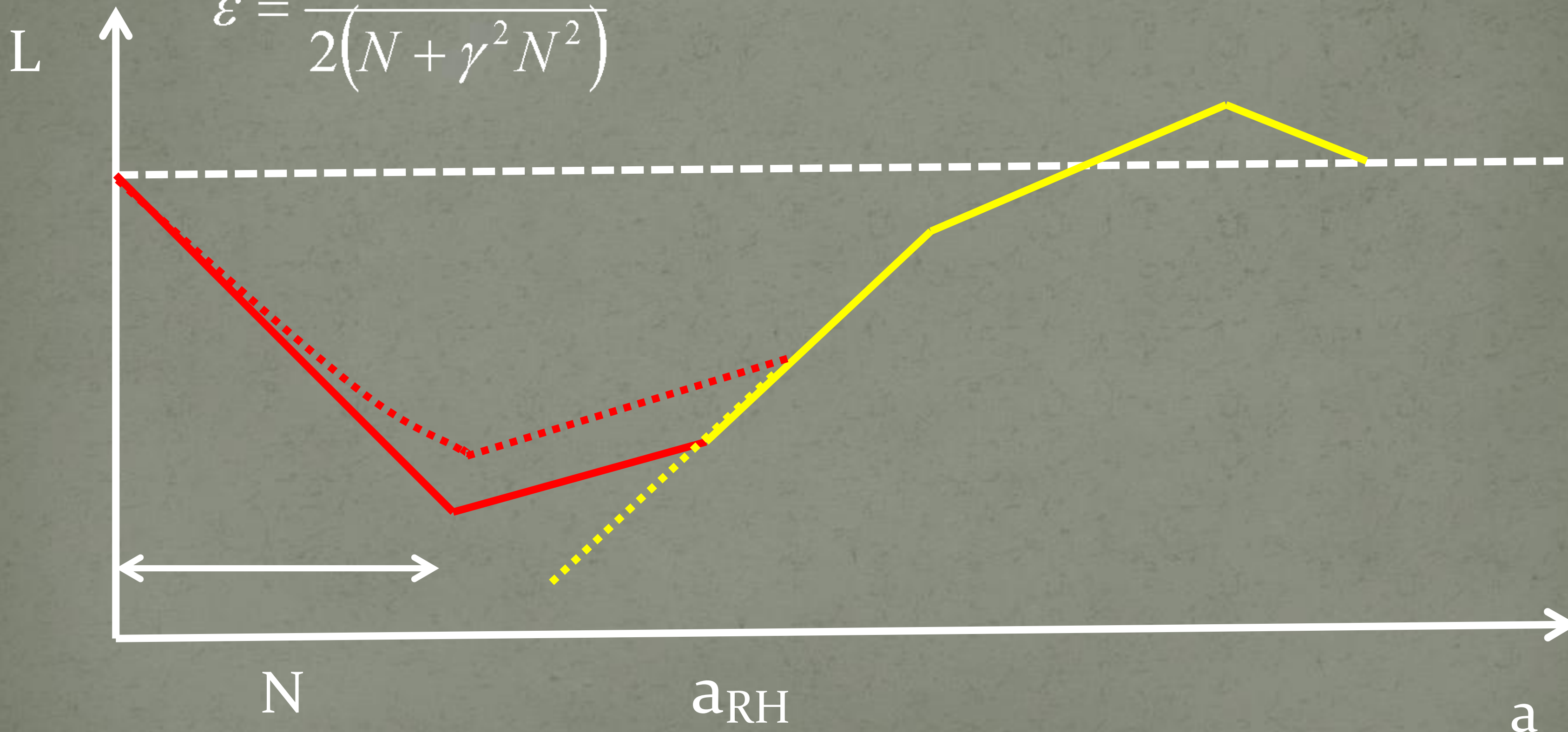


- shorter N $\rightarrow$ late reheating $\rightarrow$ lower reheating temperature

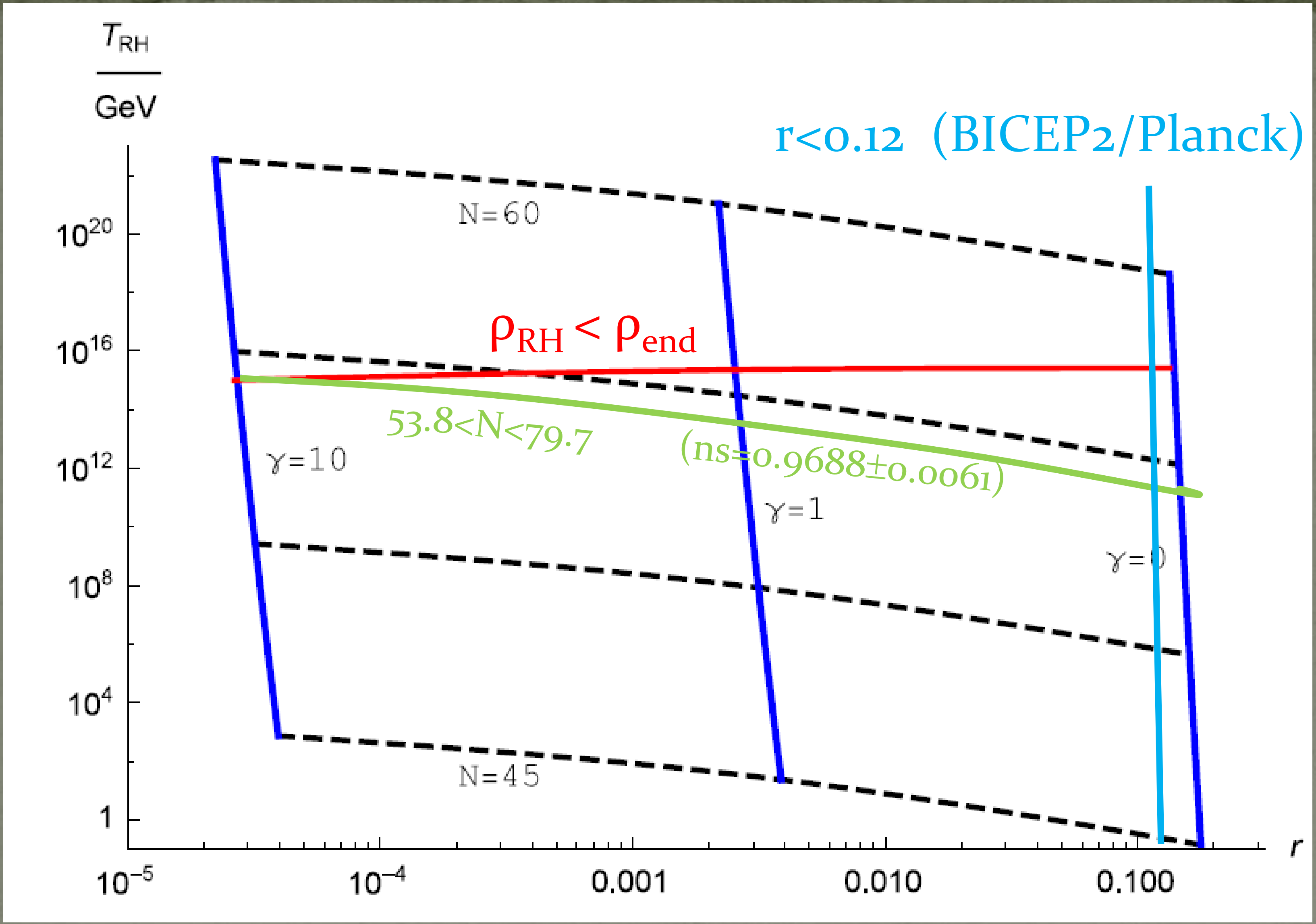
$$\gamma = \sqrt{\beta / \alpha} \geq 0$$

$$V = \frac{1}{\beta} \frac{1}{1 + \frac{1}{\gamma N}} = \frac{1}{\beta} \tanh^2\left(\frac{1}{2} \gamma (\phi - \phi_0) \right)$$

$$\varepsilon = \frac{1}{2\left(N + \gamma^2 N^2\right)}$$



- smaller γ → lower reheating temperature



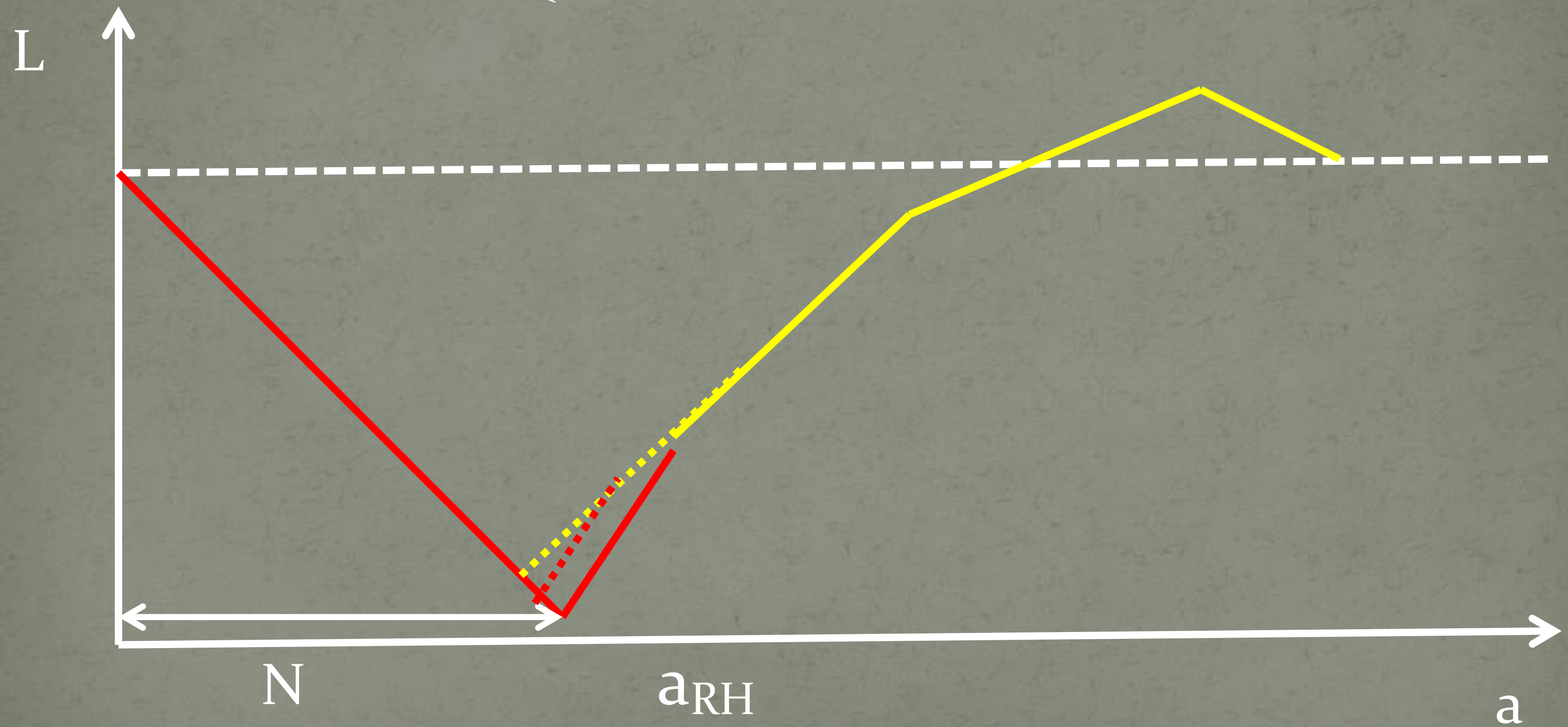
k=0.05/Mpc $\gamma=\sqrt{(\beta/\alpha)}$

$0.1<\gamma<10$ 10^{11} GeV< $T_{RH}<2\times 10^{15}$ GeV

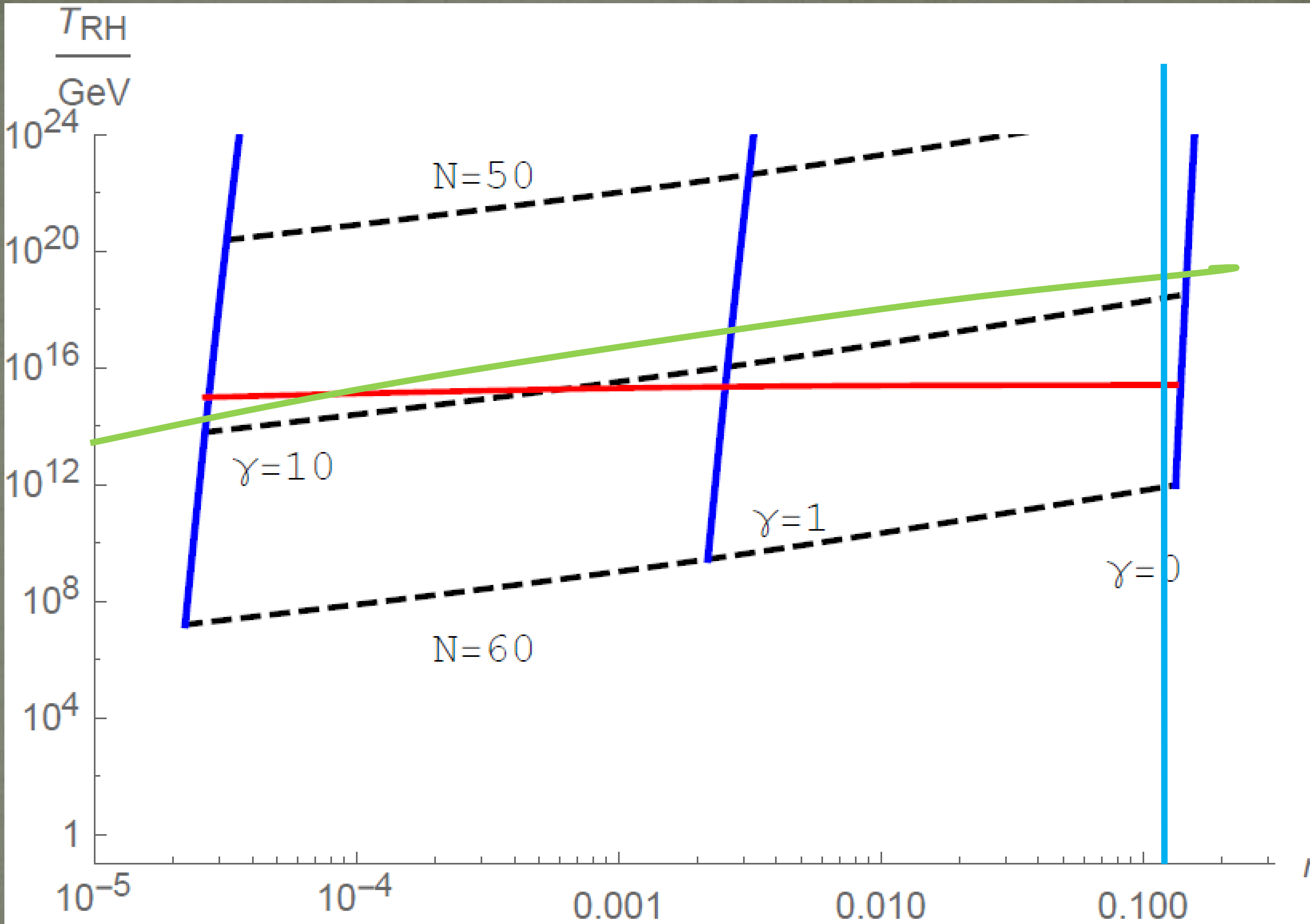
$r > 3\times 10^{-5}$

- varying the EOS during reheating
- from $w=-1/3$ to $w=1$

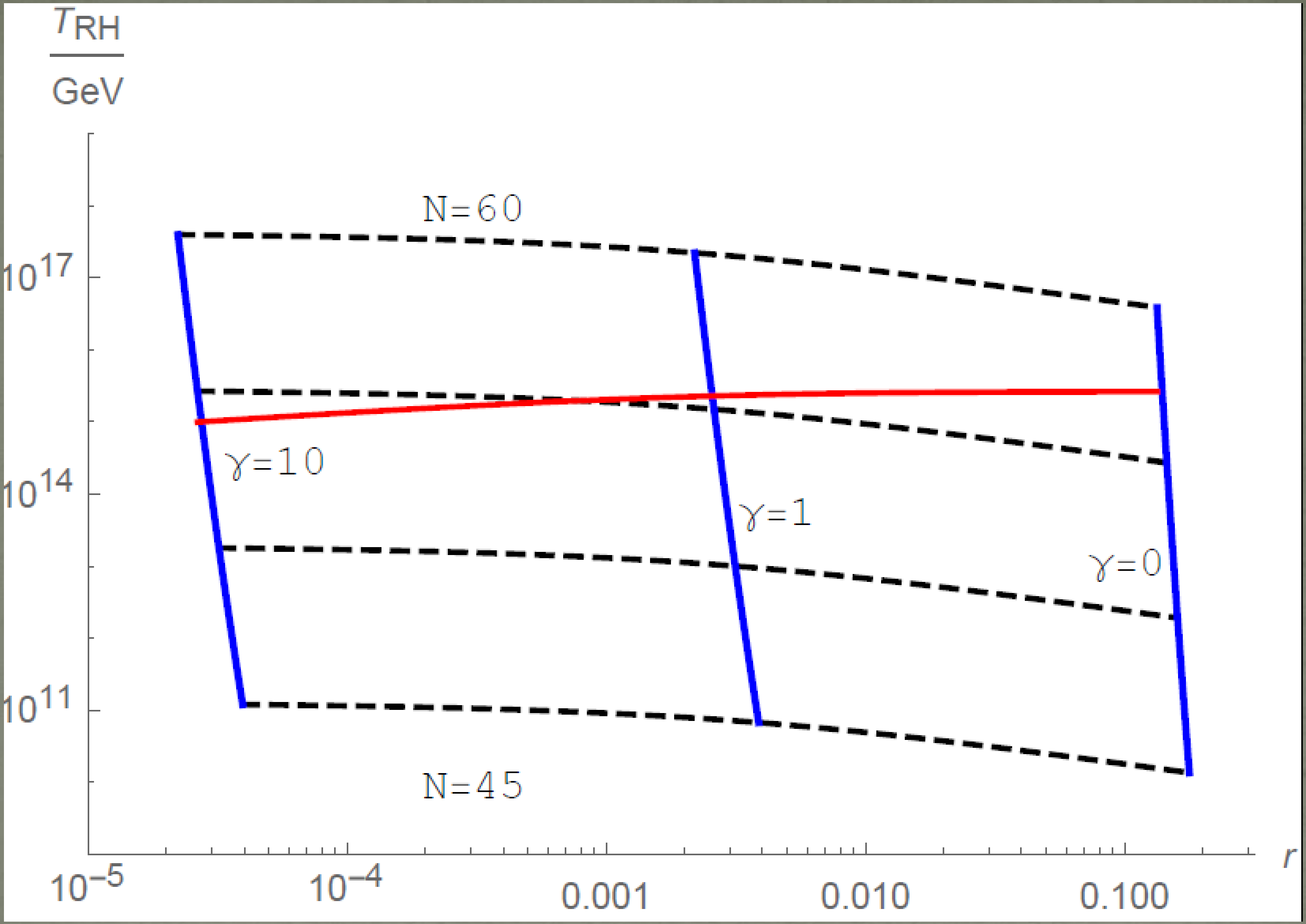
$$1/k = 1/aH \propto \begin{cases} a^{-1} & for \quad w = -1\ (inflation) \\ a^{2} & for \quad w = 1\ (kination) \\ a & for \quad w = 1/3\ (radiation) \\ a^{1/2} & for \quad w = 0\ (matter) \end{cases}$$



- shorter N → higher reheating temperature



- w=1 $0.1<\gamma$ $T_{RH}<2\times 10^{15}$ GeV



$\frac{T_{RH}}{GeV}$
N=60
10^{17}
γ=10
10^{14}
γ=1
γ=0
10^{11}
N=45
10^{-5}
10^{-4}
0.001
0.010
0.100
r

- w=-1/3

Summary

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Summary(continued)

- reheating temperature

- assuming w=0 during reheating

$0.1<\gamma<10$ 10^{11} GeV$<T_{RH}<2\times 10^{15}$ GeV

$r>3\times 10^{-5}$ for

$$V=\frac{1}{\beta}\frac{1}{1+\frac{1}{\gamma N}}=\frac{1}{\beta}\tanh^2\left(\frac{1}{2}\gamma(\phi-\phi_0)\right)$$

- reheating temperature depends strongly on the EOS