Minimal theory of massive gravity

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Introduction

- dRGT theory: deep insight into massive gravity
  [de Rham, Gabadadze, Tolley: PRL 2011]
- Difficult to find viable phenomenology
- No BD ghost [Boulware, Deser: PRD 1972]
- But other ghost are present in simple/crucial backgrounds
  [ADF, Gumrukcuoglu, Mukohyama: PRL 2012]

Motivation – Key idea

- Is it possible to make the dRGT idea work?
- The theory needs to be changed
- Bigravity, quasidilaton, …
  [de Rham etal: IJMP '14; D'Amico etal: PRD '13; ADF, Mukohyama: PLB '14]
- What if we make the theory simpler?
- Remove unwanted degrees of freedom
- Massive gravity with less than 5 dof: need to break LI

Breaking Lorentz invariance

- Usual 4D vielbein approach $g_{\mu\nu}=\eta_{AB}e^{A}{}_{\mu}e^{B}{}_{\nu}$
- Invariant under a vielbein local Lorentz transf: $e^{A}{}_{\mu}\rightarrow\Lambda^{A}{}_{C}e^{C}{}_{\mu}$
- Split 4D into 1+ 3, and remove local Lorentz transformation: this fixes a preferred frame
- Introduce the following variables $N, N_i, e^{I}{}_{j}$
- Define 3D metric $\gamma_{ij}=\delta_{MN}e^{M}{}_{i}e^{N}{}_{j}$

Initial variables

- We have 9 variables, $e^I{}_j$, 3 shifts: N_i , lapse N
- Define $N^i = \gamma^{ij} N_j , \quad N^I = e^I{}_j N^j$
- Build up ADM 4D vielbein $$e^A{}_\mu = \begin{pmatrix} N & \vec{0}^T \\ N^I & e^I{}_j \end{pmatrix}$$
- No boost: 13 vars instead of 16 (general 4D vielbein)
- Metric in ADM form: $g_{\mu\nu} = \eta_{AB} e^A{}_\mu e^B{}_\nu$

Fiducial variables

- Along the same lines fiducial variables $M, \; M_i \; E^I{}_j$
- 3D fiducial metric $\tilde{\gamma}_{ij} = \delta_{MN} E^M{}_i E^N{}_j$
- ADM 4D unboosted fiducial vielbein

$$E^A{}_\mu = \begin{pmatrix} M & \vec{0}^T \\ M^I & E^I{}_j \end{pmatrix}, \qquad M^I = E^I{}_j \tilde{\gamma}^{jk} M_k$$

- Unitary gauge: fixed-dynamics external fields

Precursor Lagrangian

- EH term for physical metric

$$\mathscr{L}_{\text{EH}} = \sqrt{-g}\, R(g), \quad M_P^2 = 2$$

- Mass term:

$$\mathscr{L}_0 = \frac{m^2}{24} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A{}_\alpha E^B{}_\beta E^C{}_\gamma E^D{}_\delta$$

$$\mathscr{L}_1 = \frac{m^2}{6} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A{}_\alpha E^B{}_\beta E^C{}_\gamma e^D{}_\delta \qquad \mathscr{L}_2 = \frac{m^2}{4} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A{}_\alpha E^B{}_\beta e^C{}_\gamma e^D{}_\delta$$

$$\mathscr{L}_3 = \frac{m^2}{6} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A{}_\alpha e^B{}_\beta e^C{}_\gamma e^D{}_\delta \qquad \mathscr{L}_4 = \frac{m^2}{24} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} e^A{}_\alpha e^B{}_\beta e^C{}_\gamma e^D{}_\delta$$

- Total

$$\mathscr{L}_{\text{TOT}} = \mathscr{L}_{\text{EH}} + \sum_{i=0}^{4} c_i \mathscr{L}_i$$

- Same in form as dRGT but asymmetrical ADM vielbein

Precursor Hamiltonian

- Consider 3D vielbein as fundamental variables
- Physical lapse and shift as Lagrange multipliers
- Canonical momentum of 3D vielbein $\pi^{jk} = \Pi^{j}{}_{I}\delta^{IJ}e_{J}{}^{k}, \quad e_{J}{}^{k}e^{I}{}_{k} = \delta_{J}{}^{I}$
- Symmetry $P^{[MN]} = e^{M}{}_{j}\Pi^{j}{}_{I}\delta^{IN} - e^{N}{}_{j}\Pi^{j}{}_{I}\delta^{IM} \approx 0$
- Hamiltonian linear in lapse and shift
- Mass term does not include shift variables

Precursor Hamiltonian/constraints

- Primary constraints: $H^{(1)} = \int d^3x \left[-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} \right]$
- Time derivative of constraints

$$\tilde{C}_\tau, \quad \tau = 1,2, \quad Y^{[MN]} = \delta^{ML} E_L{}^j e^N{}_j - \delta^{NL} E_L{}^j e^M{}_j, \qquad E_L{}^j E^M{}_j = \delta_L{}^M$$

- Precursor Hamiltonian

$$H^{(2)}_{\text{pre}} = \int d^3x \left[-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \tilde{\lambda}^\tau \tilde{C}_\tau + \beta_{MN} Y^{[MN]} \right]$$

Degrees of freedom for precursor theory

- Phase space variables: 2x9 = 18 psv: $e^I{}_j, \ \Pi^j{}_I$
- All constraints are independent and second-class

$$R_0, \ R_i, \ P^{[MN]}, \ Y^{[MN]}, \ \tilde{C}_\tau \ (\tau = 1,2)$$

- Number of dof: (18 – 1 – 3 – 3 – 3 – 2)/2 = 3
- Reason: vielbein in ADM form (breaking LI) in mass term
- Still we try to find a theory with 2 dof

Introducing further constraints

- We need to introduce extra constraints
- Without overkilling the modes
- Without killing interesting backgrounds
- Notice that on the constraint surface

$$H_{\mathrm{pre}} \approx \boldsymbol{H}_1 \equiv m^2 \int d^3x\, M\, H_1$$

Constraints

- Reconsider the time-evolution of the primary constraints

$$\{R_{0,} H_{\text{pre}}\} \approx 0\,, \quad \{R_i, H_{\text{pre}}\} \approx 0\,,$$

- But, as seen before, they introduce only 2 secondary constraints
- Then we constrain the model by imposing all 4 derivatives of primary constraints to vanish

$$C_0 \sim \dot{R}_0 \approx 0, \qquad C_i \sim \dot{R}_i \approx 0,$$

Hamiltonian of the theory

- Define then the 4 constraints (2 only are new)

$$C_0 \equiv \{R_0, \boldsymbol{H}_1\} + \frac{\partial R_0}{\partial t}, \qquad C_i \equiv \{R_i, \boldsymbol{H}_1\}$$

Therefore new Hamiltonian

$$H = \int d^3x [-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} + \lambda C_0 + \lambda^i C_i]$$

- Dof: (9x2 – 1 – 3 – 3 – 3 – 4)/2 = 2

Building blocks

- We have

$$R_0 = R_0^{\mathrm{GR}} - m^2 H_0 ,$$

$$R_0^{\mathrm{GR}} = \sqrt{\gamma}\, R[\gamma] - \frac{1}{\sqrt{\gamma}} \left(\gamma_{nl} \gamma_{mk} - \frac{1}{2} \gamma_{nm} \gamma_{kl} \right) \pi^{nm} \pi^{kl} ,$$

$$R_i = R_i^{\mathrm{GR}} = 2 \gamma_{ik} D_j \pi^{kj} ,$$

$$H_0 = \sqrt{\tilde{\gamma}} \left(c_1 + c_2 Y_I{}^I \right) + \sqrt{\gamma} \left(c_3 X_I{}^I + c_4 \right), \quad X_I{}^J = e_I{}^l E^J{}_l , \quad X_I{}^L Y_L{}^J = \delta_I{}^J ,$$

$$H_1 = \sqrt{\tilde{\gamma}} \left[c_1 Y_I{}^I + \frac{c_2}{2} \left(Y_I{}^I Y_J{}^J - Y_I{}^J Y_J{}^I \right) \right] + c_3 \sqrt{\gamma}$$

Consequences

- The theory is now given
- 14 second-class independent constraints
- The theory is defined via the Hamiltonian (breaking Lorentz invariance)
- Possible to define a Lagrangian

Cosmology

- Consider a time-dependent diagonal $M(t),\ E^I{}_j = \tilde{a}(t)\,\delta^I{}_j$
- Symmetry of $Y^{IJ} = \delta^{IM} E_M{}^l e^J{}_l$ → $e^J{}_l$ also symmetric
- On the background $N(t),\ e^I{}_j = a(t)\,\delta^I{}_j$
- The constraints $C_i \approx 0$ are trivially satisfied on FLRW
- The constraint $C_0 \approx 0$ equivalent to Bianchi identity

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0, \quad X = \tilde{a}/a, \quad H = \dot{a}/(Na)$$

Two branches

- Two branches solutions exist
- Self accelerating branch $$X = X_{\pm} = \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1}$$

  X is constant
- Normal branch $\dot{X} + N H X - M H = 0$
- For both branches $\lambda = 0$

Friedmann evolution

- Friedmann equation

$$3M_P^2H^2=\frac{m^2M_P^2}{2}\left(c_4+3c_3X+3c_2X^2+c_1X^3\right)+\rho$$

- Same background evolution of dRGT
- Self accelerating branch: effective cosmological constant
- No extra constraint: C_0 reduces to Bianchi identity

Stability of the background

- In dRGT 3 out of 5 dof are non-dynamical
- Ghosts are present (not BD ghost)
- In this theory only 2 dof exist

$$S=\frac{M_P^2}{2}\sum_{\lambda=+,x}\int d^4x\,N\,a^3\left[\frac{\dot{h}_\lambda^2}{N^2}-\frac{(\partial h_\lambda)^2}{a^2}-\mu^2 h_\lambda^2\right],$$

$$\mu^2=\frac{1}{2}m^2 X\left[(c_2X+c_3)+(c_1X+c_2)\frac{M}{N}\right]$$

Stability of background

- Only two degrees of freedom
- The 2 dof are tensor
- Stable for $\mu^2 > 0$
- Therefore it is possible to have FLRW (even de Sitter)

Phenomenology

- Only tensor modes propagate (besides matter field)
- No extra scalar/vector mode arises from gravity
- No need of screening any extra force
- No need of Vainshtein mechanism
  [Vainshtein: PLB 1972]
- No Higuchi ghost will be present (only tensor modes)
  [Higuchi: NPB 1987]

Constraints

- The self-accelerating branch induces an effective cosmological constant
- For the normal branch (for non-trivial dynamics of $M, \tilde{a}$) the background is non-trivial but no scalar dof is present
- Constraint coming from modification of emission rate of Gws from binaries $\mu < 10^{-5}$ Hz [Finn, Sutton: PRD 2002]

Conclusions

- Massive gravity extensions
- Reducing dof to only 2
- Only tensors modes remain
- FLRW becomes stable
- Phenomenology simplifies and constraints get weaker
- Gws are massive: phenomenology (sharp peak in GW spectrum)
  [Gumrukcuoglu, Kuroyanagi, Lin, Mukohyama, Tanahashi: CQG 2012]