Dark energy and non-linear power spectrum

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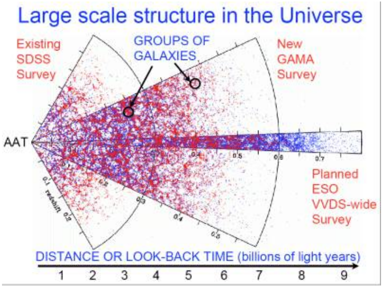
Based on S. G. Biern and JG, 1505.02972 [astro-ph.CO]

Outline

Why GR in LSS?

Planned galaxy surveys: DESI, HETDEX, LSST, Euclid, WFIRST...



Larger and larger volumes, eventually accessing the scales comparable to the horizon: beyond Newtonian gravity, fully general relativistic approach (or any modification) is necessary

Why dark energy in non-linear regime?

- DE was negligible at very early times
- DE becomes significant at later stage when non-linearities in cosmic structure are developed

Naturally DE affects the evolution of gravitational instability, so that its effects emerge more prominently at non-linear level

What are the effects of DE in non-linear regime of LSS?

Newtonian theory

3 basic equations for density perturbation $\delta \equiv \delta\rho/\bar{\rho}$, peculiar velocity $\boldsymbol{u}$ and gravitational potential Φ with a *pressureless* fluid

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\boldsymbol{u} = -\frac{1}{a}\nabla\cdot(\delta\boldsymbol{u}) \qquad \text{continuity eq}$$

$$\dot{\boldsymbol{u}} + H\boldsymbol{u} + \frac{1}{a}\nabla\Phi = -\frac{1}{a}(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} \qquad \text{Euler eq}$$

$$\frac{\Delta}{a^2}\Phi = 4\pi G\bar{\rho}\delta \qquad \text{Poisson eq}$$

Newtonian system is closed at 2nd order

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = -\frac{1}{a^2}\frac{d}{dt}\left[a\nabla\cdot(\delta\boldsymbol{u})\right] + \frac{1}{a^2}\nabla\cdot(\boldsymbol{u}\cdot\nabla\boldsymbol{u})$$

$\longrightarrow$ at linear order, $\delta_+ \propto a$ (growing) and $\delta_- \propto a^{-3/2}$ (decaying)

(Bernardeau et al. 2002)

Basic non-linear equations

Based on the ADM metric

$$ds^2 = -N^2(dx^0)^2 + \gamma_{ij}\left(N^i dx^0 + dx^i\right)\left(N^j dx^0 + dx^j\right)$$

the fully non-linear equations are (Bardeen 1980)

$$R - \overline{K}^i{}_j \overline{K}^j{}_i + \frac{2}{3}K^2 - 16\pi G E = 0$$

$$\overline{K}^j{}_{i;j} - \frac{2}{3}K_{,i} = 8\pi G J_i$$

$$\frac{K_{,0}}{N} - \frac{K_{,i}N^i}{N} + \frac{N^{;i}{}_{;i}}{N} - \overline{K}^i{}_j \overline{K}^j{}_i - \frac{1}{3}K^2 - 4\pi G(E+S) = 0$$

$$\frac{\overline{K}^i{}_{j,0}}{N} - \frac{\overline{K}^i{}_{j;k}N^k}{N} + \frac{\overline{K}_{jk}N^{i;k}}{N} - \frac{\overline{K}^i{}_k N^k{}_{;j}}{N} = K\overline{K}^i{}_j - \frac{1}{N}\left(N^{;i}{}_{;j} - \frac{\delta^i{}_j}{3}N^{;k}{}_{;k}\right) + \overline{R}^i{}_j - 8\pi G \overline{S}^i{}_j$$

$$\frac{E_{,0}}{N} - \frac{E_{,i}N^i}{N} - K\left(E + \frac{S}{3}\right) - \overline{K}^i{}_j \overline{S}^j{}_i + \frac{\left(N^2 J^i\right)_{;i}}{N^2} = 0$$

$$\frac{J_{i,0}}{N} - \frac{J_{i;j}N^j}{N} - \frac{J_j N^j{}_{;i}}{N} - KJ_i + \frac{EN_{,i}}{N} + S^j{}_{i;j} + \frac{S^j{}_i N_{,j}}{N} = 0$$

Fluid quantities: $E \equiv n_\mu n_\nu T^{\mu\nu}$, $J_i \equiv -n_\mu T^\mu{}_i$, $S_{ij} \equiv T_{ij}$

Einstein-de Sitter universe

Usually, structure formation is described in EdS

$$T_{\mu\nu} = \rho_m u_\mu u_\nu \longrightarrow J_i = S_{ij} = 0$$

- Linear growth factor is all: $D_1 = a$, $D_2 = 3D_1^2/7$ and so on
- Comoving gauge ($\gamma = 0$ and $T^0{}_i = 0$) gives identical equations to the Newtonian counterparts up to 2nd order
- Pure GR contribution appears from 3rd order and is totally sub-dominant (Jeong, JG, Noh & Hwang 2011, Biern, JG & Jeong 2014)
- In e.g. synchronous gauge ($g_{00} = -1$ and $g_{0i} = 0$) we can have another Newtonian correspondence (Hwang, Noh, Jeong, JG & Biern 2015)

Linear power spectrum is obtained by solving the Boltzmann eq (e.g. `CAMB`) and is used iteratively to obtain non-linear contributions

Putting dark energy on the table

Previous strategy is not complete

- ΛCDM power spectrum in EdS background
- Matter domination all the way

But we know the universe has been dominated by DE for a long time

$$\rho = \rho_m \longrightarrow \rho = \rho_m + \rho_{de} \quad \text{with} \quad p_{de} = w\rho_{de}$$

For simplicity

1. No DE perturbation: $\rho_{dm} = \bar{\rho}_{de}$ (cf. Park, Hwang, Lee & Noh 2009)
2. Comoving gauge: $T^0{}_i = 0$

Dark energy changes the game

DE provides different BG from both EdS and ΛCDM:

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \left(\bar{\rho}_m + \bar{\rho}_{de}\right) \quad \text{and} \quad \mathcal{H}' = -\frac{1}{2}\mathcal{H}^2 (1+3w)$$

DE permeates *all* order in perturbation: e.g. energy conservation

$$\delta' - \kappa(1-\lambda) = (\text{non-linear terms}) \text{ where } \lambda \equiv (1+w)\left(1 - \frac{1}{\Omega_m}\right)$$

Thus away from EdS ($\Omega_m = 1$) and ΛCDM ($w = -1$) the effects of general, dynamical DE are *manifest*: we use the parametrization

(Chevallier & Polarski 2001, Linder 2003)

$$w(a) = w_0 + (1-a) w_a$$

Non-linear solutions with DE

Curvature perturbation is **not** conserved: from energy constraint

$$\varphi = -\frac{\mathcal{H}^2 f}{1-\lambda}\left[1+\frac{3}{2}(1-\lambda)\frac{\Omega_m}{f}\right]\Delta^{-1}\delta \neq \text{constant}$$

Thus δ receives a) curvature evolution effects from 3rd order and b) general, dynamical DE effects from BG and linear order:

$$\delta'' + \left(\mathcal{H} + \frac{\lambda'}{1-\lambda}\right)\delta' - \frac{3}{2}(1-\lambda)\mathcal{H}^2\Omega_m\delta = \underbrace{\mathcal{N}_N + \mathcal{N}_\varphi + \mathcal{N}_{\varphi'} + \mathcal{N}_\lambda}_{=\text{non-linear source terms}}$$

	Newtonian	EdS	ΛCDM	DE
$\mathcal{N}_N$	O	O	O	O
$\mathcal{N}_\varphi$	X	O	O	O
$\mathcal{N}_{\varphi'}$	X	X	X	O
$\mathcal{N}_\lambda$	X	X	X	O

Relativistic kernels

2nd and 3rd order solutions are (Biern & JG 2015)

$$\delta_2(\boldsymbol{k},a) = D_1^2 \sum_{i=a}^{b} c_{2i}(a) \int \frac{d^3q_1 d^3q_2}{(2\pi)^3} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{q}_{12}) F_{2i}(\boldsymbol{q}_1,\boldsymbol{q}_2)\delta_1(\boldsymbol{q}_1)\delta_1(\boldsymbol{q}_2)$$

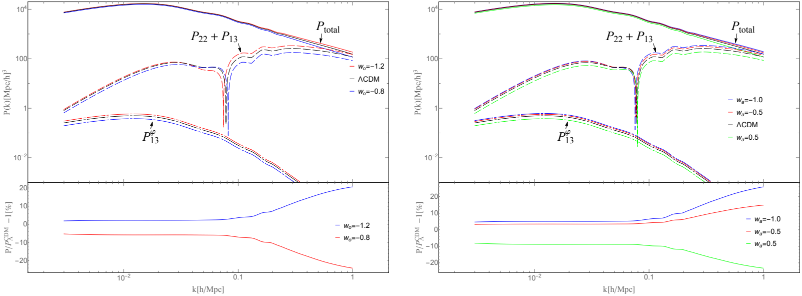
$$\delta_3(\boldsymbol{k},a) = D_1^3 \sum_{i=a}^{f} c_{3i}(a) \int \left[\cdots F_{3i} \cdots 3\ \delta_1' s\right] \qquad c_{ni} \equiv \frac{D_{ni}}{D_1^n}$$

$$+ D_1^3 \mathscr{H}^2 \sum_{i=a}^{b} c_{3i}^{\varphi}(a) \int \left[\cdots F_{3i}^{\varphi} \cdots 3\ \delta_1\text{'s}\right] \qquad c_{3i}^{\varphi} \equiv \frac{D_{3i}^{\varphi}}{D_1^3 \mathscr{H}^2}$$

In the EdS universe c's are fixed as certain numbers ($c_{2a} = 3/7\ldots$) and (also in ΛCDM) c_{ni} terms become purely Newtonian [Kamionkowski & Buchalter 1999 (2nd) and Takahashi 2008 (3rd)] and only c_{3i}^{φ} terms remain relativistic

N. B. λ is completely entangled and cannot be separated like φ

One-loop corrected power spectrum: versus ΛCDM

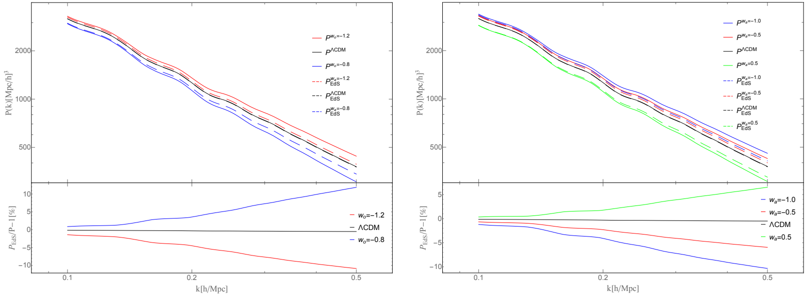


- Overall almost constant deviation on large scales ($k \lesssim 0.1h$/Mpc)
- Deviation becomes significant on $k \gtrsim 0.1h$/Mpc, close to baryon acoustic oscillations
- $w_0 > -1$ / $w_a > 0$ ($w_0 < -1$ / $w_a < 0$) give smaller (larger) $P(k)$

One-loop corrected power spectrum: versus EdS

In Newtonian studies, usually EdS power spectrum is transferred to an arbitrary DE model by replacing $a \to D_1(a)$:

$$P(k,a) = D_1^2(a)P_{11}(k) + D_1^4(a)[P_{22}(k) + P_{13}(k)]_{\mathrm{EdS}}$$



- For ΛCDM, only φ drives difference so almost identical to EdS
- For general DE, the difference notably increases from $k \approx 0.1h/\mathrm{Mpc}$

To redshift space

Observations are made i.t.o. redshift (Kaiser 1987, Heavens, Matarrese & Verde 1998

$$\delta_s = \delta_r - \partial_\parallel U + \text{higher order terms}$$

$$\text{where} \quad \delta_r = b\delta \,,\; U \equiv \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{v}}{\mathcal{H}} \quad \text{and} \quad \partial_\parallel \equiv \hat{\boldsymbol{n}}\cdot\nabla$$

Then the observable galaxy power spectrum in the redshift space

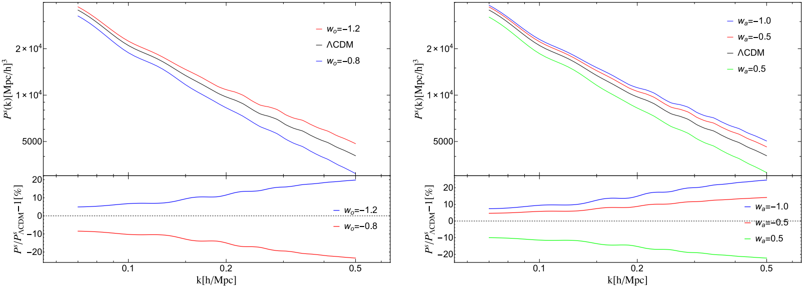
$$P_s(k,\mu,a) = P_{s11}(k,\mu,a) + P_{s22}(k,\mu,a) + P_{s13}(k,\mu,a)$$

with $\mu \equiv \hat{\boldsymbol{n}}\cdot\boldsymbol{k}/k$, thus no longer isotropic

- $\mu = 1$: line-of-sight direction, most dominant
- $\mu = 0$: perp to LoS

Thus the deviation from ΛCDM becomes larger for LoS spectrum

One-loop corrected LoS power spectrum

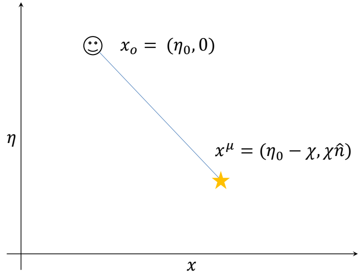


Deviation is enhanced as large as 10% at around BAO scales

$w_a = 0$ and varying w_0			$w_0 = -1$ and varying w_a			
k [h/Mpc]	$w_0 = -1.2$	$w_0 = -0.8$	k [h/Mpc]	$w_a = -1.0$	$w_a = -0.5$	$w_a = 0.5$
0.1	6.8%	-10.2%	0.1	9.5%	5.8%	-11.5%
0.2	11.6%	-15.0%	0.2	14.9%	8.8%	-15.3%
0.3	16.0%	-19.4%	0.3	20.1%	11.6%	-19.0%

Observable galaxy number density

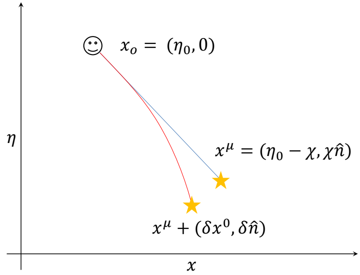
We observe as if photons come to us along a straight, unperturbed geodesic...



Observable galaxy number density

We observe as if photons come to us along a straight, unperturbed geodesic... but in fact the path is distorted due to perturbations at the locations of the observer and the source, and in between

(Yoo et al. 2009, Bonvin & Durrer 2011, Bertacca, Maartens & Clarkson 2014, Yoo & Zaldarriaga 2014...)



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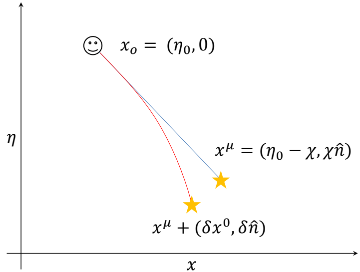

See S. G. Biern's presentation on the last day

Conclusions

- As galaxy surveys become deeper and deeper, fully GR description is relevant
- With general dark energy:
  - Dark energy background greatly affects GR contributions
  - Notable difference of a few percent near BAO scales
  - Detectable signatures of judging Λ or not
- Geodesic approach should help