"2nd APCTP-TUS workshop" TUS in Kyoto, 5th Aug 2015.

Local gravity constraints on theories beyond Horndeski

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1. Introduction

Discovery of late-time cosmic acceleration

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

The equation of state defined below characterizes dark energy.

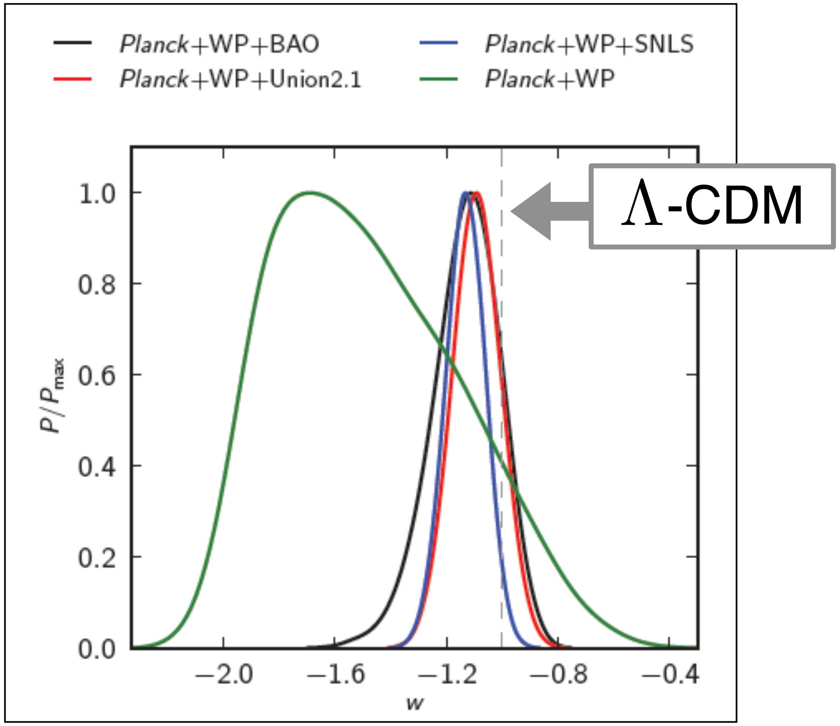
$$w \equiv P/\rho$$

Condition for acceleration :

$$w < -1/3$$

• Planck+WP+SNLS

$$w = -1.13^{+0.13}_{-0.14} \ (95\%\mathrm{CL})$$



Planck collaboration arXiv:1303.5076 [astro-ph.CO]

Dark energy problem may imply some modification of gravity on large scales.

1. Introduction

- **Several dark energy models**

・Quintessence

$$L=\frac{M_{\rm pl}^2}{2}R+X-V(\phi)\qquad \left(X\equiv-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

・Brans-Dicke gravity

non-minimal coupling

$$L=f(\phi)R+\omega(\phi)X-V(\phi)$$

・covariant Galileon

$$L=\frac{M_{\rm pl}^2}{2}R+\sum_{i=2}^{5}c_iL_i$$

non-minimal derivative coupling

$$L_2=X\,,$$
$$L_3=X\Box\phi/M^3\,,$$
$$L_4=X\left[2(\Box\phi)^2-2\phi^{;\mu\nu}\phi_{;\mu\nu}-X^2R/2\right]/M^6\,,$$
$$L_5=X\left[(\Box\phi)^3-3(\Box\phi)\phi^{;\mu\nu}\phi_{;\mu\nu}+2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}+\frac{3}{2}X^2G_{\mu\nu}\phi^{;\mu\nu}\right]/(3M^9)\,.$$

1. Introduction

Horndeski theories

$$S = \int d^4x\sqrt{-g}\sum_{i=2}^{5} L_i + S^M$$

$$G_{i,X} \equiv \partial G_i/\partial X$$

$$X = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

$$L_2 = G_2(\phi, X)\,,$$
$$L_3 = G_3(\phi, X)\Box\phi\,,$$
$$L_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right]\,,$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)\phi^{;\mu\nu}\phi_{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}\right]\,.$$

- Quintessence and K-essence $G_2 = G_2(\phi, X)\,,\quad G_3 = 0\,,\quad G_4 = \frac{M_{\rm pl}^2}{2}\,,\quad G_5 = 0$
- $f(R)$ and Brans-Dicke gravity $G_2 = G_2(\phi, X)\,,\quad G_3 = 0\,,\quad G_4 = F(\phi)\,,\quad G_5 = 0$
- covariant Galileon $G_2 = c_2 X\,,\quad G_3 = \frac{c_3}{M^3}\,,\quad G_4 = \frac{M_{\rm pl}^2}{2} + \frac{c_4}{M^6}X^2\,,\quad G_5 = \frac{c_5}{M^9}X^2$

Horndeski Lagrangians describe the most general scalar–tensor theory with **second-order** equations of motion on the **general** background.

1. Introduction

- **Horndeski theories in an ADM language on FLRW background**

$$L = A_2 + A_3 K + A_4(K^2 - \mathcal{S}) + B_4 \mathcal{R} + A_5 K_3 + B_5(\mathcal{U} - K\mathcal{R}/2)$$

$K_{\mu\nu}$: extrinsic curvature

$\mathcal{R}_{\mu\nu}$: intrinsic curvature

$$K \equiv K^\mu_\mu\,, \quad \mathcal{S} \equiv K^\mu_\nu K^\nu_\mu\,,$$
$$\mathcal{R} \equiv \mathcal{R}^\mu_\mu\,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu} K^{\mu\nu}\,,$$
$$K_3 = 3H(2H^2 - 2HK + K^2 - \mathcal{S})$$

Horndeski theories satisfy the following relations:

$$A_4 = 2X B_{4,X} - B_4\,, \quad A_5 = -X B_{5,X}/3$$

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$$K \equiv K^{\mu}_{\mu}\,, \quad \mathcal{S} \equiv K^{\mu}_{\nu} K^{\nu}_{\mu}\,,$$
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Gleyzes, Langlois, Piazza, and Vernizzi (GLPV) generalized Horndeski theory in such a way that the above relations are not necessarily satisfied.

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, PRL(2015)

1. Introduction

- **Action describing GLPV theories in a covariant form**

$$S=\int d^4x\sqrt{-g}\sum_{i=2}^{5}L_i+\int d^4x\sqrt{-g}\,L_m(g_{\mu\nu},\Psi_m)\,.$$

$$\begin{aligned}
L_2 &= A_2(\phi,X)\,,\\
L_3 &= \left[C_3(\phi,X)+2XC_{3,X}(\phi,X)\right]\Box\phi+XC_{3,\phi}(\phi,X)\,,\\
L_4 &= B_4(\phi,X)R-\frac{B_4(\phi,X)+A_4(\phi,X)}{X}\left[(\Box\phi)^2-\phi^{;\mu\nu}\phi_{;\mu\nu}\right]\\
&\quad+\frac{2\left[B_4(\phi,X)+A_4(\phi,X)-2XB_{4,X}(\phi,X)\right]}{X^2}\left(\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}\Box\phi-\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\sigma}\phi^{;\nu\sigma}\right)\,,\\
L_5 &= G_5(\phi,X)G_{\mu\nu}\phi^{;\mu\nu}-|X|^{3/2}A_5(\phi,X)\left[(\Box\phi)^3-3(\Box\phi)\phi^{;\mu\nu}\phi_{;\mu\nu}+2\phi_{;\mu\nu}\phi^{;\sigma\nu}\phi^{\mu}_{;\sigma}\right]\\
&\quad+\frac{XB_{5,X}(\phi,X)+3A_5(\phi,X)}{|X|^{5/2}}\\
&\quad\times\left[(\Box\phi)^2\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu}-2\Box\phi\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu\sigma}\phi^{;\sigma}-\phi_{;\mu\nu}\phi^{;\mu\nu}\phi_{;\sigma}\phi^{;\sigma\gamma}\phi_{;\gamma}+2\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu\sigma}\phi^{;\sigma\gamma}\phi_{;\gamma}\right].
\end{aligned}$$

$$\left(A_3=2|X|^{3/2}C_{3,X}+2\sqrt{|X|}B_{4,\phi}\right)$$

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$$L_2=A_2(\phi,X)\,,$$

$$L_3=\left[C_3(\phi,X)+2XC_{3,X}(\phi,X)\right]\Box\phi+XC_{3,\phi}(\phi,X)\,,$$

$$L_4=B_4(\phi,X)R-\frac{B_4(\phi,X)+A_4(\phi,X)}{X}\left[(\Box\phi)^2-\phi^{;\mu\nu}\phi_{;\mu\nu}\right]$$
$$+\frac{2\left[B_4(\phi,X)+A_4(\phi,X)-2XB_{4,X}(\phi,X)\right]}{X^2}\left(\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}\Box\phi-\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\sigma}\phi^{;\nu\sigma}\right)\,,$$

$$L_5=G_5(\phi,X)G_{\mu\nu}\phi^{;\mu\nu}-|X|^{3/2}A_5(\phi,X)\left[(\Box\phi)^3-3(\Box\phi)\phi^{;\mu\nu}\phi_{;\mu\nu}+2\phi_{;\mu\nu}\phi^{;\sigma\nu}\phi^{;\mu}_{;\sigma}\right]$$
$$+\frac{XB_{5,X}(\phi,X)+3A_5(\phi,X)}{|X|^{5/2}}$$
$$\times\left[(\Box\phi)^2\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu}-2\Box\phi\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu\sigma}\phi^{;\sigma}-\phi_{;\mu\nu}\phi^{;\mu\nu}\phi_{;\sigma}\phi^{;\sigma\gamma}\phi_{;\gamma}+2\phi_{;\mu}\phi^{;\mu\nu}\phi_{;\nu\sigma}\phi^{;\sigma\gamma}\phi_{;\gamma}\right]\,.$$

In Horndeski theories, they vanish.

How far can we go beyond Horndeski theories?

1. Introduction

- **GLPV theories on a spherically symmetric background**

・**Kase and Tsujikawa, PRD (2014)**
On the flat cosmological background, the deviation from Horndeski theories dose not appear at the background level.

・**Kobayashi, Watanabe and Yamauchi, PRD (2015)**
In GLPV theories, the Vainshtein mechanism tend to be broken in the high density region.

・**Saito, Yamauchi, Mizuno, Gleyzes and Langlois, JCAP (2015)**
The breaking of the Vainshtein mechanism affects to stellar structure.

・**Kase et al. PRD (2014)**
On the spherically symmetric configuration, the deviation from Horndeski theories appears explicitly even at the background level.

Local gravity experiments may put constraints on the deviation from Horndeski theories.

2. Basic equations

- **EOMs on the spherically background**

・Action

$$S = \int d^4x\sqrt{-g}\sum_{i=2}^{5} L_i + \int d^4x\sqrt{-g}\, L_m(g_{\mu\nu}, \Psi_m)\,.$$

・Metric

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,,$$

・In the following we focus on GLPV theories with $L_5 = 0$
since it tends to prevent the success of the screening mechanism.

Kimura et al. PRD (2012),
Koyama et al. PRD (2013),
Kase and Tsujikawa, JCAP (2013).

2. Basic equations

- **EOMs on the spherically background**

e.g., (0,0) component

$$\left(\frac{4e^{-2\Phi}A_4}{r} - \mathcal{C}_1\right)\Phi' + \mathcal{C}_2 - \frac{2A_4}{r^2}\left(e^{-2\Phi} - 1 - \alpha_{\rm t}\right) = -\rho_m \,,$$

$$\alpha_{\rm t} \equiv -\frac{B_4}{A_4} - 1$$

$\mathcal{C}_i$: functions of A_j and their derivatives

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In the cosmological background this quantity represents the deviation of the tensor propagation speed from 1.

2. Basic equations

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e.g., (0,0) component

$$\left(\frac{4e^{-2\Phi}A_4}{r}-\mathcal{C}_1\right)\Phi'+\mathcal{C}_2-\frac{2A_4}{r^2}\left(e^{-2\Phi}-1-\alpha_{\rm t}\right)=-\rho_m\,,$$

In GR, the dominant contributions is of the same order as this term.

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These terms should be small in order to satisfy local gravity constraints.

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In order to confront GLPV theories with solar system constraints, we adopt

- **Weak gravitational background** $(|\Phi| \ll 1\,,\ |\Psi| \ll 1)$
- **Parameters** $\epsilon_1 \equiv r\mathcal{C}_1/A_4\,,\ \epsilon_2 \equiv r^2\mathcal{C}_2/A_4\,, \ldots$ **satisfy** $|\epsilon_i| \ll 1$

2. Basic equations

- **EOMs on the spherically background**

➡ Basic equations under the weak field approximation

$$(r\Phi)' \simeq -\frac{\rho_m r^2}{4A_4} + \frac{r^2}{4A_4}\left[A_2 - \phi'\left(A_{3,\phi} + 2\phi'' A_{3,X}\right) + \frac{4\phi'}{r}\left(A_{4,\phi} + 2\phi'' A_{4,X}\right)\right] - \frac{1}{2}\alpha_{\rm t}\,,$$

$$\Psi' \simeq \frac{\Phi}{r} - \frac{r}{4A_4}\left(A_2 - 2\phi'^2 A_{2,X}\right) + \frac{\phi'^2 A_{3,X}}{A_4} - \frac{\phi'^2 A_{4,X}}{A_4 r} + \frac{\alpha_{\rm H}}{2r}\,,$$

$$\Box\phi = \mu_4\rho_m + \mu_5$$

μ_i : Functions in terms of A_{2-4} , B_4 and their derivatives

$$\alpha_{\rm H} \equiv \frac{2XB_{4,X} - B_4}{A_4} - 1$$

:which is 0 in Horndeski theories

Solving the third equation and substituting the solution into the first and second equations, one can estimate the gravitational potentials.

3. Application to specific models

- **Simplest model beyond the Horndeski domain**

$$A_2 = -\frac{1}{2}X\,, \quad C_3 = 0\,, \quad A_4 = -\frac{1}{2}M_{\rm pl}^2\,, \quad B_4 = \frac{1}{2}M_{\rm pl}^2 F_0\,,$$

F_0 : a constant representing a deviation from the Horndeski domain

($F_0 = 1$ in the Horndeski domain)

$$\Box\phi = \mu_4\rho_m + \mu_5 \quad \Rightarrow \quad \phi'' + \frac{2}{r}\phi'\left(1 - \frac{\rho_m r^2}{4M_{\rm pl}^2}\right) = 0$$

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$$\Rightarrow \begin{cases} \phi'(r) = \dfrac{C_1}{r^2} & (r \gg r_g) \\ \phi'(r) = \dfrac{C_2}{r^2}\exp\left(\dfrac{\rho_m r^2}{4M_{\rm pl}^2}\right) & (r \ll r_g\,,\ \rho_m = \text{constant})\,. \end{cases}$$

r_g : Schwarzschild radius

Demanding a regularity condition $\phi'(0) = 0$ we need to set $C_2 = 0$.
Matching the two solutions $r = r_g$, it follows that $C_1 = 0$.

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$$\phi'(r) = 0\,.$$

$$(r\Phi)' \simeq -\frac{\rho_m r^2}{4A_4} + \cancel{\frac{r^2}{4A_4}\left[A_2 - \phi'\left(A_{3,\phi} + 2\phi'' A_{3,X}\right) + \frac{4\phi'}{r}\left(A_{4,\phi} + 2\phi'' A_{4,X}\right)\right]} - \frac{1}{2}\alpha_{\rm t}\,,$$

$$\Psi' \simeq \frac{\Phi}{r} \cancel{- \frac{r}{4A_4}\left(A_2 - 2\phi'^2 A_{2,X}\right) + \frac{\phi'^2 A_{3,X}}{A_4} - \frac{\phi'^2 A_{4,X}}{A_4 r}} + \frac{\alpha_{\rm H}}{2r}\,,$$

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$$\Psi' \simeq \frac{\Phi}{r} - \cancel{\frac{r}{4A_4}\left(A_2 - 2\phi'^2 A_{2,X}\right) + \frac{\phi'^2 A_{3,X}}{A_4} - \frac{\phi'^2 A_{4,X}}{A_4 r}} + \boxed{\frac{\alpha_{\rm H}}{2r}}\,, \quad \alpha_{\rm H} = \alpha_{\rm t} = F_0 - 1$$

Even though the field derivative vanishes everywhere, the gravitational law still can be different from GR.

3. Application to specific models

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$$A_2 = -\frac{1}{2}X\,, \quad C_3 = 0\,, \quad A_4 = -\frac{1}{2}M_{\rm pl}^2\,, \quad B_4 = \frac{1}{2}M_{\rm pl}^2 F_0\,,$$

$$\Phi = \frac{r_g}{2r} - \frac{1}{2}\alpha_{\rm t}\,, \qquad \Psi = -\frac{r_g}{2r}\,, \qquad \left(r_g \equiv \frac{1}{M_{\rm pl}^2}\int_0^r \rho_m \tilde{r}^2 d\tilde{r} \right)$$

- Post-Newtonian parameter

$$\gamma \equiv -\frac{\Phi}{\Psi} = 1 - \alpha_{\rm t}\frac{r}{r_g}$$

In order to satisfy solar system constraints $|\gamma - 1| < 2.3 \times 10^{-5}\,,$

$$|\alpha_{\rm H}| = |\alpha_{\rm t}| < 2.3 \times 10^{-5}\frac{r_g}{r}$$

Considering the Sun $(r_g = 3 \times 10^5 {\rm cm})$ and taking the radius $r = 1\ {\rm Au} = 1.5 \times 10^{13} {\rm cm}$, we obtain the bound

$$|\alpha_{\rm H}| = |F_0 - 1| < 5 \times 10^{-13}$$

3. Application to specific models

- **GLPV theories with a scalar-matter coupling**

$$A_2 = -\frac{1}{2}X\,, \quad C_3 = 0\,, \quad A_4 = -\frac{1}{2}M_{\rm pl}^2 e^{-2q_1\phi/M_{\rm pl}}\,, \quad B_4 = \frac{1}{2}M_{\rm pl}^2 e^{-2q_2\phi/M_{\rm pl}}\,.$$

$$\frac{d}{dr}\left(r^2\phi'\right) \simeq \left(\frac{\phi' r}{2} + q_2 M_{\rm pl}\right)\frac{\rho_m r^2}{M_{\rm pl}^2} + M_{\rm pl}\beta_{\rm H}\,,$$

$\beta_{\rm H} \simeq 2(q_1 - q_2)$: which is related with $\partial\alpha_{\rm H}/\partial\phi$

Integrating the above equation in the regime $r \gg r_g$ it leads

$$\Rightarrow \begin{cases} \phi'(r) = \dfrac{q_2 M_{\rm pl} r_g}{r^2} + \dfrac{\beta_{\rm H} M_{\rm pl}}{r}\,, \\ \phi(r) = -\dfrac{q_2 M_{\rm pl} r_g}{r} + \beta_{\rm H} M_{\rm pl} \ln\dfrac{r}{r_c}\,, \end{cases}$$

r_c : an integration constant

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diverges at $r = \infty$

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Using these solutions, we finally obtain the following expression of the post-Newtonian parameter ($|\gamma - 1| < 2.3 \times 10^{-5}$,)

$$\gamma - 1 \simeq -4q_2^2 + 2q_2\beta_{\rm H}\frac{r}{r_g}\boxed{\ln\frac{r}{r_c}} + \dots\,.$$

Considering the solar system $r \sim 10^{12}{\rm cm}$ and the Schwarzschild radius of the Sun $r_g \sim 10^5{\rm cm}$, it is at most of the order of 10 by choosing $r_c = H_0^{-1} \sim 10^{28}{\rm cm}\,.$

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$$|q_2| \lesssim 10^{-3}$$

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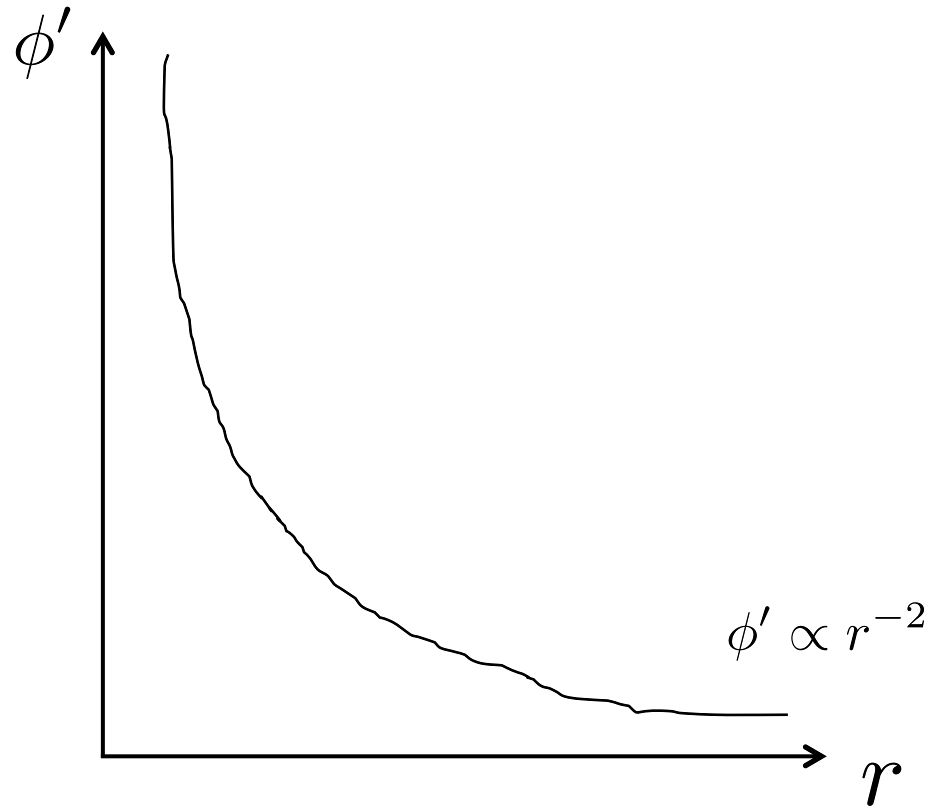
$$|q_2| \lesssim 10^{-3} \qquad |\beta_{\rm H}| \lesssim 10^{-10}$$

Not only the deviation parameter $\alpha_{\rm H}$ from Horndeski domain but also its variation $\beta_{\rm H}$ is tightly constrained.

3. Application to specific models

- **What happens if the Vainshtein mechanism is at work? (in progress)**



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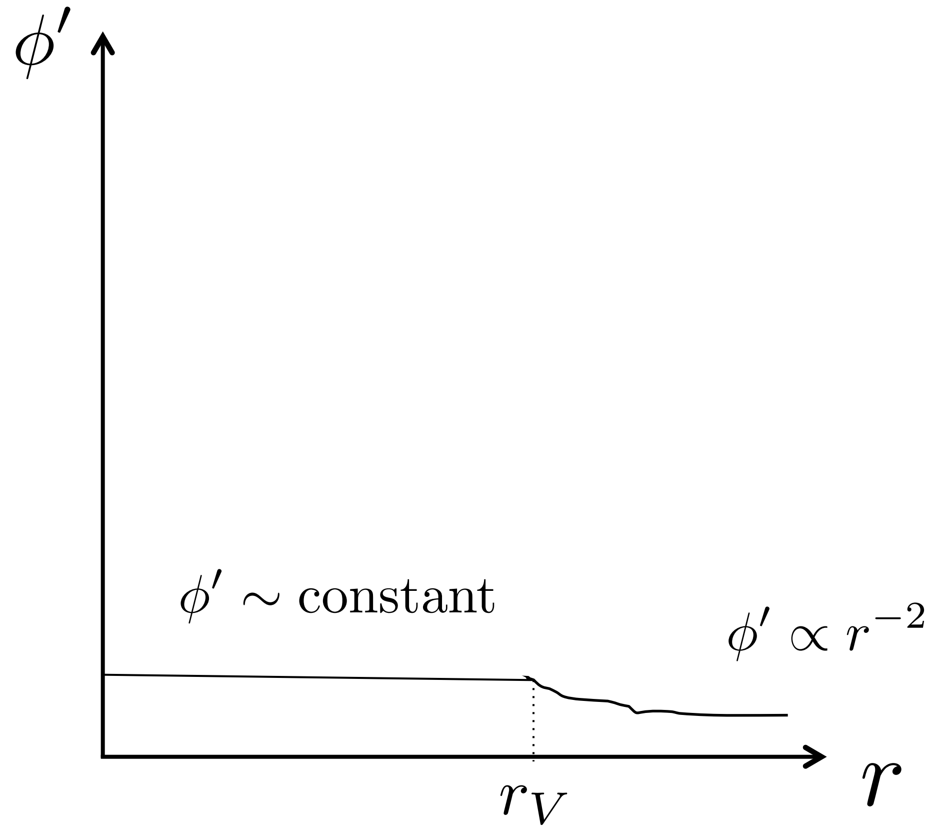
- **What happens if the Vainshtein mechanism is at work? (in progress)**

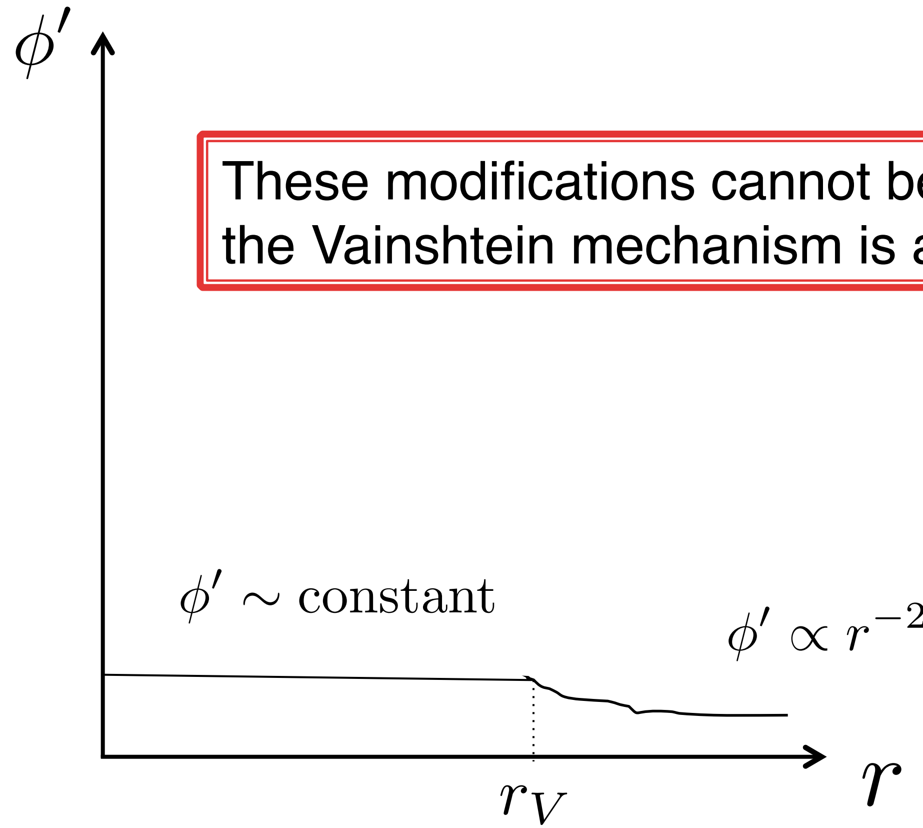


$$(r\Phi)' \simeq -\frac{\rho_m r^2}{4A_4} + \frac{r^2}{4A_4}\left[A_2 - \phi'\left(A_{3,\phi} + 2\phi'' A_{3,X}\right) + \frac{4\phi'}{r}\left(A_{4,\phi} + 2\phi'' A_{4,X}\right)\right] - \frac{1}{2}\alpha_{\mathrm{t}}\,,$$

$$\Psi' \simeq \frac{\Phi}{r} - \frac{r}{4A_4}\left(A_2 - 2\phi'^2 A_{2,X}\right) + \frac{\phi'^2 A_{3,X}}{A_4} - \frac{\phi'^2 A_{4,X}}{A_4 r} + \frac{\alpha_{\mathrm{H}}}{2r}\,,$$

3. Application to specific models

- **What happens if the Vainshtein mechanism is at work? (in progress)**



These modifications cannot be screened even if the Vainshtein mechanism is at work!!

$$(r\Phi)' \simeq -\frac{\rho_m r^2}{4A_4} + \frac{r^2}{4A_4}\left[A_2 - \phi'\left(A_{3,\phi}+2\phi''A_{3,X}\right) + \frac{4\phi'}{r}\left(A_{4,\phi}+2\phi''A_{4,X}\right)\right] - \frac{1}{2}\alpha_{\rm t},$$

$$\Psi' \simeq \frac{\Phi}{r} - \frac{r}{4A_4}\left(A_2 - 2\phi'^2 A_{2,X}\right) + \frac{\phi'^2 A_{3,X}}{A_4} - \frac{\phi'^2 A_{4,X}}{A_4 r} + \frac{\alpha_{\rm H}}{2r},$$

4. Conclusions

- We studied local gravity constraints on GLPV theories (in progress).
- We derived basic equations on the spherically symmetric background under the weak field approximation.
- For the simplest model beyond the Horndeski domain we showed that the parameter α_{H}, representing the deviation from Horndeski theories, is tightly constrained by using the bound of local gravity experiments.
- We also considered a simple model with a scalar-matter coupling in GLPV theories. In this case we found that the local gravity experiments give tight constraints.
- Even if the Vainshtein mechanism is at work, the deviation from the Horndeski domain should be quite small since there are some terms responsible for modification of gravity which is not screened.