Gravitational Waves from 2nd order perturbations

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Inflation

Leading theory of the early Universe

Predictions

scalar perturbations Φ

→ seed of the large scale structure

testable by CMB, galaxy survey, etc.

tensor perturbations h_{ij}

testable by CMB B-modes, GW experiments

Inflation

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Predictions

scalar perturbations Φ

→ seed of the large scale st

testable by CMB, galaxy sur

2nd order

$\Phi\Phi$

tensor perturbations h_{ij} MUST exist

testable by CMB B-modes, GW experiments

GWs from 2nd order perturbations of Φ

Evolution equation for GWs

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}$$

source term $\Phi \equiv \Phi^{(1)}, \ \Psi \equiv \Psi^{(1)}$

$$\begin{aligned}
\mathcal{S}_{ij} \equiv\ & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi \\
& - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\
& - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) \\
& - \frac{2c_s^2}{3w\mathcal{H}^2}[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi]\partial_i\partial_j(\Phi - \Psi)
\end{aligned}$$

$P^{(1)} \equiv c_s^2\rho^{(1)}, \quad w \equiv P^{(0)}/\rho^{(0)}$

$\hat{T}_{ij}{}^{lm}$: projection to the TT gauge

$$ds^2 = a^2(\eta)[-(1+2\Phi)d^2\eta + [(1-2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j]$$

GWs from 2nd order perturbations of Φ

Evolution equation for GWs

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source term $\Phi \equiv \Phi^{(1)}, \; \Psi \equiv \Psi^{(1)}$

→ If $\Pi^{(1)}=0$, $\Phi = \Psi$

Evolution equation for Φ

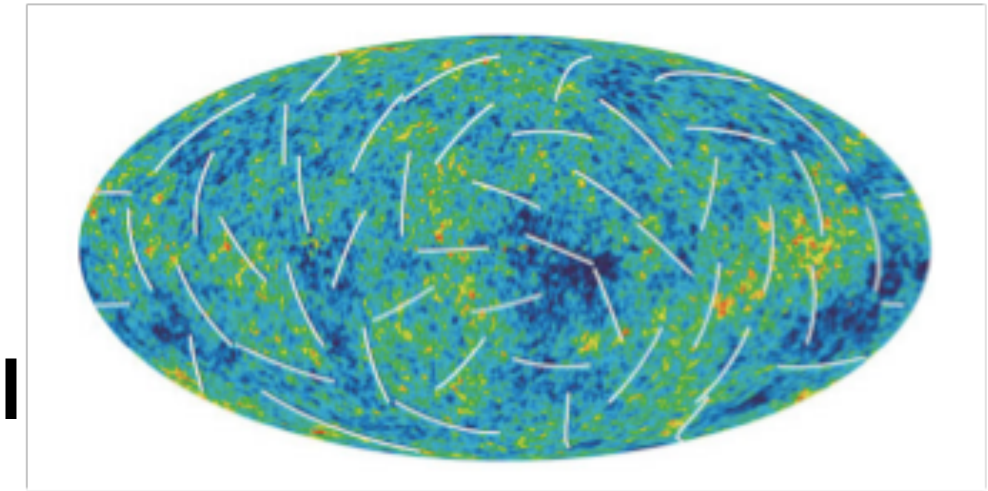
$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = 4\pi G a^2 \delta P$$

For kη>1

$$\Phi = \begin{cases} \text{constant (MD)} \\ \propto a^{-2} \text{ (RD)} \end{cases}$$

→ More GWs in Matter-dominated phase

GW experiments



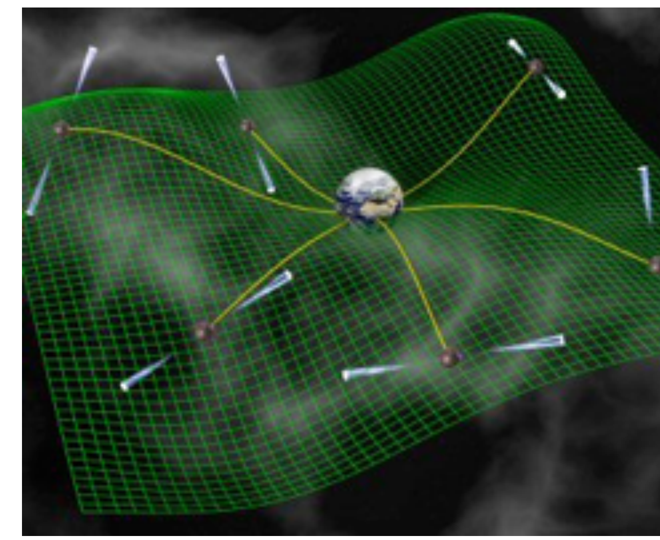
WMAP Three Year Polarized CMB Sky (http://wmap.gsfc.nasa.gov/)

- CMB B-mode polarization: **Planck, CMBpol**
- Pulsar timing: **SKA (2020-)**
- Direct detection

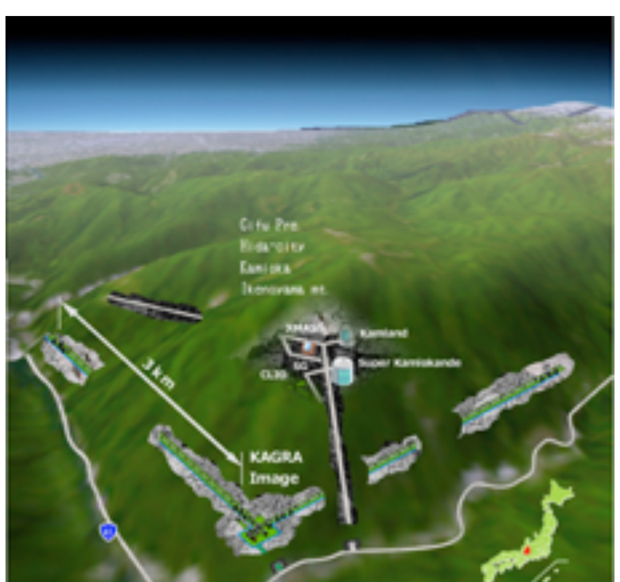
  Ground：**Advanced-LIGO、KAGRA、**

  Advanced-Virgo、IndIGO (2017-)

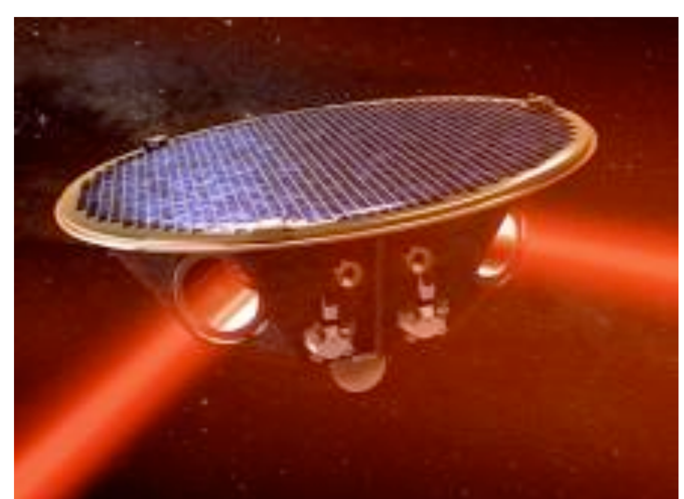
  Space：**eLISA/NGO (2034)、DECIGO/BBO**



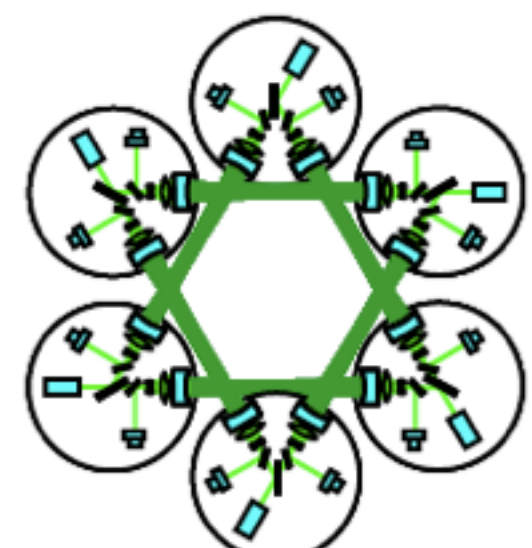
PTA image (NRAO)



KAGRA image (http://gwcenter.icrr.u-tokyo.ac.jp/)

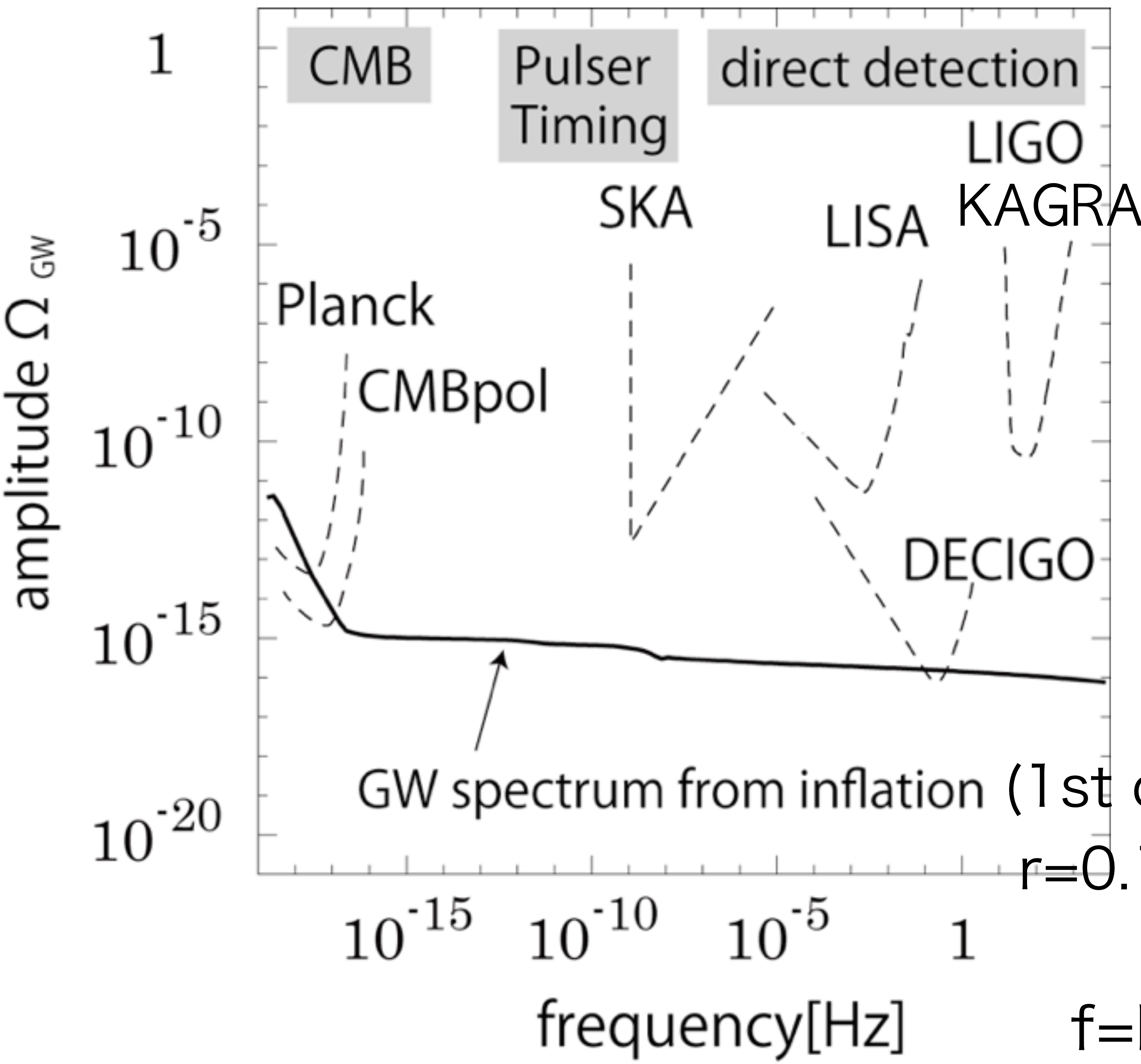


eLISA image (http://elisa-ngo.org/)



DECIGO image, S. Kawamura et al, J. Phys.: Conf. Ser. 122, 012006 (2006)

Sensitivity curves of GW experiments



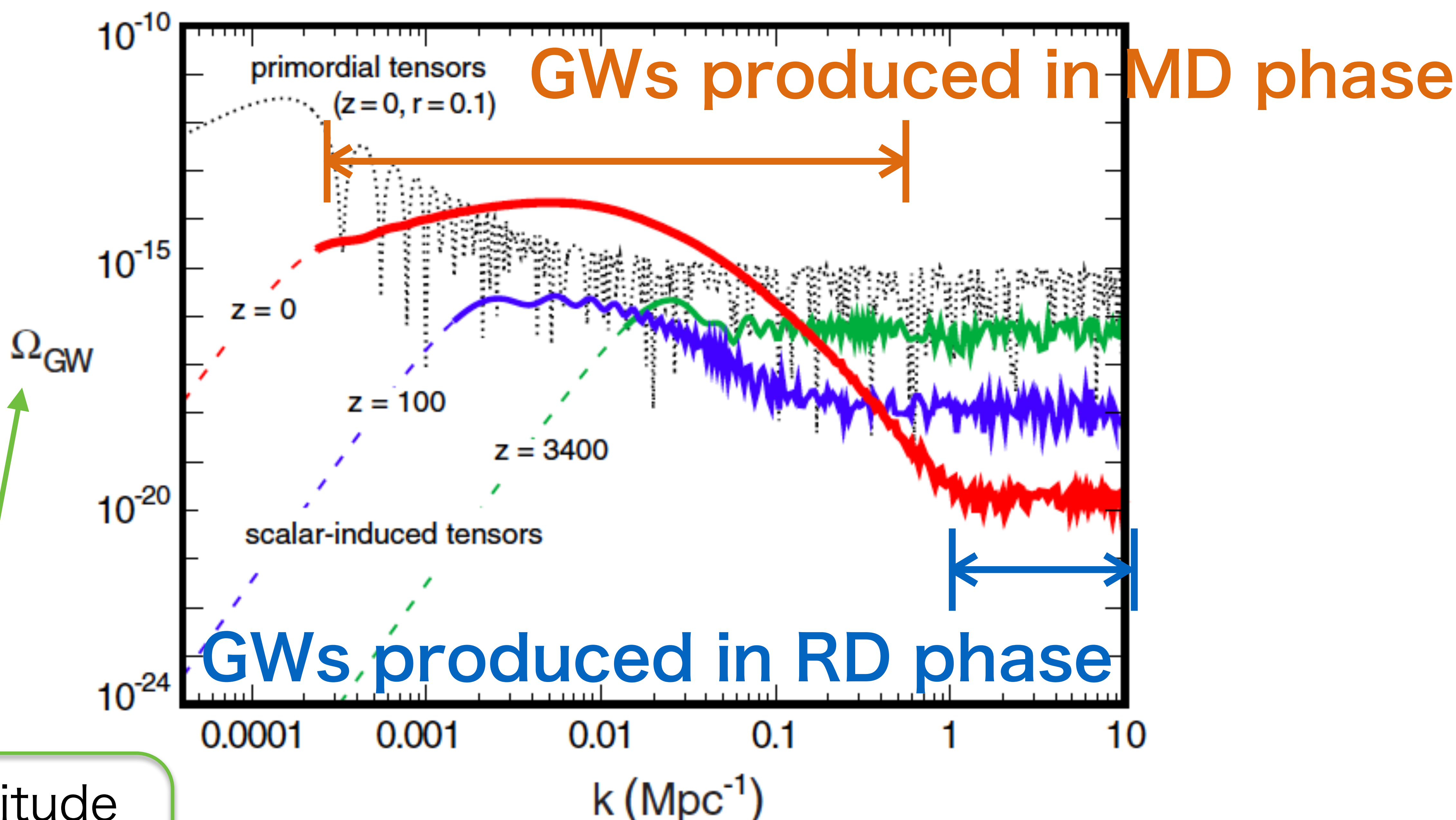
(1st order)

r=0.14

f=k/2π

Detectability of 2nd order GWs

on CMB scale



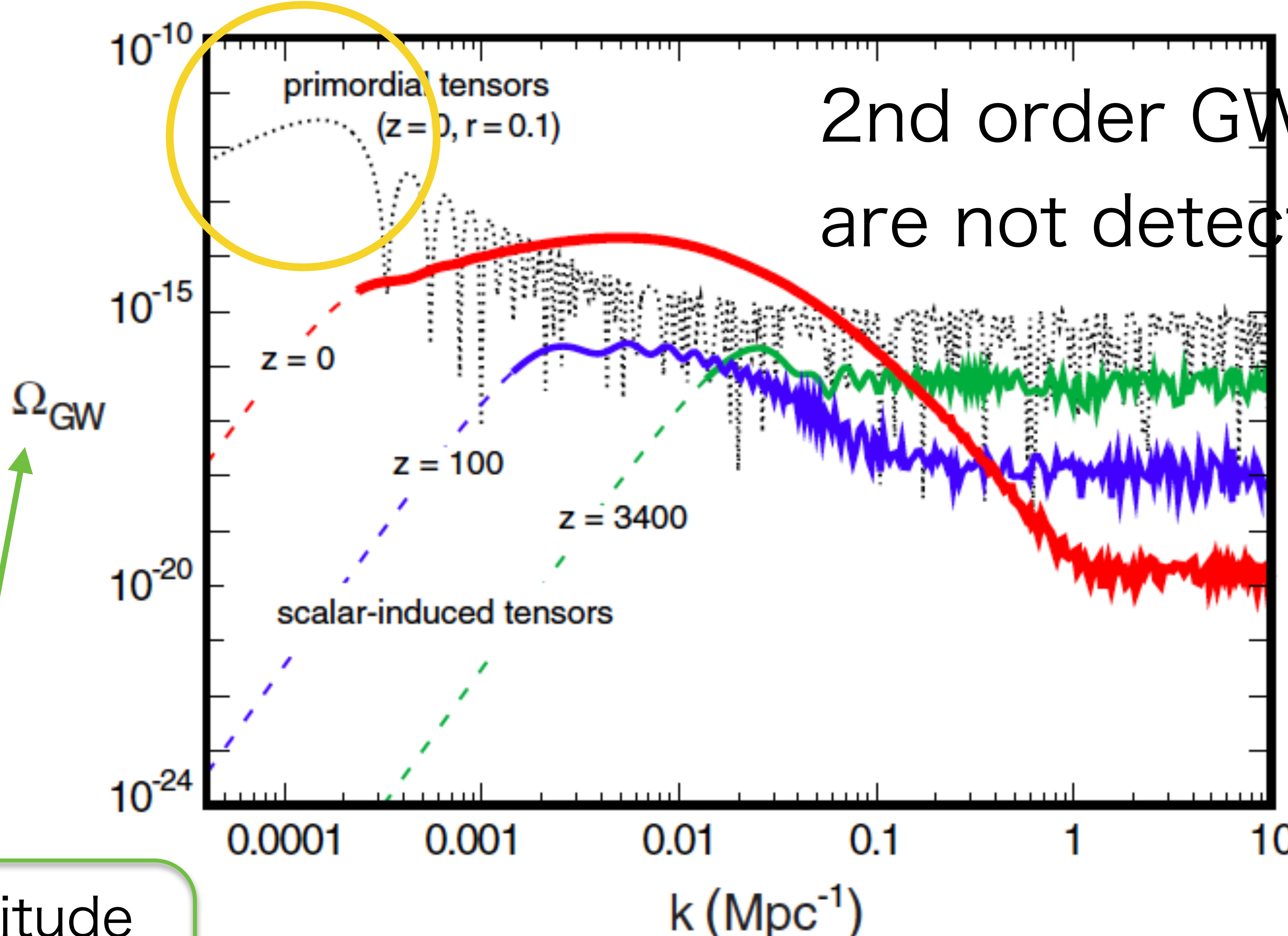
GW amplitude

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$

Baumann et al. PRD 76, 084019 (2007)

Detectability of 2nd order GWs

CMB B-mode　　on CMB scale



2nd order GWs are not detectable

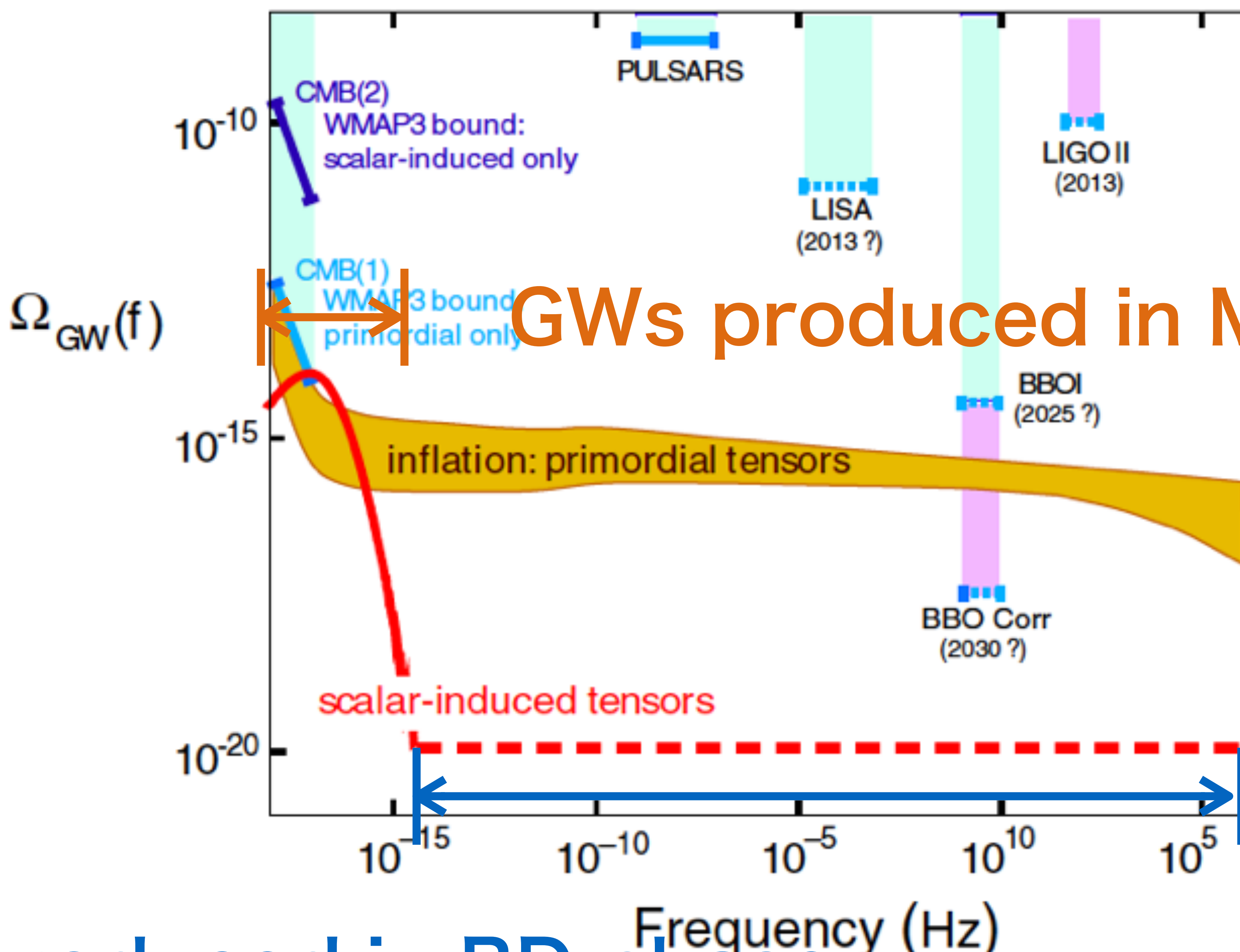
GW amplitude

$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k}$$

Baumann et al. PRD 76, 084019 (2007)

Detectability of 2nd order GWs

on small scales (direct detection experiments)



GWs produced in MD phase

GWs produced in RD phase

Baumann et al. PRD 76, 084019 (2007)

Detectability of 2nd order GWs

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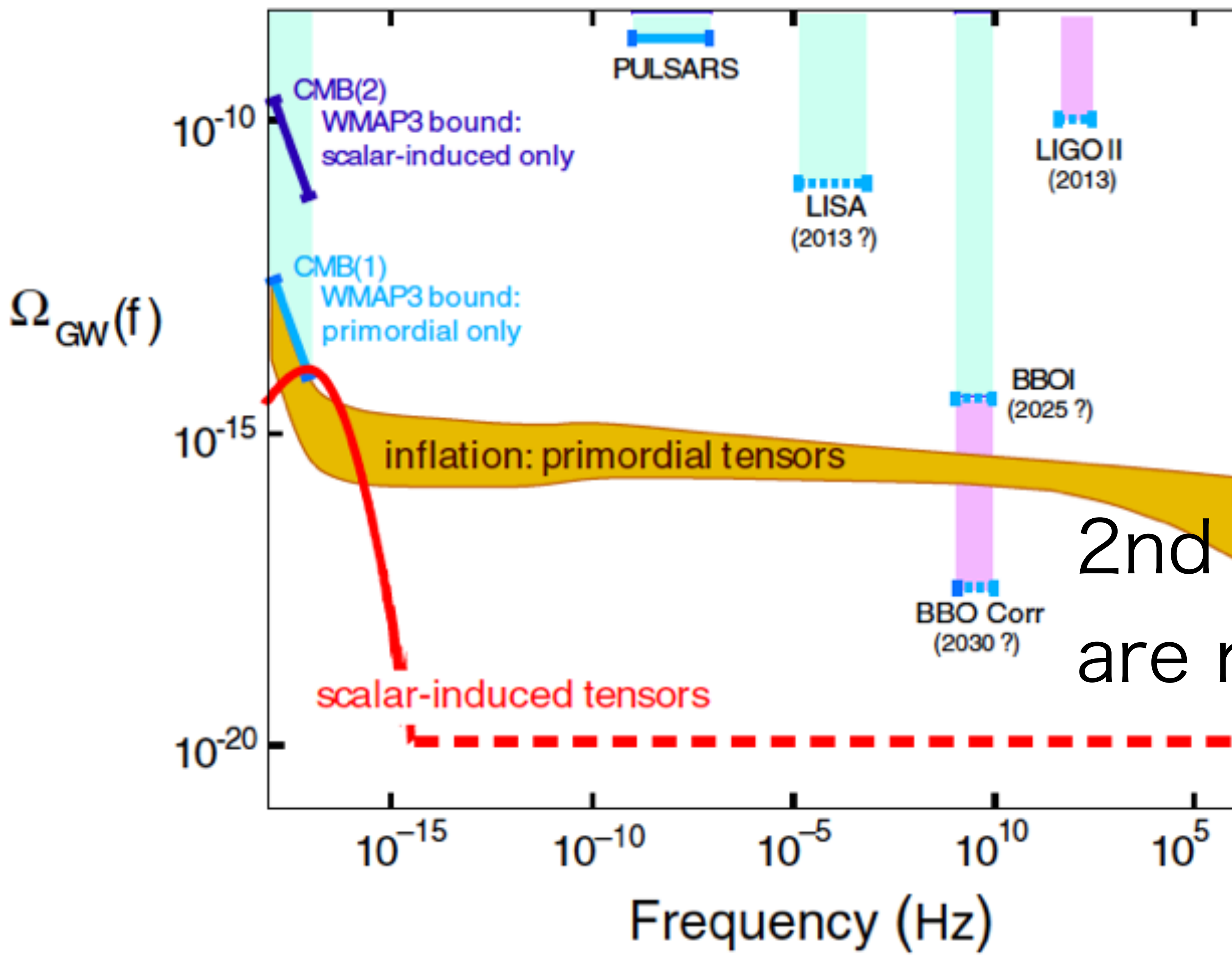


2nd order GWs are not detectable but…

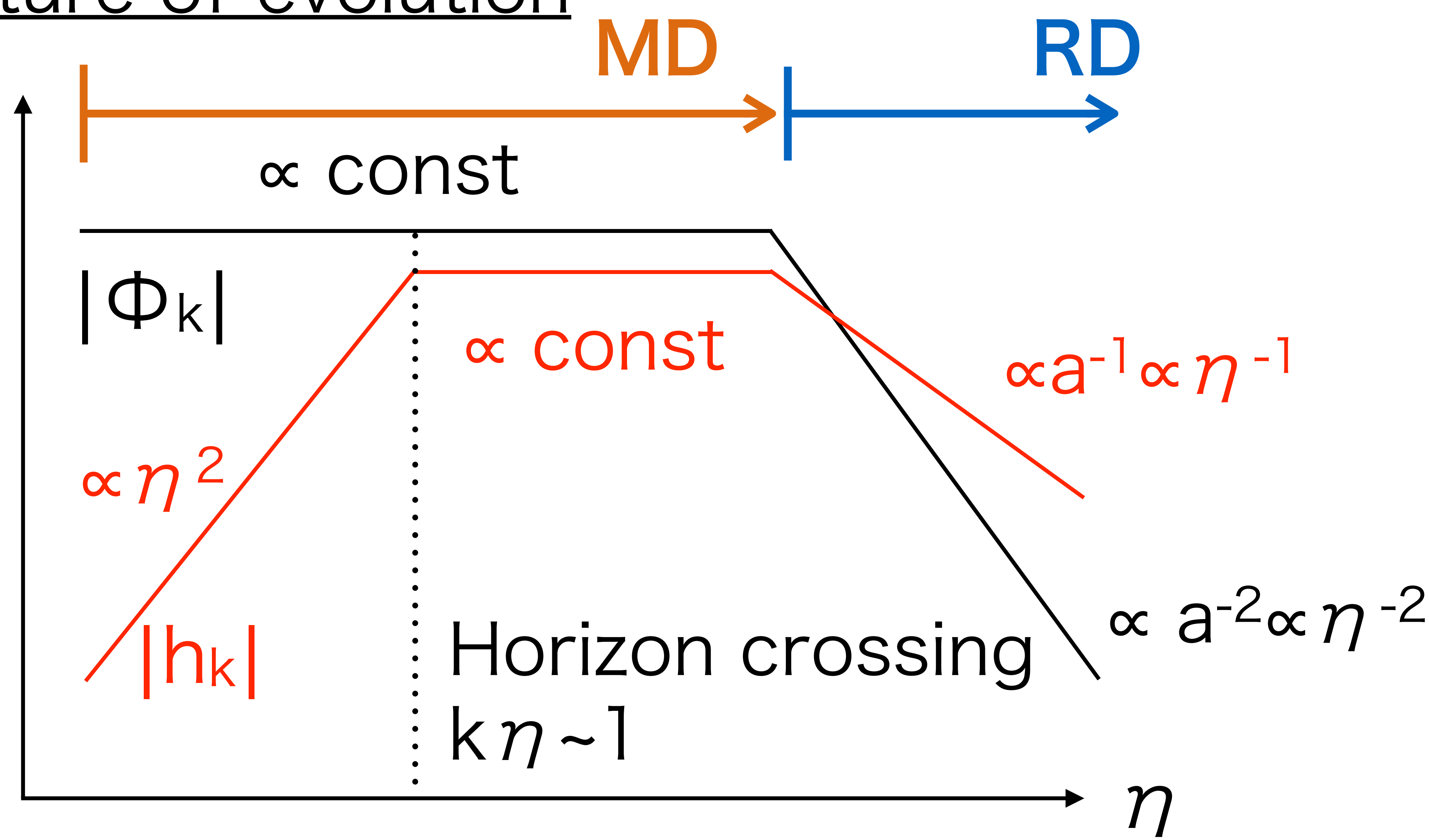
Baumann et al. PRD 76, 084019 (2007)

2nd order GWs in early matter phase

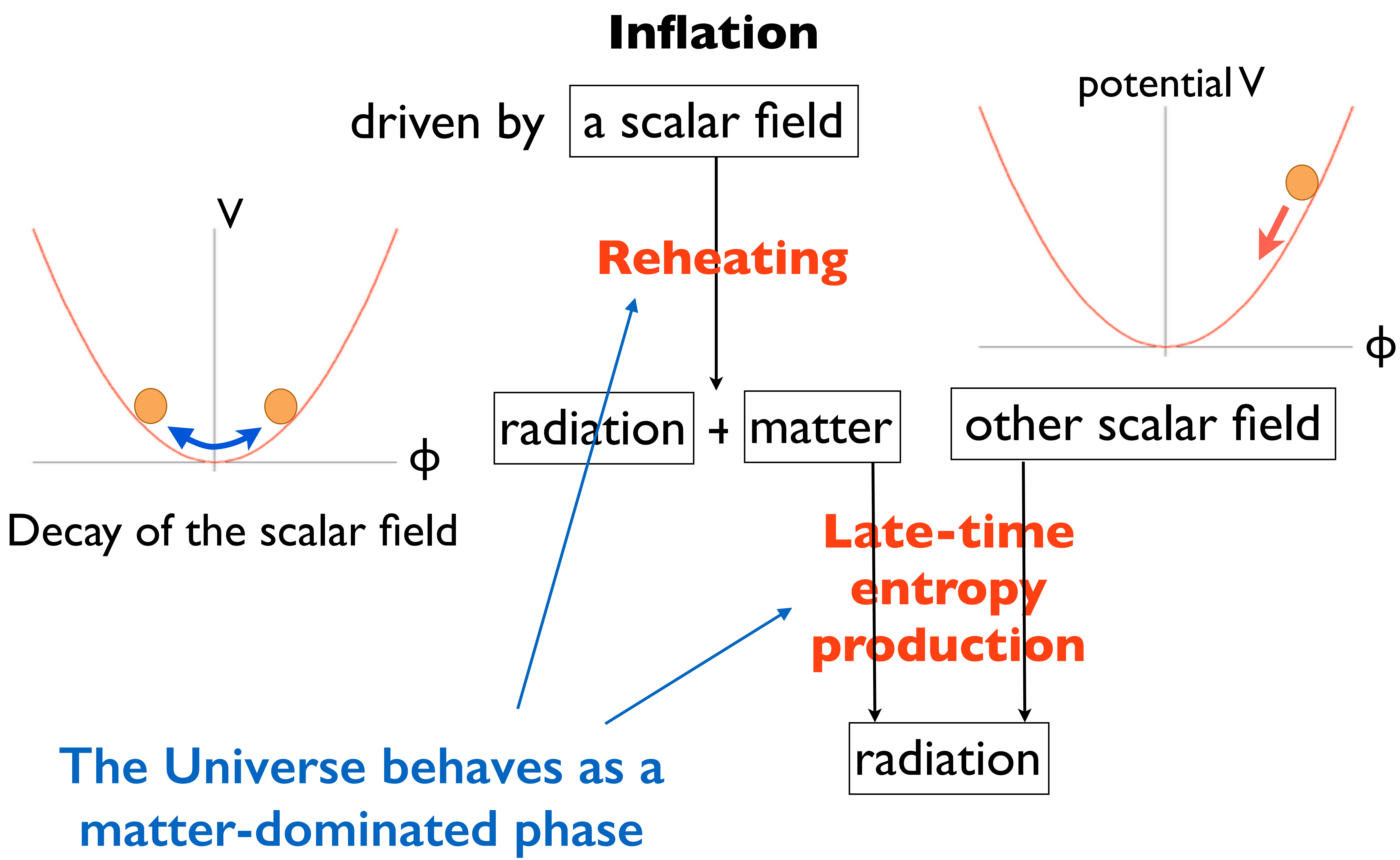
Large GWs are expected on small scales if the Universe has experienced early MD phase

Assadullahi and Wands, PRD 79, 083511 (2009)

Rough picture of evolution



2nd order GWs in early matter phase



2nd order GWs in early matter phase

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GW amplitude

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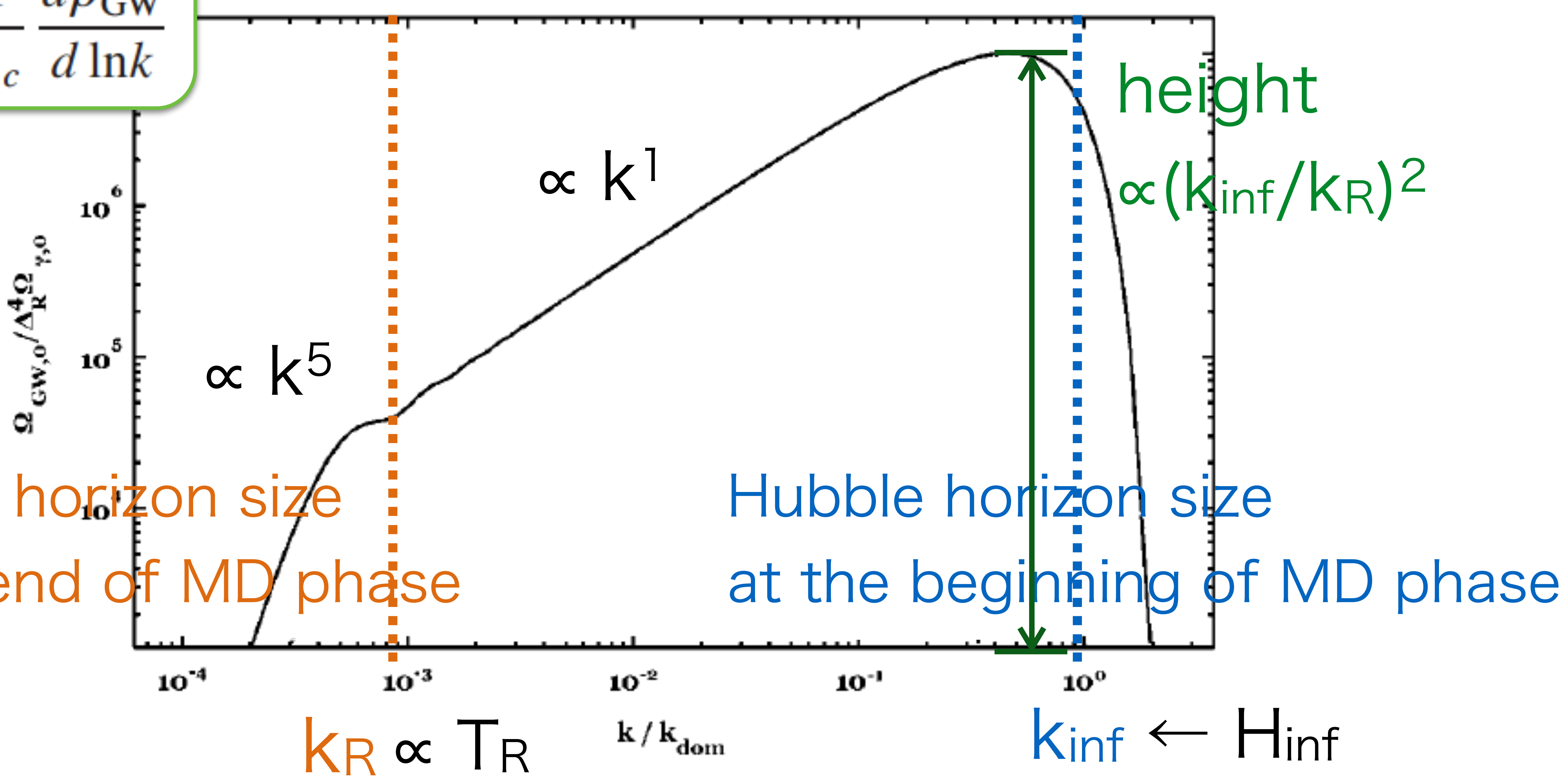
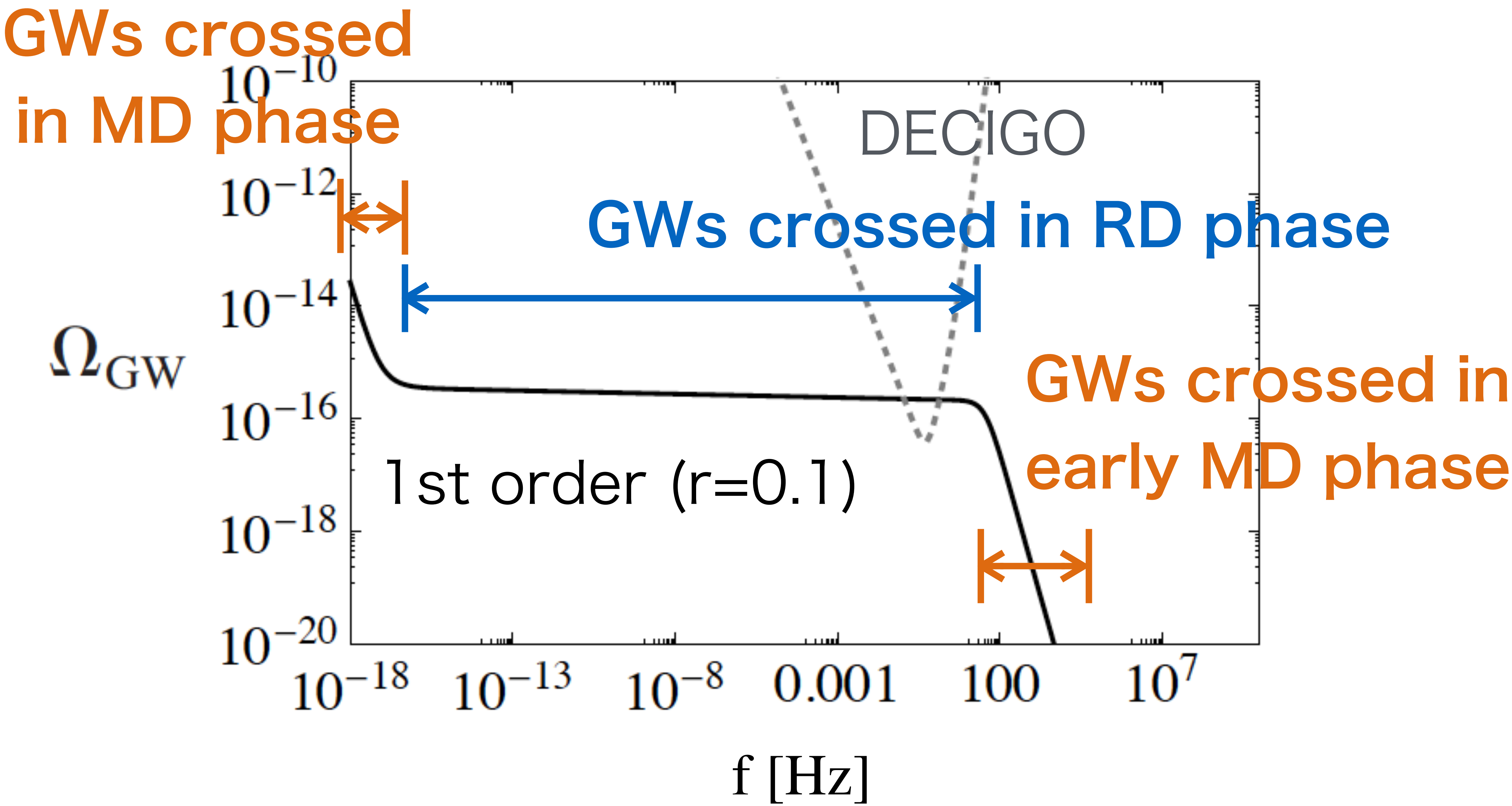
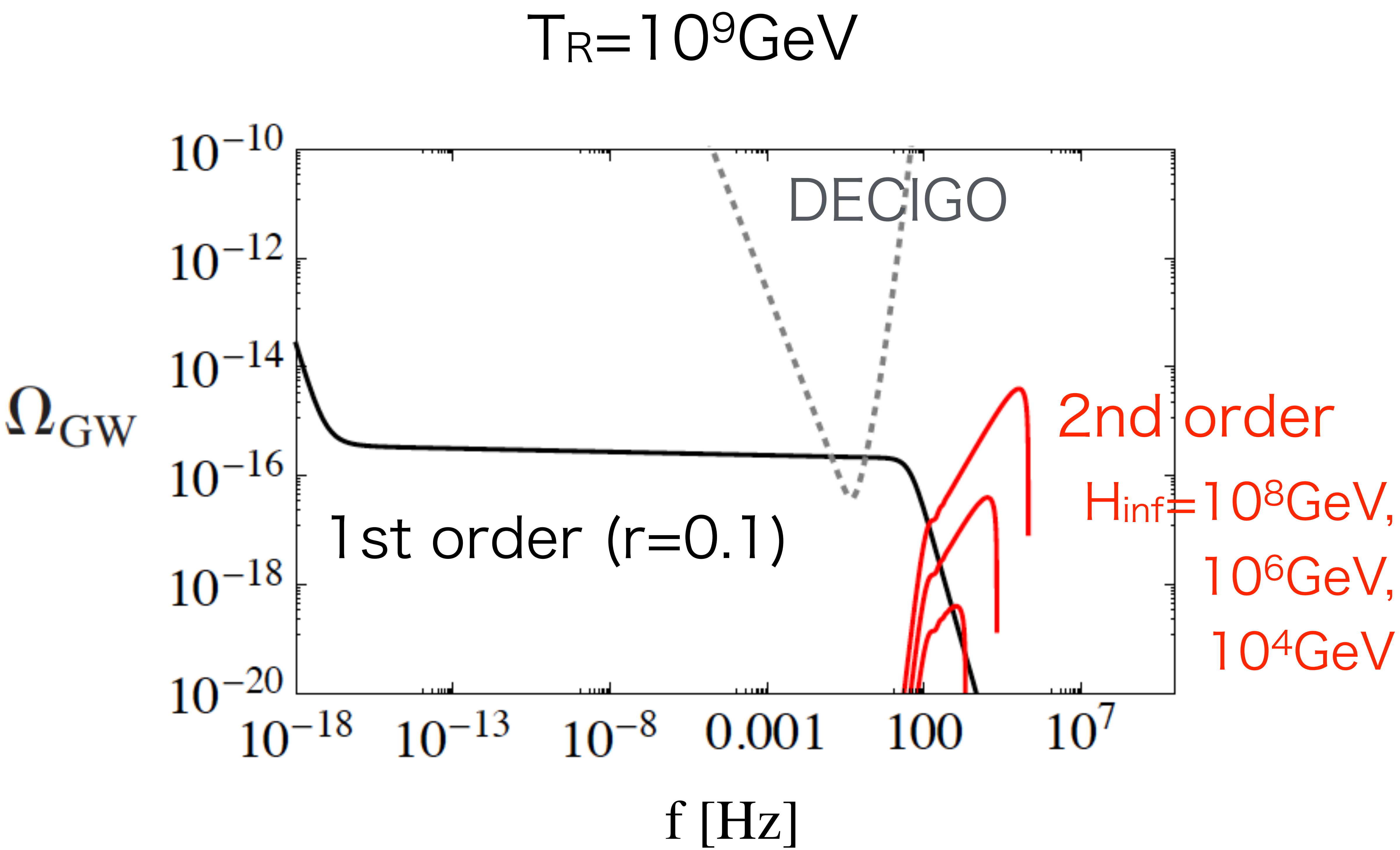


Comparison with 1st order GWs

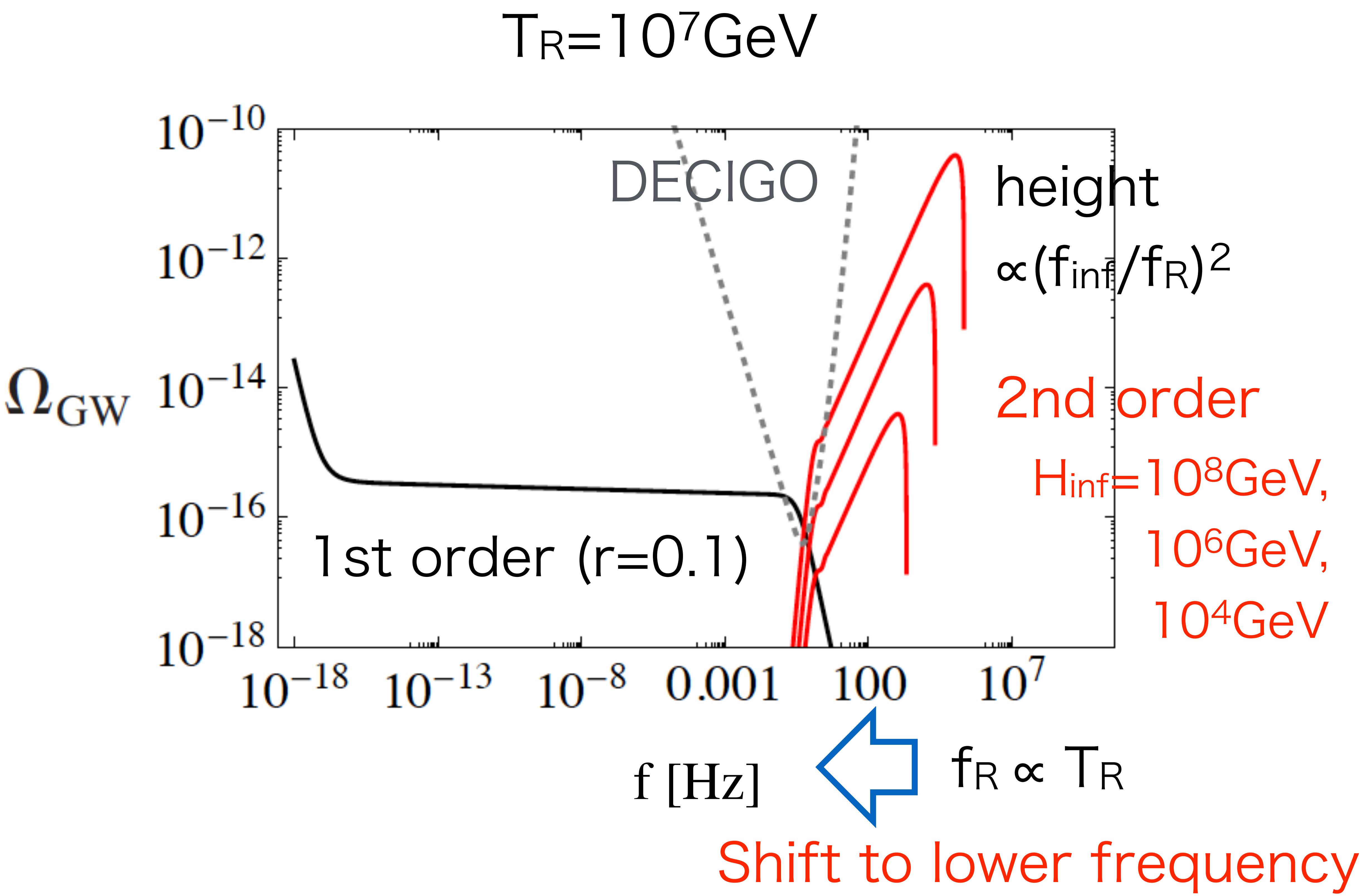


1st order GWs decay more in MD phase

Comparison with 1st order GWs

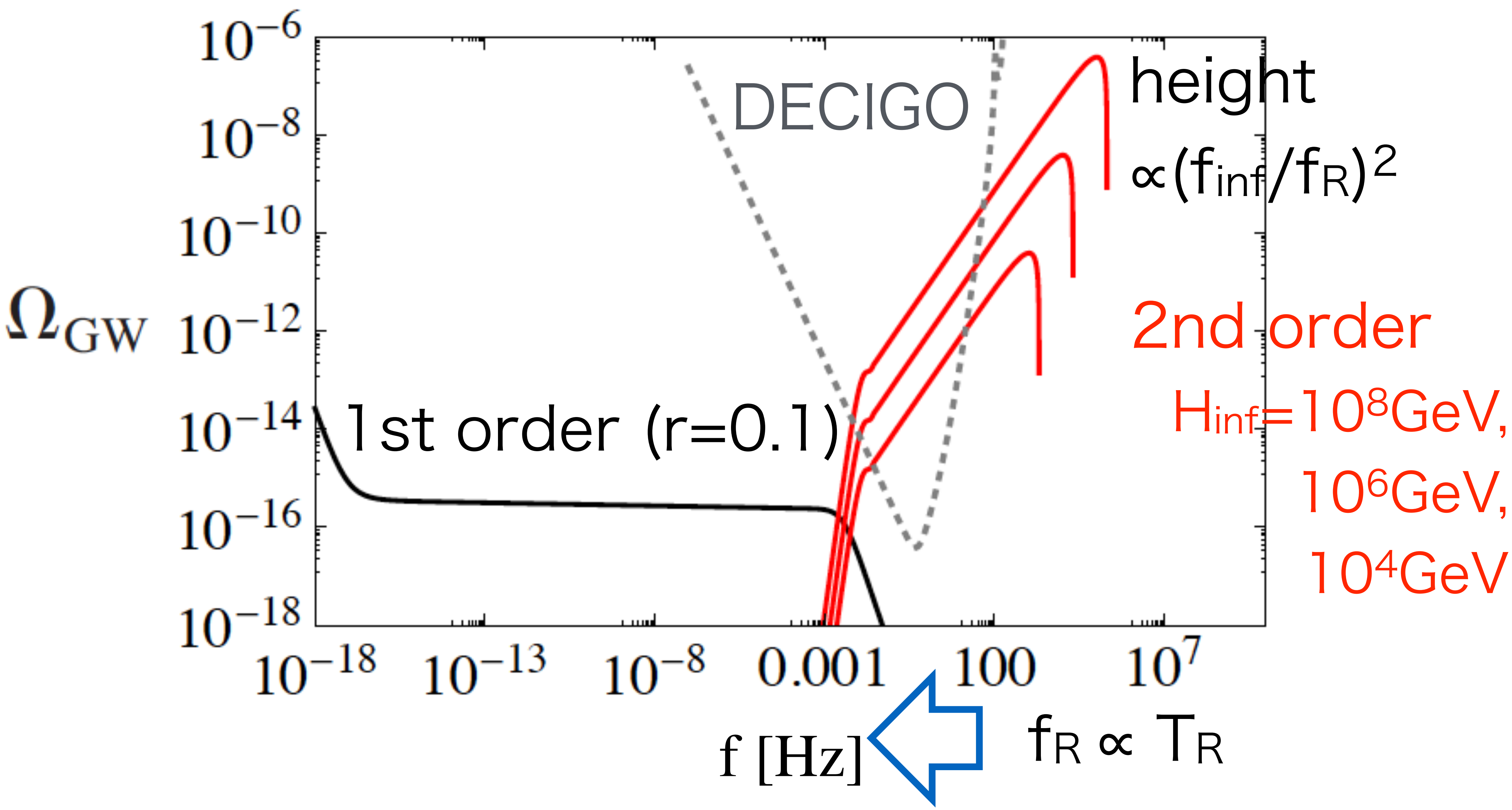


Comparison with 1st order GWs



Comparison with 1st order GWs

$T_R = 10^5$GeV



2nd order GWs dominate for $T_R < 10^7$GeV

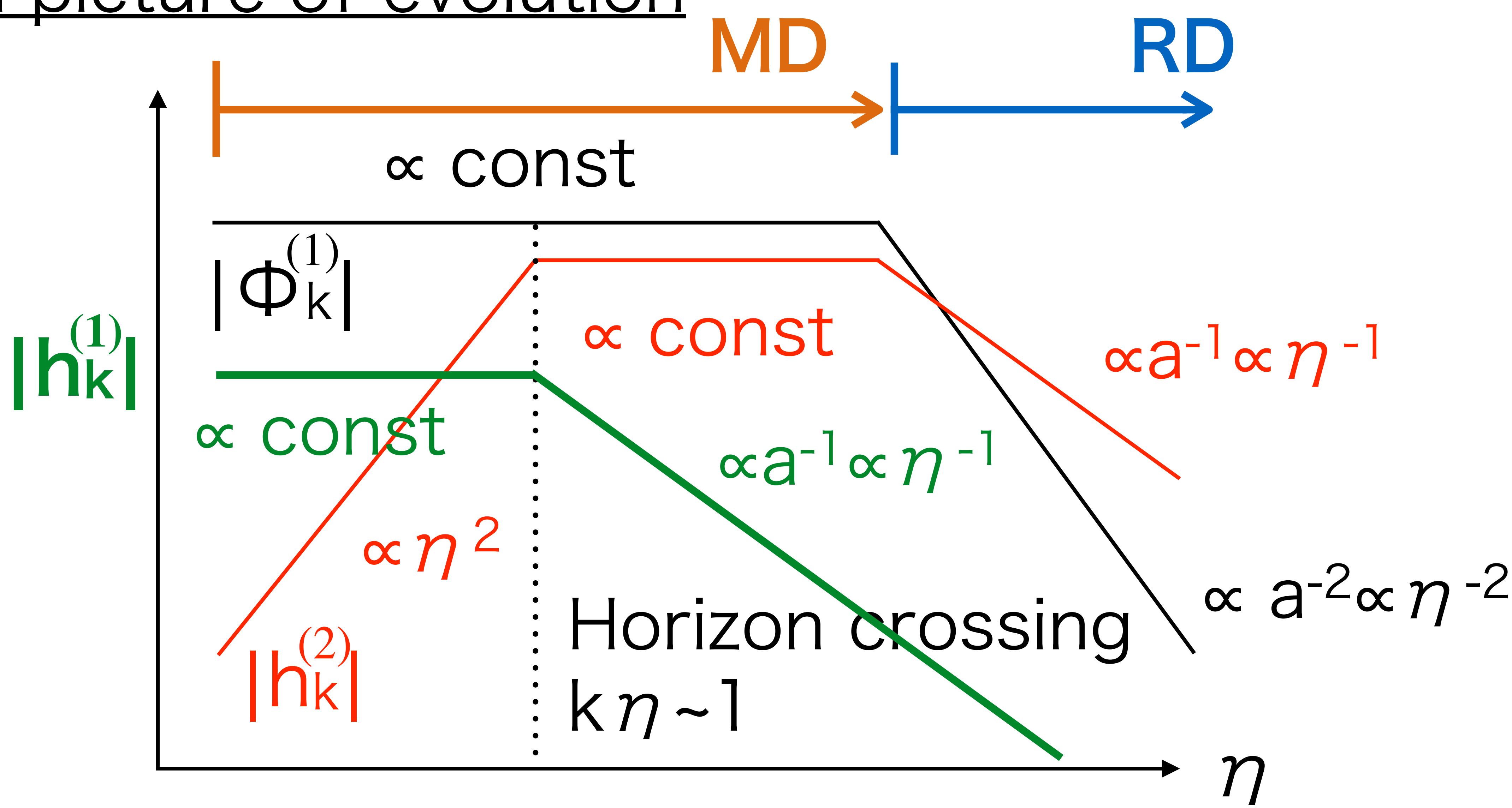
My work

$\Phi^{(1)}\Phi^{(1)}$ $\Phi^{(1)}\Psi^{(1)}$ $\Psi^{(1)}\Psi^{(1)}$
$\Phi^{(1)}h^{(1)}$? $\Psi^{(1)}h^{(1)}$? $h^{(1)}h^{(1)}$?

Do 1st order GWs affect?

$$h''^{(2)}_{ij} + 2\mathcal{H}h'^{(2)}_{ij} - \nabla^2 h^{(2)}_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}$$

Rough picture of evolution



Motivation

Φ is not necessarily larger than h in small scale

eg. Even in standard slow-roll prediction

$$P_{\Phi,prim}(k_{CMB}) \sim 2\times10^{-9}$$

$$P_{h,prim}(k_{CMB}) \sim 2\times10^{-10} \quad (\text{for } r=0.1)$$

If one include tilt of the spectrum

$$P_{\Phi,prim}(k_{DECIGO}=0.1\text{Hz}) \sim P_{\Phi,prim}(k_{CMB})(k_{DECIGO}/k_{CMB})^{n_s-1}$$

$$P_{h,prim}(k_{DECIGO}=0.1\text{Hz}) \sim P_{h,prim}(k_{CMB})(k_{DECIGO}/k_{CMB})^{n_t}$$

$$\begin{cases} n_s-1=-0.04 \\ n_t=-r/8=-0.0125 \\ k_{DECIGO}/k_{CMB} \sim 10^{17} \end{cases}$$

$$P_{\Phi,prim}(k_{DECIGO}) \sim 4\times10^{-10}$$

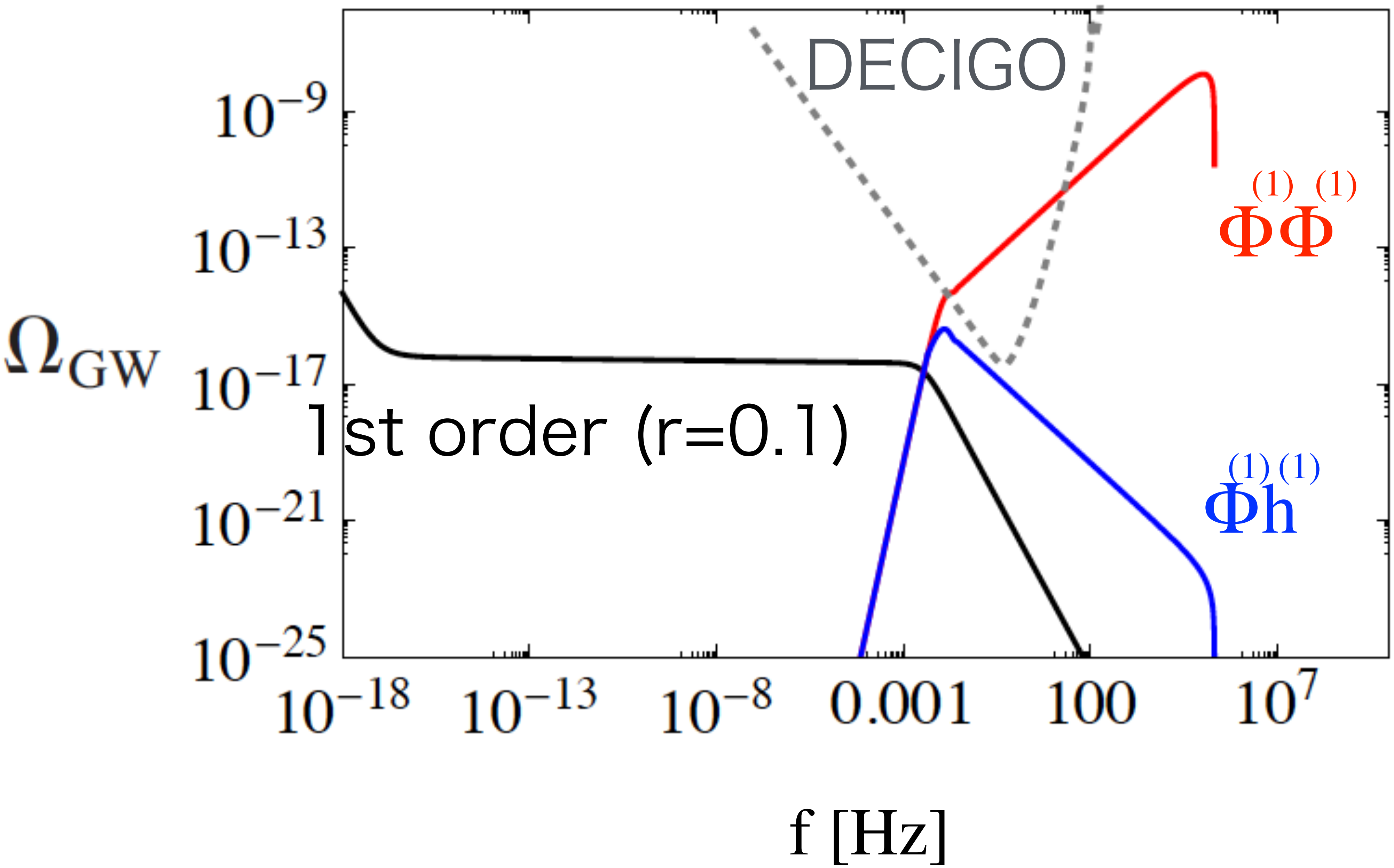
$$P_{h,prim}(k_{DECIGO}) \sim 1.2\times10^{-10}$$

↑ difference becomes smaller

Result (Preliminary)

$T_R = 10^5$GeV
$H_{inf} = 10^8$GeV

$P_{\Phi,prim}(k_{DECIGO}) \sim 4\times10^{-10}$
$P_{h,prim}(k_{DECIGO}) \sim 1.2\times10^{-10}$



comparable contribution at low frequency

Conclusions

- 2nd order GWs are robust predictions of inflationary theory, but the amplitude is typically small.
- They are enhanced if the Universe experiences an early matter dominated phase, and become detectable by DECIGO (and complementary with 1st order GWs).
- We have calculated the effect of $O(h\Phi)$ contributions.
- Models with small Φ and large h at small scales (eg. large running) would be interesting to investigate in our formulation.