On Acceleration of the Universe

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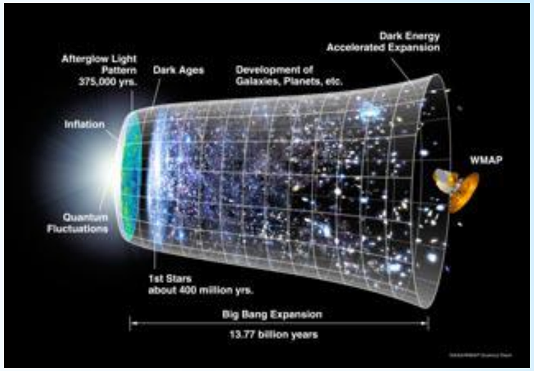
Big mystery in cosmology

Acceleration of cosmic expansion



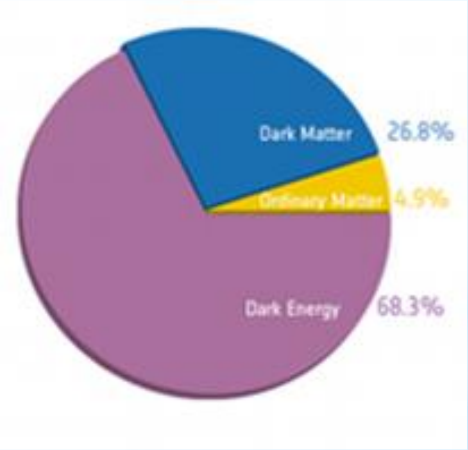
- Inflation: early stage of the Universe

  Inflaton ?

- Present Acceleration

  cosmological constant

$$\Lambda \sim 10^{-120} m_{PL}^2$$



⇒
- Dark Energy
- Modified gravity

Two Comments

[I] Matter couplings

[2] Negative cosmological constant

[I] Matter couplings

When we discuss acceleration by some unknown field (or modification of gravity), we ignore matter fields.

Matter: (1) Particles in Standard Model

(2) Perfect Fluid with $P = w\rho \quad (w \geq 0)$

Its energy density will drop when the Universe expands

However, if there exists some couplings between matter and field (or gravity), dynamics may change.

(1) Coupling with gauge field

What is an inflaton ϕ?

✧ top-down superstring (or 10D supergravity)

In compactification,
we naturally expect a dilaton or moduli coupling. $\exp[-\alpha\phi]$

This coupling may spoil the inflationary models unless the moduli is fixed.

Townsend (2003) $V = V_0 \exp[-\alpha\phi] \quad (V_0 \geq 0)$

Flux compactification $\alpha \geq \sqrt{6}$

Hyperbolic compactification $\sqrt{2} \leq \alpha \leq \sqrt{6}$

⇨ No accelerated expansion

Note: power-law inflationary solution if $\alpha < \sqrt{2}$

scale factor $a = t^p$ with $p = \dfrac{2}{\alpha^2}$

There exists another natural ingredient in the unified theories : **gauge fields**

Abelian [U(1)] or non-Abelian [e.g. SU(2)] gauge fields

Heteroticstring theory $E_8 \times E_8$

Flux compactification **U(1) multiplet**

In effective 4D action,

moduli coupling may appear: $\frac{1}{4}\exp[\lambda\phi]\mathbf{F}^2$

Hull-Townsend (1995) : $\lambda = 0, \sqrt{2/3}, \sqrt{2}, \sqrt{6}$

If VEVs of gauge fields exist, it will change the dynamics of a scalar field.

$$\frac{1}{4} f^2(\phi) \mathbf{F}^2$$

- U(1) field → **Anisotropic Inflation**

Kanno, Soda, Watanabe (2009), Watanabe, Kanno, Soda (2010) $\frac{1}{4}\exp\left[c\phi^2\right] F_{\mu\nu}^2$

Kanno. Soda, Watabnabe (2010) $\exp\left[\lambda\phi\right] F_{\mu\nu}^2$

- **U(1) multiplet** with the same gauge-kinetic coupling
- **Non-Abelian gauge field**

The isotropic inflationary universe is an attractor.
Anisotropic inflation can be possible as a transient state

KM & K. Yamamoto (2013)

Inflation with Gauge Fields

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2}(\nabla\phi)^2 - V_0 e^{-\alpha\phi} - \frac{1}{4} e^{\lambda\phi} F^{(\mathrm{a})}_{\mu\nu} F^{(\mathrm{a})\mu\nu} \right]$$

$$F^{(\mathrm{a})}_{\mu\nu} = \partial_\mu A^{(\mathrm{a})}_\nu - \partial_\nu A^{(\mathrm{a})}_\mu + g_{YM} \epsilon_{\mathrm{abc}} A^{(\mathrm{b})}_\mu A^{(\mathrm{c})}_\nu$$:SU(2) Yang-Mills field

Isotropic and homogeneous universe

FLRW metric $ds^2 = -dt^2 + a^2(t) d\boldsymbol{x}^2$

YM potential $A^{(\mathrm{a})}_i = A(t)\delta^{(\mathrm{a})}_i$ $A^{(\mathrm{a})}_0 = 0$

Scalar field $\phi(t)$

Basic equations:

- **YM equation**

$$\ddot{A} + H\dot{A} + \lambda\dot{\phi}\dot{A} + 2g_{YM}^2\frac{A^3}{a^2} = 0$$

electric component $E := -\frac{\dot{A}}{a}$ magnetic component $B = g_{YM}\frac{A^2}{a^2}$

YM energy density $\rho_{YM} = \rho_E + \rho_B$ $\rho_E = \frac{3}{2}e^{\lambda\phi}E^2$ $\rho_B = \frac{3}{2}e^{\lambda\phi}B^2$

$$\dot{\rho}_E = -(4H + \lambda\dot{\phi})\rho_E - 4(\dot{A}/A)\rho_B$$

$$\dot{\rho}_B = -(4H - \lambda\dot{\phi})\rho_B + 4(\dot{A}/A)\rho_B$$

- **scalar field equation**

$$\ddot{\phi} + 3H\dot{\phi} - \alpha V - \lambda\left(\rho_E - \rho_B\right) = 0$$

- **Einstein equations**

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V + \rho_{YM}\right] \qquad \dot{H} = -\left[\frac{1}{2}\dot{\phi}^2 + \frac{2}{3}\rho_{YM}\right]$$

U(1) triplet

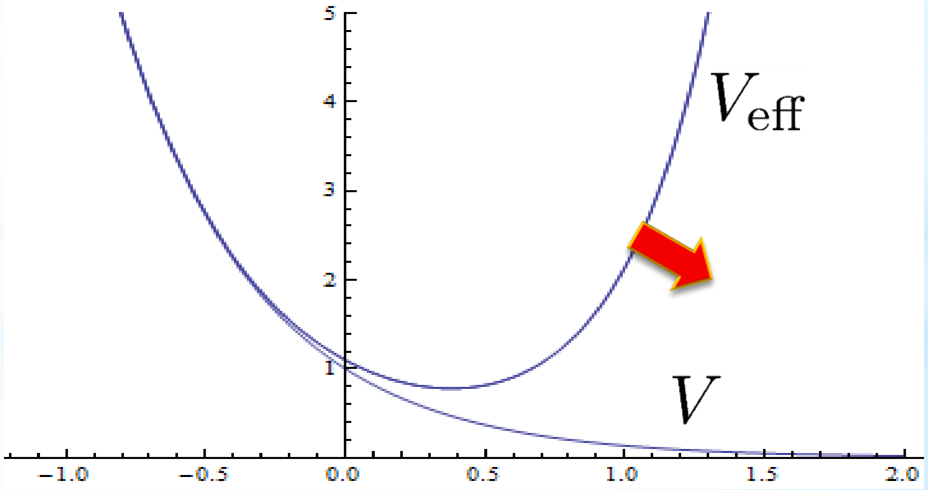
no non-linear coupling

$$\dot{\rho}_E = -(4H + \lambda\dot{\phi})\rho_E$$
$$\dot{\rho}_B = -(4H - \lambda\dot{\phi})\rho_B$$

$$\rho_E = \rho_{E0}\frac{e^{-\lambda(\phi-\phi_0)}}{(a/a_0)^4}$$
$$\rho_B = \rho_{B0}\frac{e^{\lambda(\phi-\phi_0)}}{(a/a_0)^4}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_{\rm eff}}{\partial\phi} = 0 \qquad V_{\rm eff} = V_0 e^{-\alpha\phi} + \frac{1}{a^4}\left(C_E e^{-\lambda\phi} + C_B e^{\lambda\phi}\right)$$

$$C_E \neq 0 \ \&\ \lambda < 0$$
$$C_B \neq 0 \ \&\ \lambda > 0$$



power-law solution with larger power exponent

power-law solutions

$$a \propto t^p \qquad \phi = \frac{2}{\alpha} \ln t + \phi_0$$

$$\rho_E = \frac{C_E}{a^4} \exp[-\lambda\phi] \qquad \rho_B = \frac{C_B}{a^4} \exp[\lambda\phi]$$

Inflation

- The case with electric field (E_{U1})

$$p = \frac{1}{2}\left(1 - \frac{\lambda}{\alpha}\right) \qquad C_B = 0 \qquad \lambda < \alpha - \frac{4}{\alpha} \ \& \ \lambda < 0 \qquad \lambda < -\alpha$$

- The case with magnetic field (B_{U1})

$$p = \frac{1}{2}\left(1 + \frac{\lambda}{\alpha}\right) \qquad C_E = 0 \qquad \lambda > -\alpha + \frac{4}{\alpha} \ \& \ \lambda > 0 \qquad \lambda > \alpha$$

- The scalar field dominance (S_{U1})

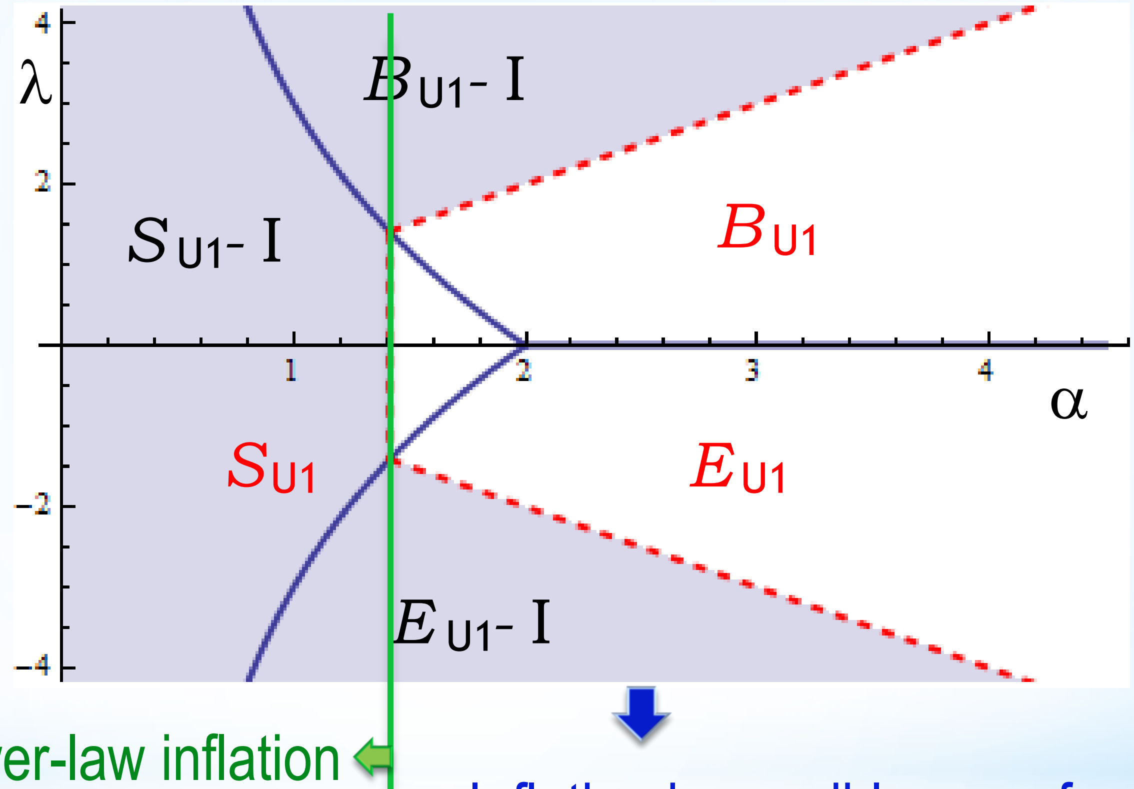
$$p = \frac{2}{\alpha^2} \qquad C_E = C_B = 0 \qquad \alpha - \frac{4}{\alpha} < \lambda < -\alpha + \frac{4}{\alpha} \qquad \alpha < \sqrt{2}$$

E-B duality $\quad E_{U1} \Leftrightarrow B_{U1}$

phase diagram

each phase is an attractor

$$-\frac{1}{4}e^{\lambda\phi}\mathbf{F}^2$$



conventional power-law inflation

Inflation is possible even for $\alpha > \sqrt{2}$

$$V = V_0 e^{-\alpha\phi}$$

YM field

Both electric and magnetic components exist
Non-linear coupling

→ complicated

There exist the corresponding inflationary phases

- The case with dominant electric component (E_{YM}-I)

$$p = \frac{1}{2}\left(1 - \frac{\lambda}{\alpha}\right) \qquad \lambda < \alpha - \frac{4}{\alpha} \ \& \ \lambda < -\alpha$$

- The case with dominant magnetic component (B_{YM}-I)

$$p = \frac{1}{2}\left(1 + \frac{\lambda}{\alpha}\right) \qquad \lambda > -\alpha + \frac{4}{\alpha} \ \& \ \lambda > \alpha$$

- The scalar field dominance (S_{YM}-I)

$$p = \frac{2}{\alpha^2} \qquad \alpha - \frac{4}{\alpha} < \lambda < -\alpha + \frac{4}{\alpha} \ \& \ \alpha < \sqrt{2}$$

If

$\rho_B \ll \rho_E$

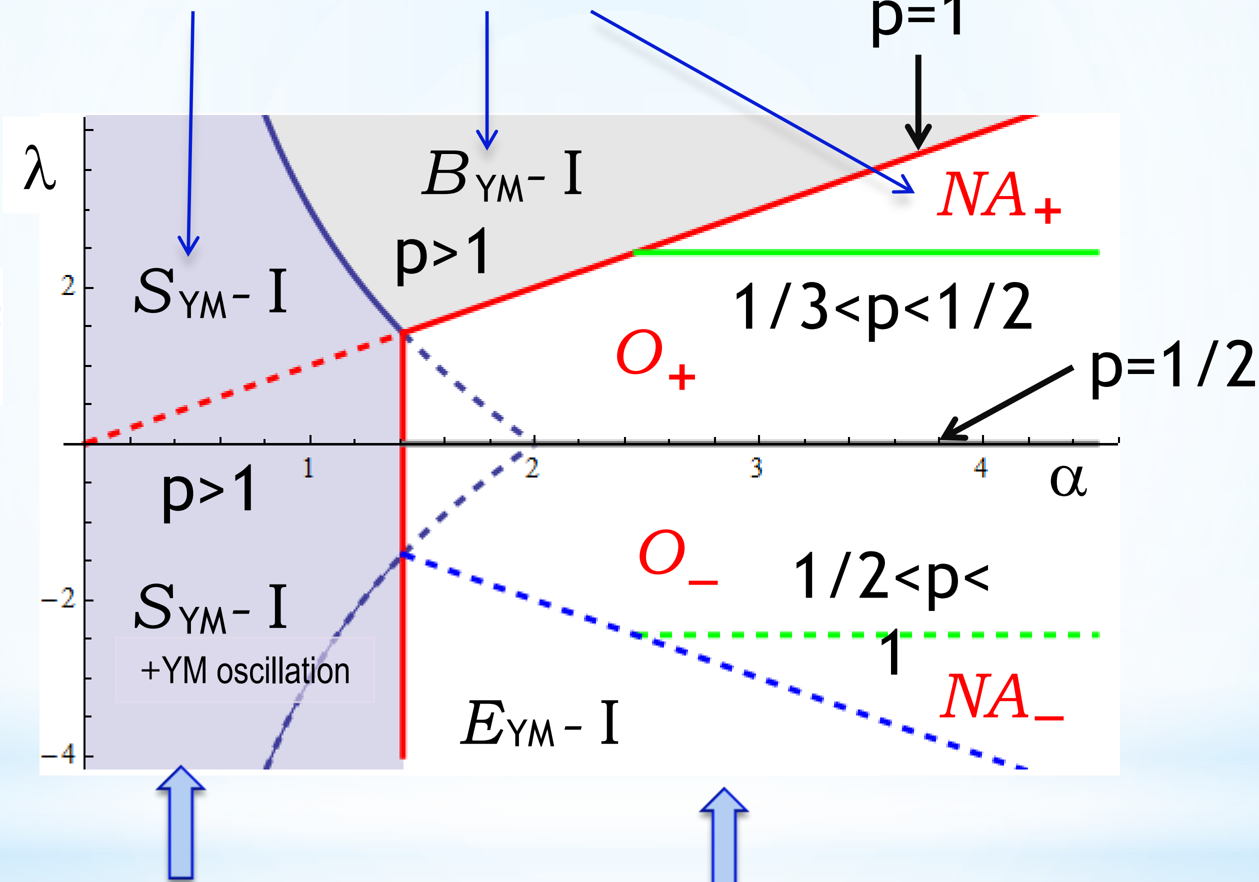
$\rho_B \gg \rho_E$

$\rho_B, \rho_E \ll \rho_\phi$

phase diagram

Stable fixed points

$$-\frac{1}{4}e^{\lambda\phi}\mathbf{F}^2$$



Inflationary spacetime +YM oscillations

Asymptotic spacetime +YM oscillations

$$V = V_0 e^{-\alpha\phi}$$

- The power-law inflation with the gauge field is possible even for a steep potential such as $\alpha > \sqrt{2}$ which is expected in the higher-dimensional unified theories.

(2) Coupling with perfect fluid

KM, Y. Fujii (09)

Modified gravity (e.g. scalar tensor theory)

MODEL

$$S_J = \int d^4x\sqrt{-g}\left[\frac{\xi}{2}\phi^2 R(g) - \frac{\epsilon}{2}(\nabla\phi)^2 - V(\phi)\right] + \int d^4x\sqrt{-g}L_{\mathrm{m}}(\psi, g)$$

⇕ **conformal transformation**

$\mathbf{g} \rightarrow \mathbf{g}\ \mathbf{exp}(2\zeta\kappa\sigma)$

$\zeta = \sqrt{\xi/(\varepsilon+6\xi)}$

Einstein gravity (g) + scalar field σ

$U = V \exp(-4\zeta\kappa\sigma)$

Dynamics without matter is well-known

But, coupling with matter is important

$V = V_0$ (constant)

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\left[\frac{1}{2}\left(\frac{d\sigma}{dt}\right)^2 + U + \rho\right]$$

$$\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + \frac{\partial U}{\partial \sigma} = \zeta\kappa(\rho - 3P)$$

$$\frac{d\rho}{dt} + 3\gamma H\rho = -\zeta\kappa(4-3\gamma)\frac{d\sigma}{dt}\rho$$

$$P = (\gamma - 1)\rho$$

Two fixed points

FP1 Scalar field dominant

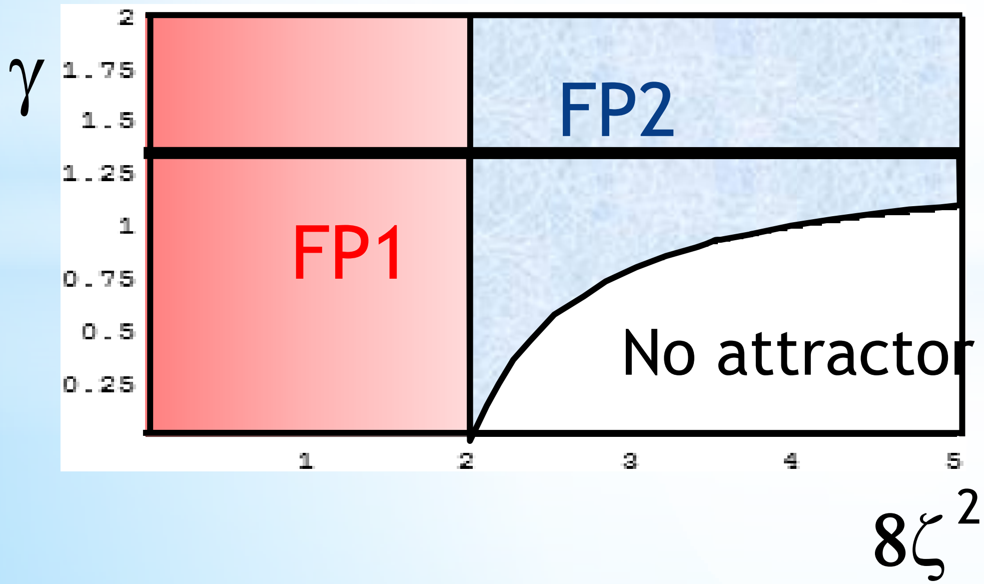
$$a \propto t^{\frac{1}{8\zeta^2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

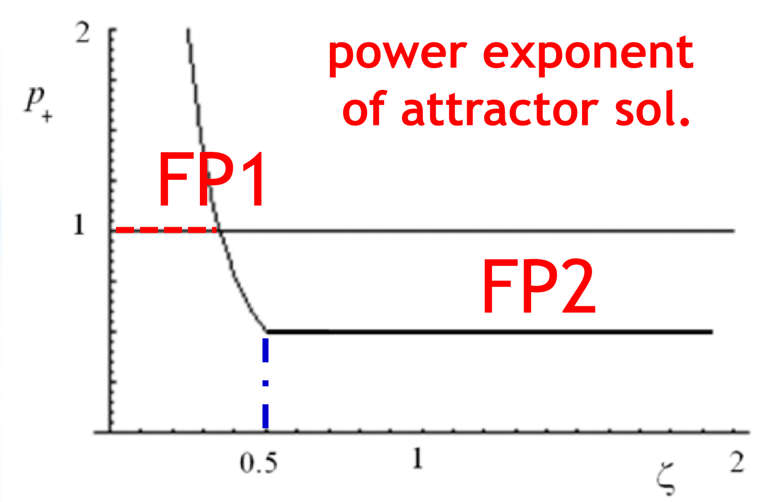
FP2 Scaling solution

$$\left(\frac{\rho}{V}\right)_2 = \frac{2(4\zeta^2 - 1)}{2 - \gamma - 2(4-3\gamma)\zeta^2} \quad \text{const}$$

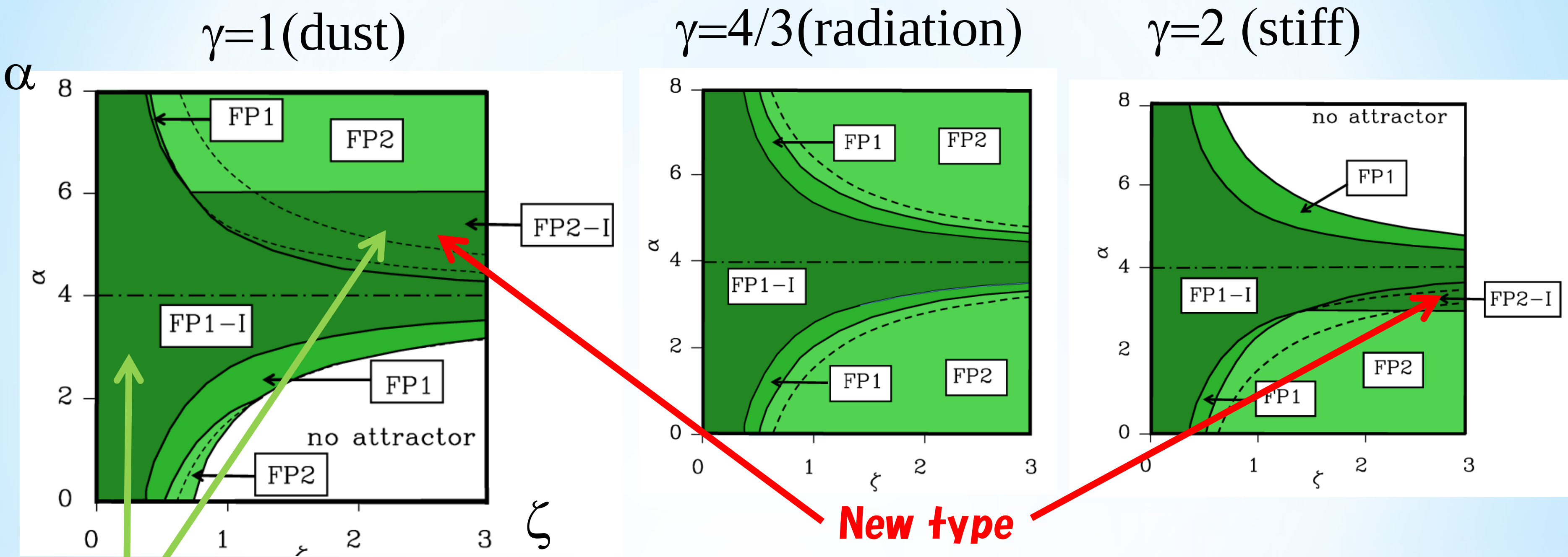
$$a \propto t^{\frac{1}{2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

Minkowski in Jordan frame





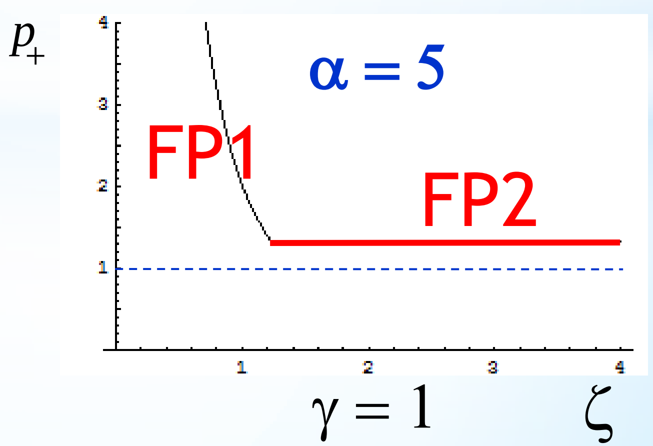
power-law potential $V = (\kappa\phi)^{\alpha} V_0$



Power-law inflation

Inflation with a steep potential

power exponent
of attractor sol.



$a \propto t^p$

[2] Negative cosmological constant

Supergravity (Superstring)

⇨ cosmological constant ≤ 0

⇩

Accelerating universe

effective cosmological constant > 0

(1) Quantum corrections

(2) KKLT compactification

Heterotic superstring theory

Quantum corrections

R.R. Metsaev A.A. Tseytlin, ('87)

$$S=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g}e^{-2\phi}\Big[R+4(\nabla\phi)^2+\alpha_2R^2_{ABCD}\Big] \qquad \alpha_2=\frac{\alpha'}{8}$$

ghosts

$$R^2_{(\mathrm{GB})}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$$

B. Zwiebach ('85)

Ambiguity in the effective action due to field redefinition

$$S=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left[R+4(\nabla\phi)^2+\alpha_2\left(R^2_{(\mathrm{GB})}-\frac{1}{16}(\nabla\phi)^4\right)\right]$$

Negative cosmological constant + higher-curvature terms

KM & Ohta (14)

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} \left[R - 2\Lambda + \alpha_2 R^2_{(\mathrm{GB})} \right]$$

$$ds_D^2 = -dt^2 + e^{2u_1} ds_p^2 + e^{2u_2} ds_q^2 \qquad D = 1 + p + q$$

Accelerating universe $\quad u_1 = Ht\,, \quad u_2 = \mathrm{constant}$

EH action
$$H = \sqrt{\frac{2\Lambda}{p(p+q-1)}}$$

$$A_q := \sigma_q e^{-2u_2} = \frac{2\Lambda}{(q-1)(p+q-1)}$$

$\Lambda > 0\,, \quad \sigma_q > 0$ ⇨ unstable

EH+GB

$$H^2 = \frac{1}{p_2}\Big\{ - p[1-2(q-1)(p-q+1)A_q] + \Big[p^2[1-2(q-1)(p-q+1)A_q]^2 + 8p_2(q-1)A_q[1+2(q-2)_3A_q]\Big]^{1/2}\Big\}$$

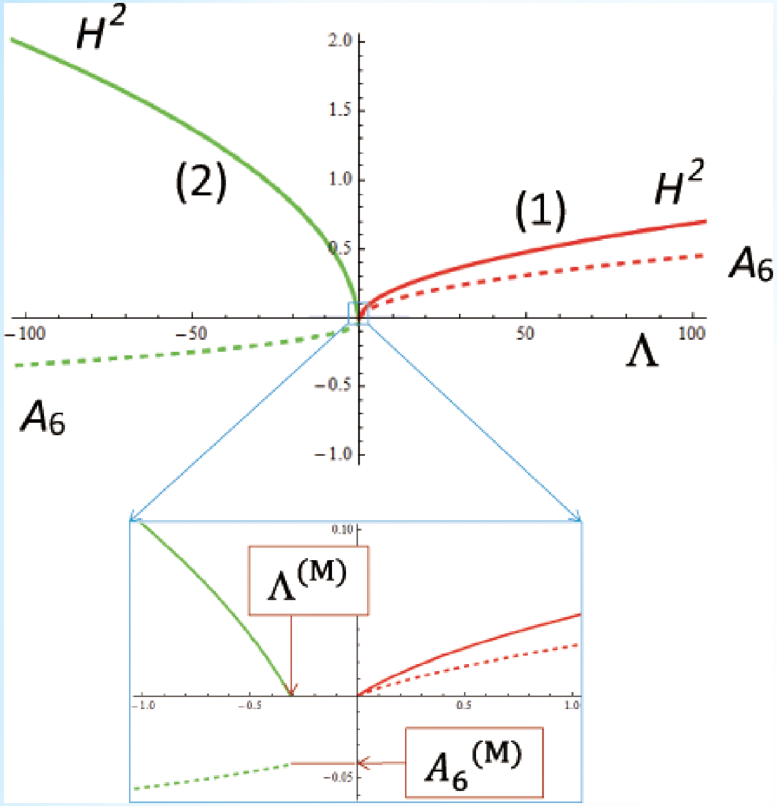
$$p_2 := p(p-1)(p-2)$$

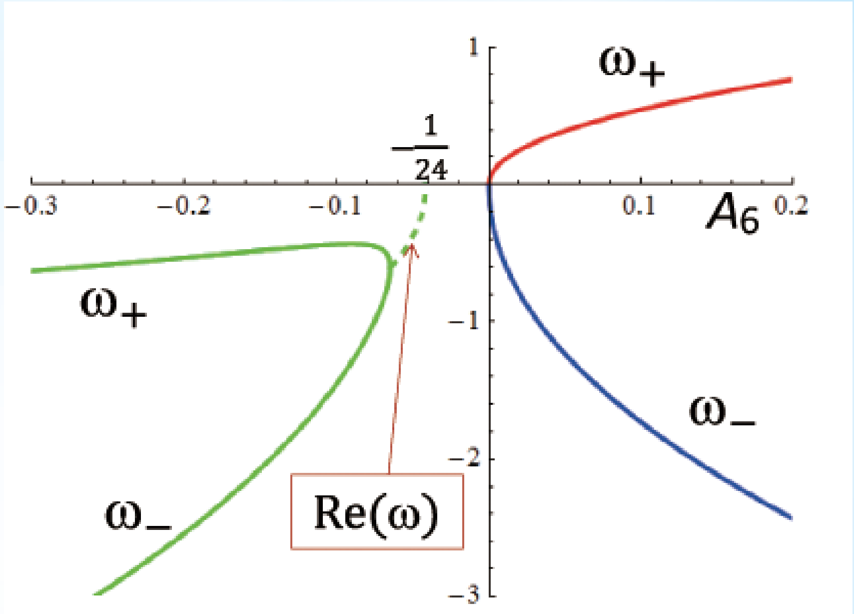
two branches:

$A_q \geq 0$ $\sigma_q \geq 0$ *q*-sphere

$$A_q \leq A_q^{(\mathrm{M})} := -\frac{1}{2(q-2)(q-3)}$$

$\sigma_q \leq 0$ *q*-hyperbolic space





Branch (2)

Λ: negative

stable

Branch (1)

Λ: positive

unstable

de Sitter solution with GB term is stable if Λ is negative.

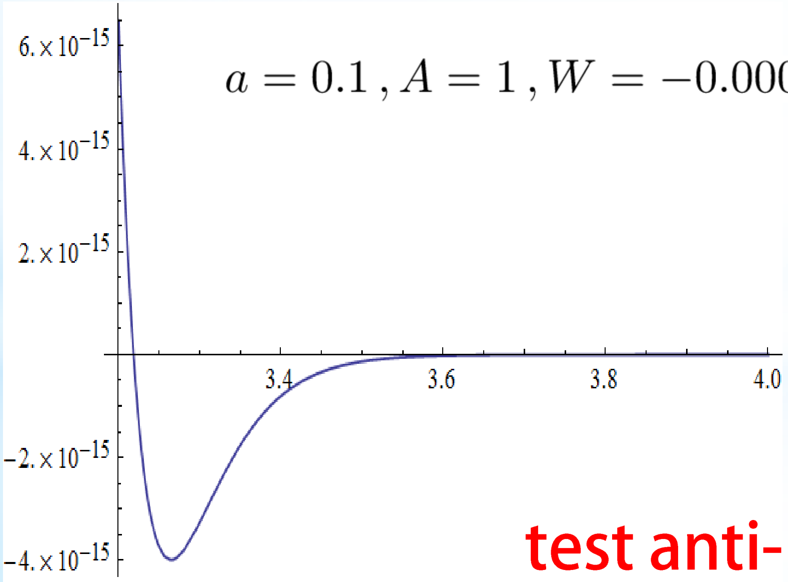
(2) KKLT compactification   fixing moduli

CY compactification:

$$d\bar{s}_g^2 = S^{-6} ds_g^2(x) + S^2 d\tilde{s}_{\rm CY}^2(y)$$

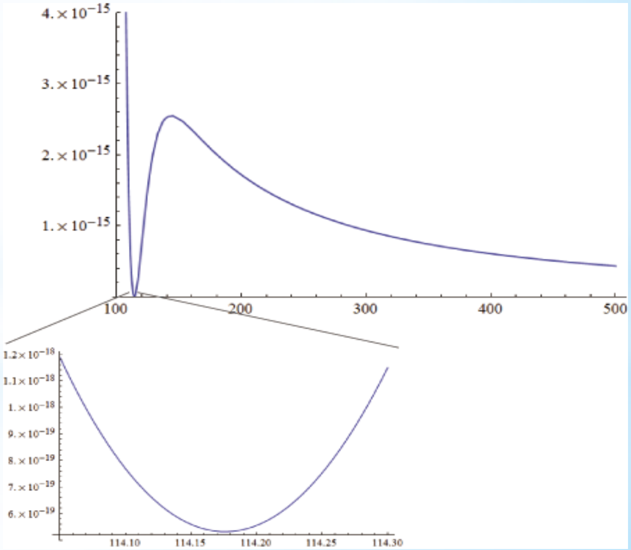
$$S = \exp\left[\frac{1}{4}\sqrt{\frac{2}{3}}\phi_g\right]$$

$$V_g(\phi_g) = \frac{a_g A_g}{2} e^{-2\sqrt{2/3}\phi_g} \exp[-a_g e^{\sqrt{2/3}\phi_g}] \left[W_g + \frac{A_g}{3}\exp[-a_g e^{\sqrt{2/3}\phi_g}]\left(3 + a_g e^{\sqrt{2/3}\phi_g}\right)\right]$$



AdS

test anti-brane →



dS

- Two types of strings

g-string & f-string ⟹ bigravity theory in 10-dim

two metrics & twin matter fluid
g, f

Interactions ?

- similar interactions to ghost-free bigravity $\bar{\gamma}^A{}_B = \left[\sqrt{g^{-1}f}\right]^A{}_B$

$$S_{\rm I} = \frac{\bar{m}^2}{\bar{\kappa}^2}\int d^D X\sqrt{-\bar{g}}\Big[\frac{b_0}{D!}\epsilon_{AB\cdots C}\epsilon^{AB\cdots C} + \frac{b_1}{(D-1)!}\epsilon_{AB\cdots C}\epsilon^{PB\cdots C}\bar{\gamma}^A{}_P + \cdots$$
$$+\frac{b_k}{k!(D-k)!}\epsilon_{A_1A_2\cdots A_kA_{k+1}\cdots A_D}\epsilon^{B_1B_2\cdots B_kA_{k+1}\cdots A_D}\bar{\gamma}^{A_1}{}_{B_1}\bar{\gamma}^{A_2}{}_{B_2}\cdots\bar{\gamma}^{A_k}{}_{B_k} + \cdots$$
$$+\frac{b_D}{D!}\epsilon_{A_1A_2\cdots A_D}\epsilon^{B_1B_2\cdots B_D}\bar{\gamma}^{A_1}{}_{B_1}\bar{\gamma}^{A_2}{}_{B_2}\cdots\bar{\gamma}^{A_D}{}_{B_D}\Big]$$

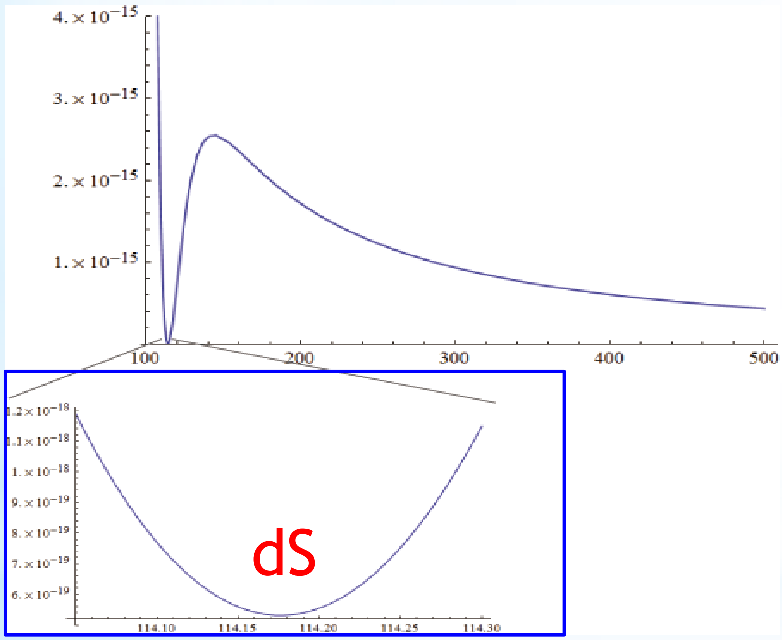
$b_0, \cdots, b_D$: coupling constants

Not need to introduce anti-branes

$$V_I = Be^{-\sqrt{\frac{3}{2}}\phi}$$

$$B = \frac{m^2}{\kappa^2}(B_0 + 4B_1 + 6B_2 + 4B_3 + B_4)$$

$$B_k = b_k + 6b_{k+1} + 15b_{k+2} + 20b_{k+3} + 15b_{k+4} + 6b_{k+5} + b_{k+6} \quad (k = 0 - 4)$$



$$V_g(\phi_g) + V_I(\phi_g)$$

$$a = 0.1\,, A = 1\,, W = -0.0001\,, B = 4.853 \times 10^{-12}$$

Interactions between three forms

CY ⇨ VEV of three forms $H_{abc}\,,\quad F_{def}$

$$S_{\rm I} = g_{\rm int}\int d^{10}X\sqrt{-\bar{g}}\bar{\epsilon}_{a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}}\bar{\epsilon}^{b_1b_2b_3b_4b_5b_6b_7b_8b_9b_{10}}$$

$$\times\bar{\Gamma}^{a_1a_2a_3a_4}{}_{b_1b_2b_3b_4}H^{(g)\,a_5a_6a_7}F^{(g)\,a_8a_9a_{10}}H^{(f)}_{b_5b_6b_7}F^{(f)}_{b_8b_9b_{10}}$$

$$\bar{\Gamma}^{a_1a_2a_3a_4}{}_{b_1b_2b_3b_4} = \frac{b_0}{4!}\delta^{a_1}{}_{b_1}\delta^{a_2}{}_{b_2}\delta^{a_3}{}_{b_3}\delta^{a_4}{}_{b_4} + \frac{b_1}{3!}\bar{\gamma}^{a_1}{}_{b_1}\delta^{a_2}{}_{b_2}\delta^{a_3}{}_{b_3}\delta^{a_4}{}_{b_4}$$

$$+\frac{b_2}{4}\bar{\gamma}^{a_1}{}_{b_1}\bar{\gamma}^{a_2}{}_{b_2}\delta^{a_3}{}_{b_3}\delta^{a_4}{}_{b_4} + \frac{b_3}{3!}\bar{\gamma}^{a_1}{}_{b_1}\bar{\gamma}^{a_2}{}_{b_2}\bar{\gamma}^{a_3}{}_{b_3}\delta^{a_4}{}_{b_4}$$

$$+\frac{b_4}{4!}\bar{\gamma}^{a_1}{}_{b_1}\bar{\gamma}^{a_2}{}_{b_2}\bar{\gamma}^{a_3}{}_{b_3}\bar{\gamma}^{a_4}{}_{b_4}$$

$$V_{\rm I} = \lambda\exp\left[-\sqrt{\frac{3}{2}}\phi_g\right]$$

the same as the previous interaction term

$$\lambda = g_{\rm int}H_gF_gH_fF_f\left(b_0+b_1+b_2+b_3+b_4\right)$$

Does this explain smallness of the "graviton mass" ?

[I] Matter couplings

Matter coupling may change the dynamics

[2] Negative cosmological constant

Two examples to find de Sitter solution

Thank you for your attention