2nd APCTP-TUS workshop on dark energy

2015. 8. 5

Modified Gravity inside Astrophysical Bodies

Shuntaro Mizuno (Waseda)

Saito, Yamauchi, SM, Gleyzes, Langlois, JCAP06(2015) 008

Contents

1. Introduction
2. Breaking of Vainshtein mechanism in beyond Horndeski
3. Impact of the breaking on the stellar structure
4. Summary and discussions

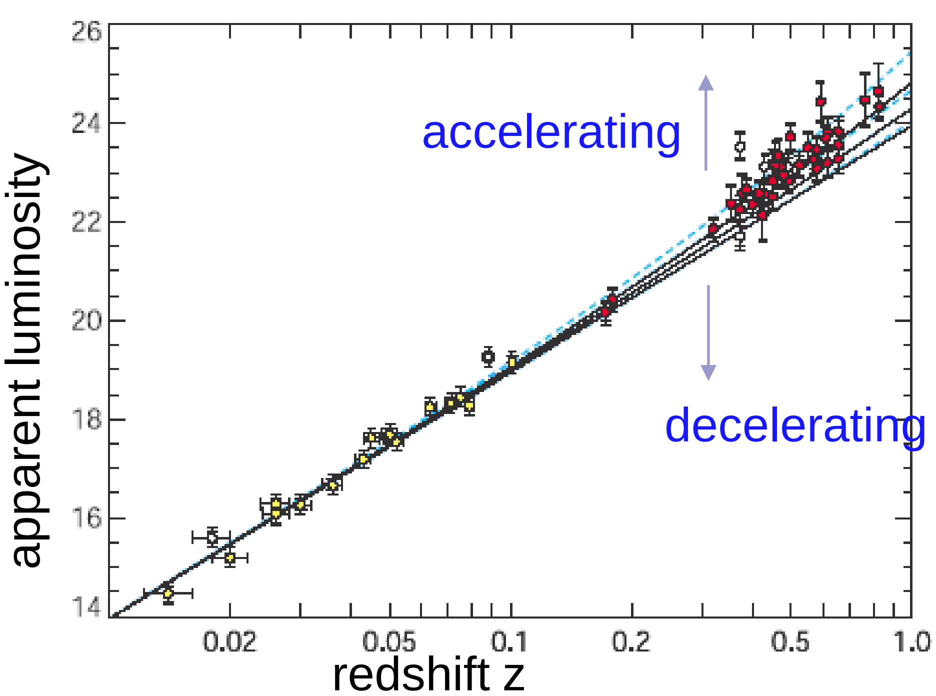
1. Introduction

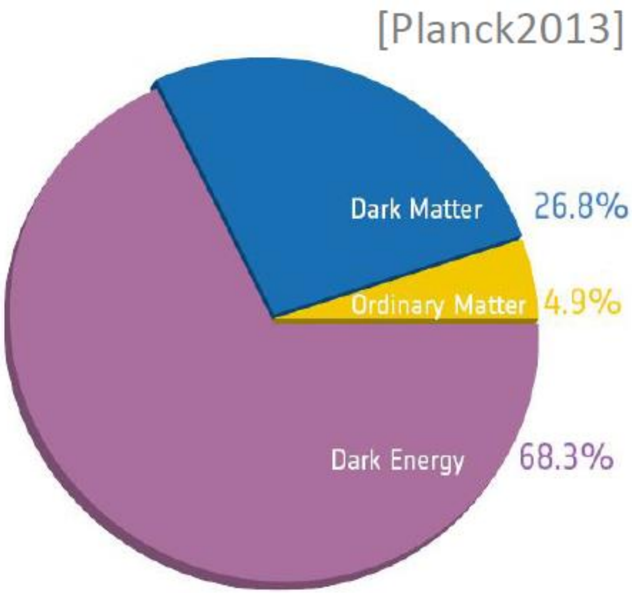
Cosmic acceleration

・Expansion of the current Universe is accelerating.

⇨ Dark energy or Modified gravity ?

・Successful modified gravity should have the deviation from GR only on cosmological (IR) scales and must not spoil the success of it in the solar-system experiments.





Modified gravity theories

- GR: massless spin 2 particle (graviton)

In general, to modify gravity on large scales, a new d.o.f is introduced.

- Scalar-tensor theory, f(R) gravity, Galileon, ,,,,

  light scalar d.o.f universally coupled to matter

  It also mediates a new long-range force (fifth force) on small scales

- Massive gravity

  a mass term for the graviton

  massless multiplet (2 d.o.f) ⇨ massive multiplet (5 d.o.f)

The helicity 0 mode works in a similar way

Fifth force

- Prototype Brans-Dicke model as a concrete example

$$\mathcal{L} = \sqrt{-g}\left(\varphi R - \omega \frac{1}{\varphi} g^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi + L_{\rm matter}\right)$$

⇒ $$\Box \varphi = \frac{1}{6+4\omega} T$$ the scalar field couples with matter !!

⇒ $$V_\varphi(r) = -G_s \frac{M}{r}$$

with $G_s = \dfrac{G_N}{3+2\omega}$

But for relativistic objects, this coupling does not work.

⇒ Constraints $\omega > 40000$ from solar-system experiment

Screening mechanisms

- Poisson equation for the scalar field (non-relativistic objects)

$$\nabla^2 \phi \quad + \cdots = 4\pi G_s \rho$$

interaction terms are important to screen the fifth force around non-relativistic objects

- Chameleon mechanism

f(R) theory

The scalar field becomes massive

- Vainshtein mechanism

Galileon, Massive gravity

The scalar field strongly coupled with derivative interaction

2. Breaking of Vainshtein mechanism in beyond Horndeski

Behavior of metric perturbations

- Metric perturbation in Newtonian gauge

$$\mathrm{d}s^2 = -\left(1 + 2\Phi(\boldsymbol{x})\right)\mathrm{d}t^2 + \left(1 - 2\Psi(\boldsymbol{x})\right)\mathrm{d}\boldsymbol{x}^2$$

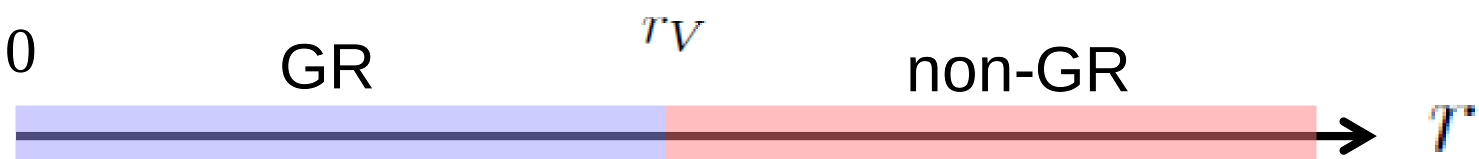
- General Relativity

$$\nabla\Phi = \nabla\Psi \left(= \frac{G_N M}{r^2} \right)$$

- Scalar-tensor theories

$$\phi(\mathbf{x}) = \phi_0 + \pi(\mathbf{x}) \quad \Rightarrow \quad \nabla\Phi - \nabla\Psi \sim \nabla\pi$$

- Desirable behavior



$$\nabla\Phi = \nabla\Psi \gg \nabla\pi \qquad\qquad \nabla\Phi \neq \nabla\Psi$$

Horndeski theory

Covariant Galileon

Horndeski, `74,
Kobayashi, Yamaguchi, Yokoyama, `11
Deffayet,Gao, Steer, Zahariade, `11

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + \frac{\partial G_4}{\partial X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

- The most general scalar-tensor theory with second-order field equations
- Having 4 arbitrary functions of ϕ and $X = -1/2(\nabla\phi)^2$
- Because of the nonlinear derivative interaction terms,

Vainshtein mechanism works around non-relativistic objects

Beyond Horndeski theory

Gleyzes, Langlois, Piazza, Vernizzi, `14

$$\mathcal{L} = \mathcal{L}_{\text{Horndeski}} + \mathcal{L}_{\text{beyond}}$$

$$\begin{aligned}\mathcal{L}_{\text{beyond}} = & F_4(\phi, X)\Big\{\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\Box\phi - \nabla^\mu\nabla_\mu\nabla_\lambda\phi\nabla^\nu\nabla_\nu\nabla^\lambda\phi + X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\Big\} \\ & + F_5(\phi, X)\Big\{(\Box\phi)^2\,\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi - 2\Box\phi\nabla^\mu\phi\nabla_\mu\nabla_\lambda\phi\nabla^\nu\phi\nabla_\nu\nabla^\lambda\phi \\ & - (\nabla_\mu\nabla_\nu\phi)^2\,\nabla^\rho\phi\nabla^\lambda\phi\nabla_\rho\nabla_\sigma\phi + 2\nabla^\mu\phi\nabla_\mu\nabla^\lambda\phi\nabla_\lambda\nabla^\rho\phi\nabla_\rho\nabla_\lambda\phi\nabla^\lambda\phi \\ & + \frac{2}{3}X\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}\end{aligned}$$

- This generalization gives higher order field equations.
- But # of initial conditions remain same for the time hypersurfaces that coincide with the uniform scalar field hypersurfaces.

⇒ No new d.o.f. is introduced !!

Strategy for analyzing Vainshtein mechanism

Kobayashi, Watanabe, Yamauchi, PRD91 (2015)6, 064013

Kimura, Kobayashi, Yamamoto, PRD85 (2012), 024023

De Fellice, Kase, Tsujikawa, PRD85 (2012), 044059

Narikawa, Kobayashi, Yamauchi, Saito, PRD87 (2013), 124006

Koyama, Niz, Tasinato, PRD88 (2013), 021502(R)

for Horndeski theory

- Constructing an effective theory on small scales
- Finding a spherically symmetric solution

⇨ Algebraic eq. for π' ⇨ Φ', Ψ'

New terms from beyond Horndeski theory

$$\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm Horndeski} + \mathcal{L}^{\rm eff}_{\rm beyond}$$

$$\mathcal{L}^{\rm eff}_{\rm beyond} = \frac{4a\xi_t}{\mathcal{M}} \dot{\Psi} \nabla^2 \pi + \frac{2\alpha_t}{a\Lambda^3 \mathcal{M}} \dot{\Psi} \mathcal{E}_3^{\rm Gal} - \frac{4\alpha_*}{a\Lambda^3} \nabla^i \Psi \nabla^j \pi \nabla_i \nabla_j \pi - \frac{4\beta_*}{a^3\Lambda^6} \nabla^i \Psi \nabla^j \pi \left[\left(\nabla^2 \pi\right) \nabla_i \nabla_j \pi - \nabla_i \nabla^k \pi \nabla_k \nabla_j \pi \right]$$

$$\xi_t \,, \alpha_t \,, \alpha_* \,, \beta_* \propto X$$

$$\mathcal{M}^2 = \Lambda^3 / M_{\rm Pl}$$

- $\mathcal{L}_{\rm beyond}$ also generate the same Horndeski terms in $\mathcal{L}^{\rm eff}_{\rm Horndeski}$, which can be absorbed in the redefinition of the coefficients.
- Only when $\dot{\phi} \neq 0$, these new terms appear.
- From now on, we concentrate on the third term.

Impact of new term on spherical overdensity

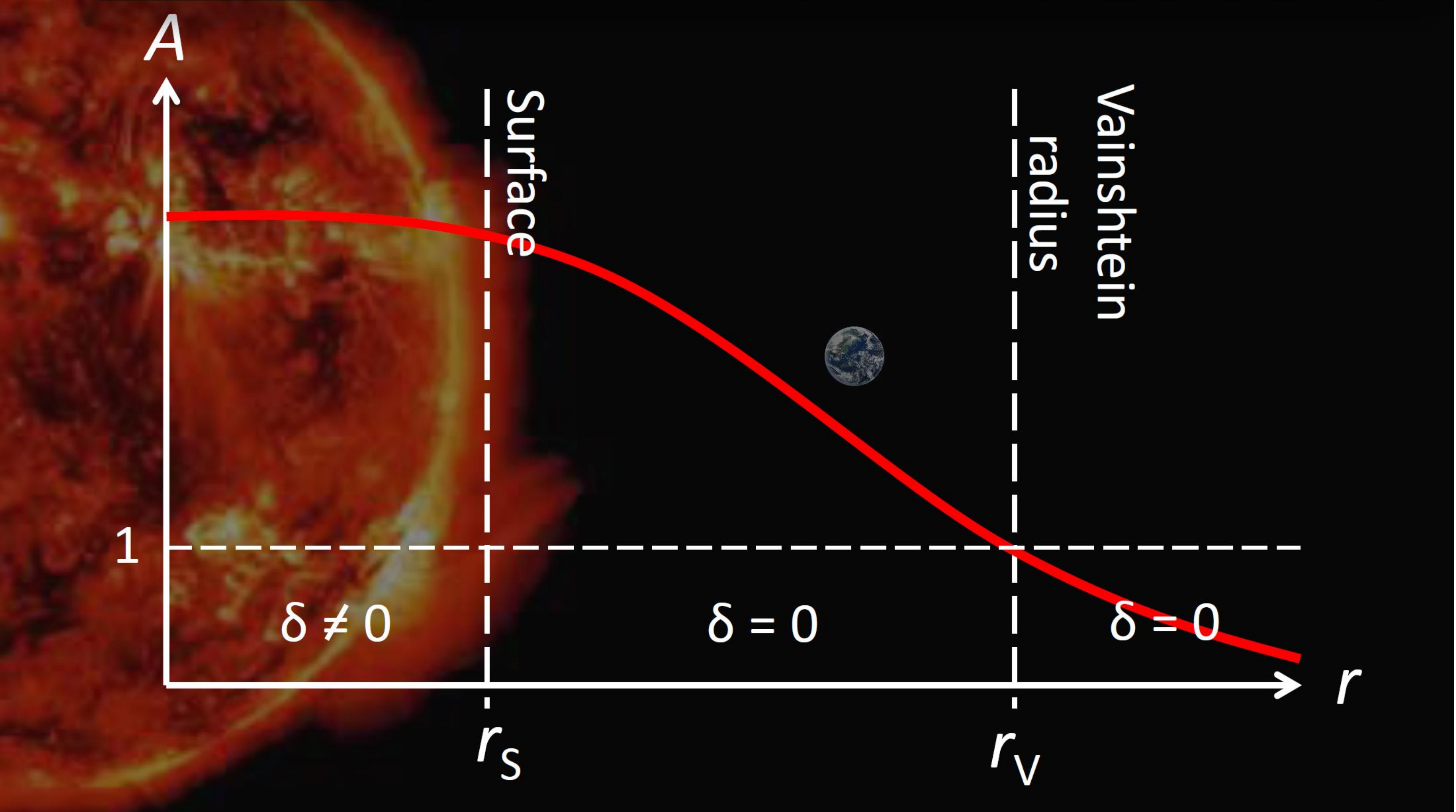
In terms of $x = \frac{1}{\Lambda^3} \frac{\pi'}{a^2 r}$ $A = \frac{1}{M_{\rm Pl}\Lambda^3} \frac{M(t,r)}{8\pi r^3}$

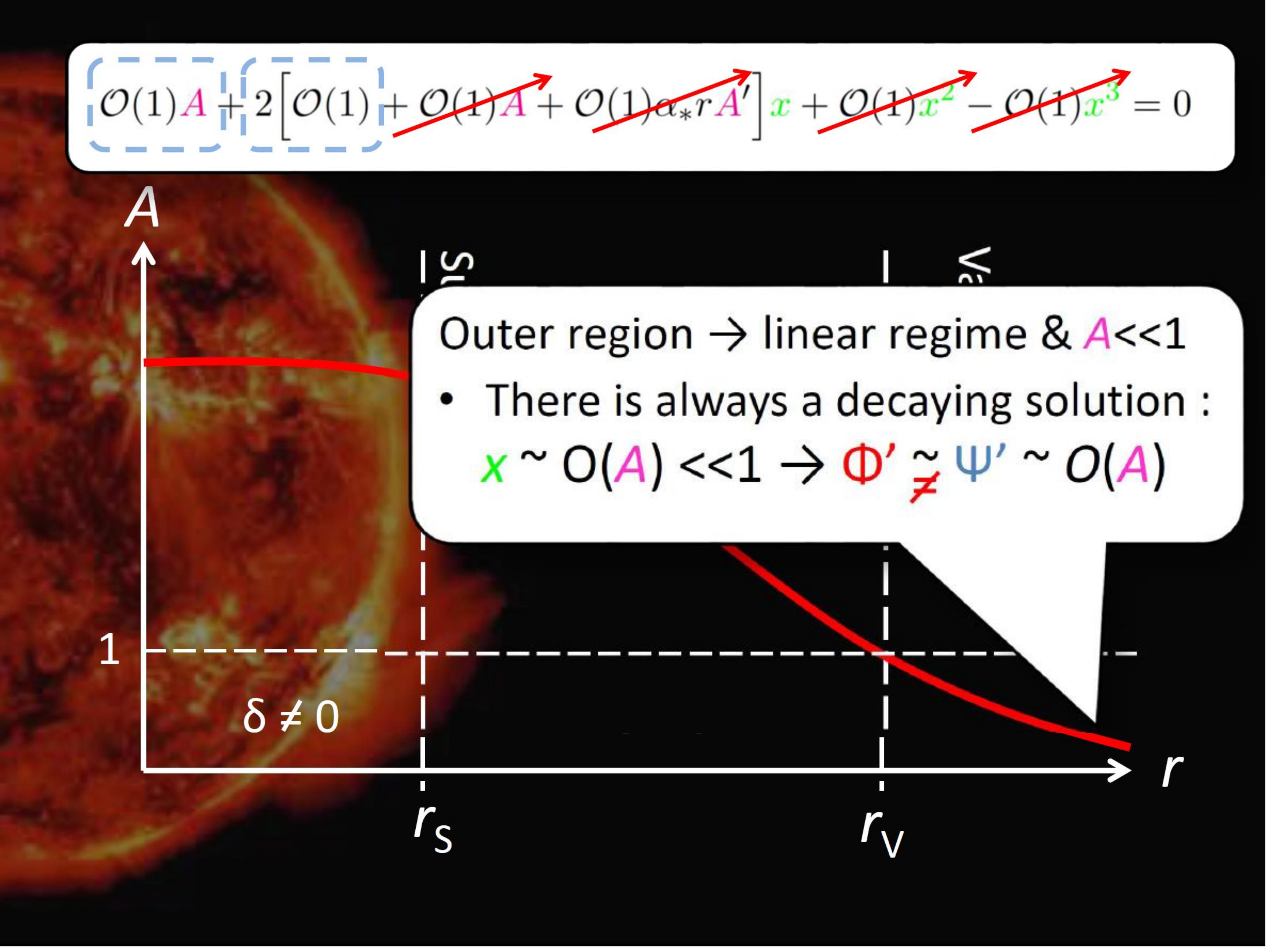
Algebraic equation for *x* can be written as

$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$

Inner solution is determined not only the enclosed mass (M∝A) but also from the local energy density (A'~ρ)!

$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$





$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$
A
Outer region → linear regime & A<<1
• There is always a decaying solution :
$x \sim O(A) \ll 1 \rightarrow \Phi' \not\simeq \Psi' \sim O(A)$
1
$\delta \neq 0$
r
r_S
r_V

$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right] x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$

Inner region → nonlinear regime & A, A', A''>>1

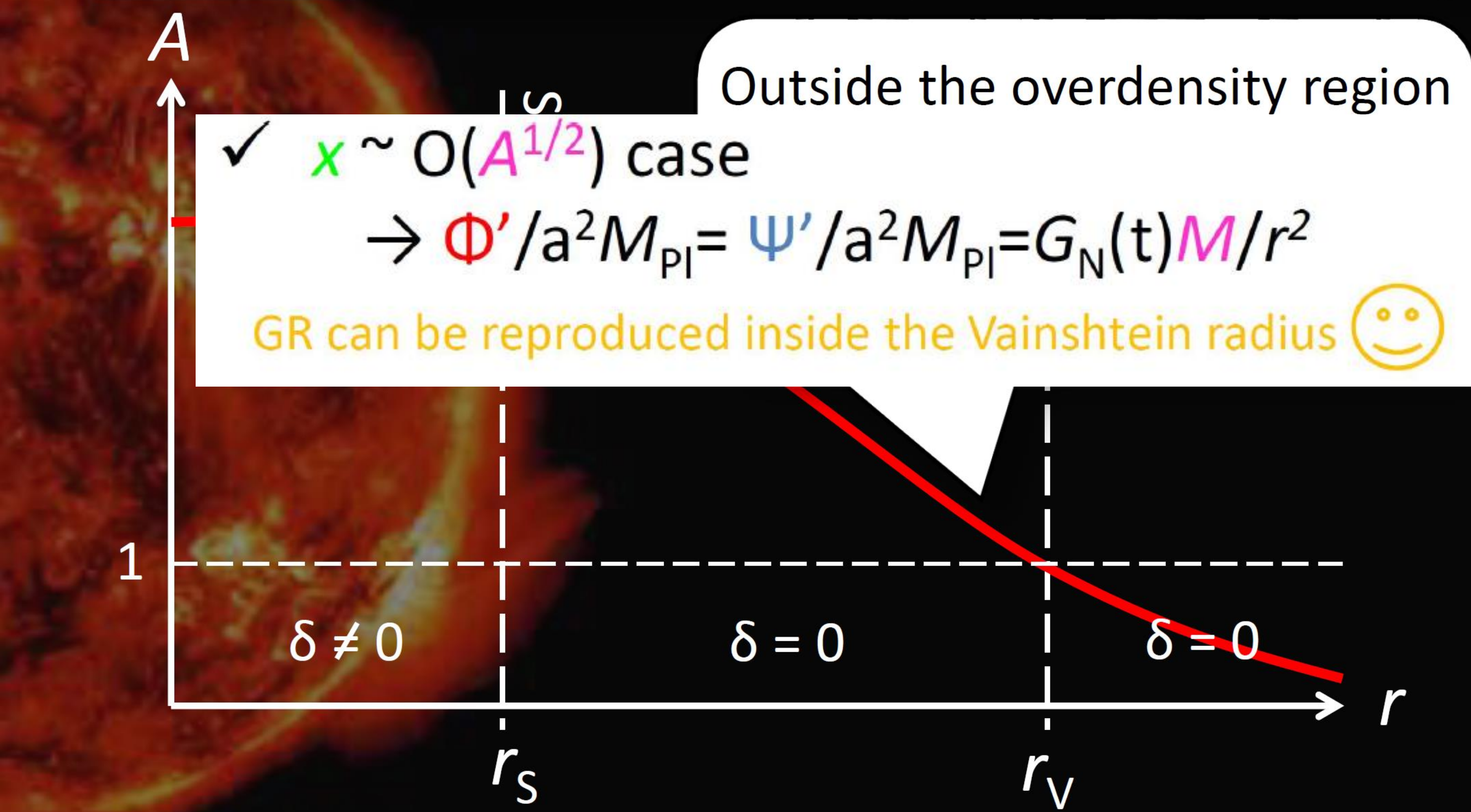
$$x^2 = \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'$$

Plugging the concrete expression of the coefficients,

$$\Rightarrow \begin{cases} \dfrac{\Phi'}{a^2 M_{\rm Pl}} = \dfrac{G_{\rm N} M}{r^2} - \mathcal{O}(1)\alpha_*^2 \dfrac{M''}{8\pi M_{\rm Pl}^2} \\ \dfrac{\Psi'}{a^2 M_{\rm Pl}} = \dfrac{G_{\rm N} M}{r^2} - \mathcal{O}(1)\alpha_* \dfrac{M'}{8\pi M_{\rm Pl}^2 r} \end{cases}$$

New contribution

$$\mathcal{O}(1)A + 2\left[\mathcal{O}(1) + \mathcal{O}(1)A + \mathcal{O}(1)\alpha_* r A'\right]x + \mathcal{O}(1)x^2 - \mathcal{O}(1)x^3 = 0$$



3. Impact of the breaking on the stellar structure

Saito, Yamauchi, SM, Gleyzes, Langlois, JCAP06(2015) 008

Model for stellar interiors

Static, spherically symmetric, polytropic model

- Euler equation (hydrostatic equilibrium)

$$\frac{dP}{dr} = -\rho \frac{d\Phi}{dr}$$

only the gravitational law is modified !!

- Poisson equation

$$\frac{d\Phi}{dr} = G_{\rm N} \left(\frac{\mathcal{M}}{r^2} - \epsilon \frac{d^2 \mathcal{M}}{dr^2} \right)$$

with

$$\mathcal{M}(r) = 4\pi \int_0^r r'^2 \rho(r') dr'$$

- Equation of state

$$P = K \rho^{1+\frac{1}{n}}$$

n : polytropic index

- 1 for neutron stars
- 3 for main sequence stars like our Sun

Modified Lane-Emden (MLE) equation

- Intro. dimensionless variable ξ and function $\chi(\xi)$

$$\xi = \frac{r}{r_c} \quad r_c = \sqrt{\frac{(n+1)K\rho_c^{-1+\frac{1}{n}}}{4\pi G_N}}$$

$$\rho = \underline{\rho_c}\,[\chi(\xi)]^n$$

energy density at the center of a star

- Closed equation for the density

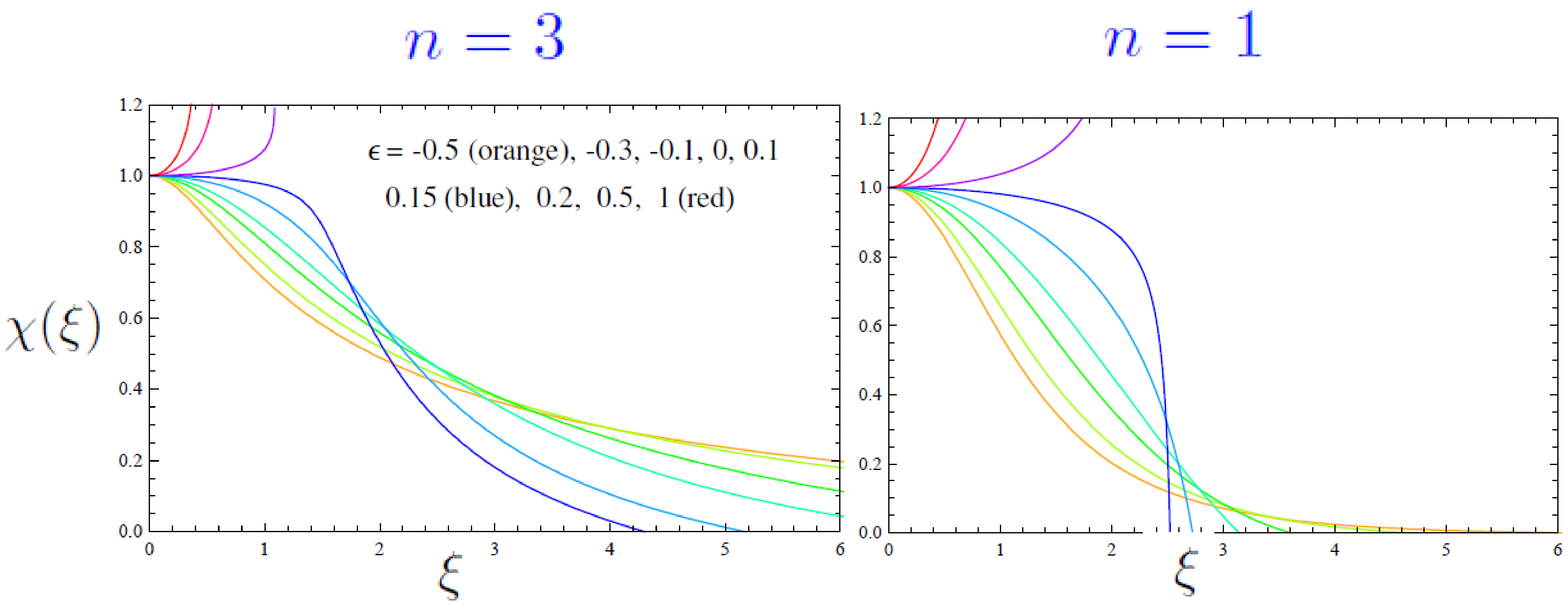
$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d}{d\xi}\left(\chi - \epsilon\xi^2\chi^n\right)\right] = -\chi^n$$

This reduces to the standard Lane-Emden equation for $\epsilon = 0$

- Boundary condition s for $\chi(\xi)$ at the center of a star $\xi = 0$

$$\chi(0) = 1, \; (d\chi/d\xi)\big|_{\xi=0} = 0$$

Numerical solutions for MLE equation



- For $\epsilon > 1/6$, the density blows up and never approaches to zero.
- For $\epsilon < 1/6$, the force is always attractive.

For smaller ϵ , gravity becomes stronger in the inner region but weaker in the outer region

Analytic solution for MLE equation

- MLE equation with n=1

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\chi-\epsilon\xi^2\chi\right)\right]=-\chi$$

It is well known that there is an analytic solution for $\epsilon = 0$

$$\chi(\xi)=\frac{\sin\xi}{\xi}$$

We find that analytic solutions exist for arbitrary ϵ

$$\chi(\xi)={}_2F_1\left[\frac{5}{4}-\frac{1}{4}\sqrt{\frac{4+\epsilon}{\epsilon}},\ \frac{5}{4}+\frac{1}{4}\sqrt{\frac{4+\epsilon}{\epsilon}},\ \frac{3}{2};\ \epsilon\xi^2\right]$$

${}_2F_1$: hypergeometric function

$${}_2F_1(0,5/2,3/2,\xi^2/6)=1$$

holds identically !!

Universal bound on the modification (1)

- Expansion of $\rho(r)$ and $P(r)$ near the center $r=0$

$$\rho = \rho_c + \frac{1}{2}\rho_2 \frac{r^2}{R^2} + \cdots , \qquad P = P_c + \frac{1}{2} P_2 \frac{r^2}{R^2} + \cdots$$

inserting these expansions into MLE eq. , at lowest order in r

$$P_2 = -\frac{4\pi G_N \rho_c^2 R^2}{3}(1-6\epsilon)$$

pressure increases for $\epsilon > 1/6$

- Modified Poisson equation

$$\frac{d\Phi}{dr} = G_N \left(\frac{\mathcal{M}}{r^2} - \epsilon \frac{d^2\mathcal{M}}{dr^2} \right) \qquad \mathcal{M} \simeq \frac{4\pi}{3}\rho_c r^3 + \mathcal{O}(r^5)$$

$$\simeq (1-6\epsilon) G_N \frac{\mathcal{M}}{r^2}$$

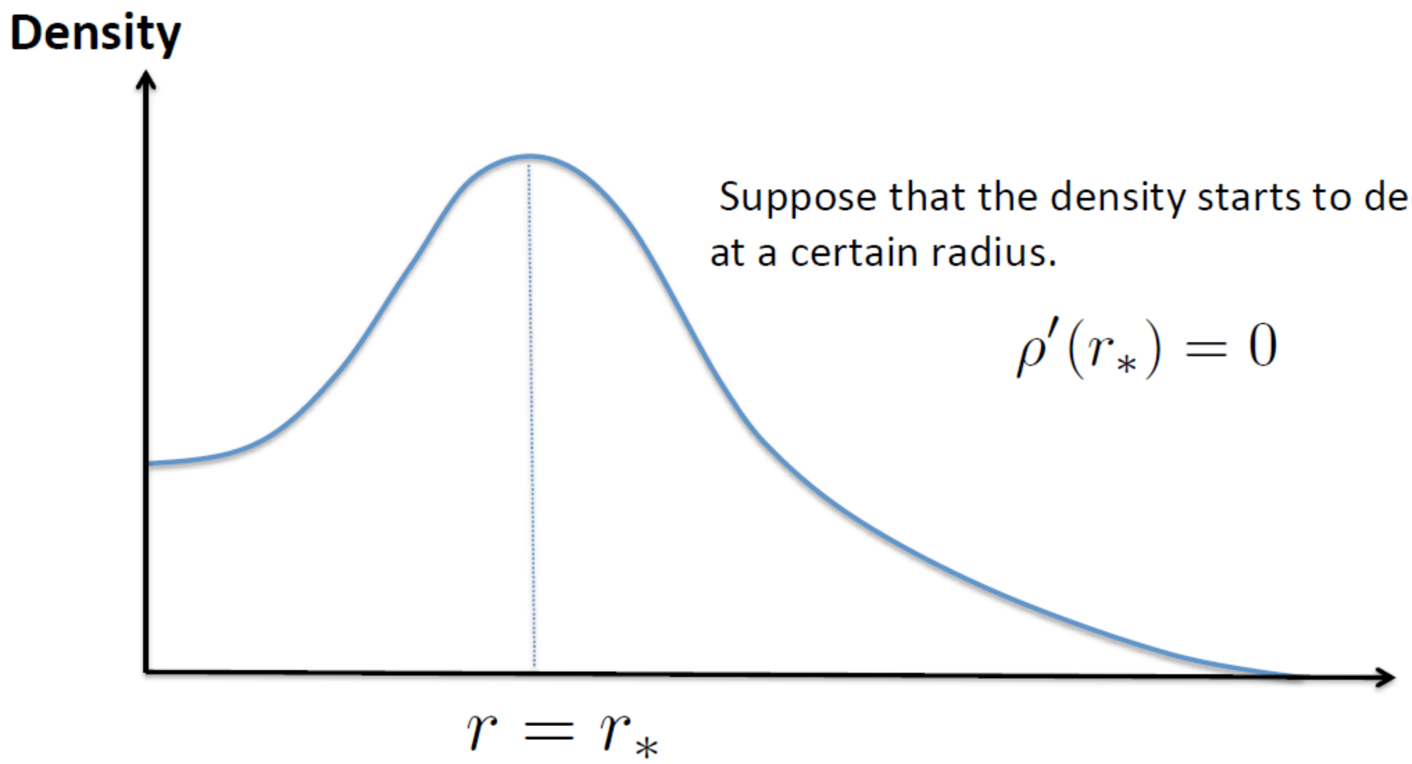
gravity becomes repulsive for $\epsilon > 1/6$

Universal bound on the modification (2)

- For a physically reasonable EoS, one expects P increases with ρ

  ⇨ The density is increasing near the center when $\epsilon > 1/6$

- We will show that this property continues for higher radii.



Universal bound on the modification (3)

$$\mathcal{M}(r_*) = 4\pi \int_0^{r_*} r^2 \rho(r) dr < \frac{4\pi}{3} \rho(r_*) r_*^3$$

$$\frac{\mathrm{d}^2 \mathcal{M}}{\mathrm{d}r^2}(r_*) = 4\pi \left[2\rho(r_*) r_* + \frac{\mathrm{d}\rho}{\mathrm{d}r}(r_*) r_*^2 \right] = 8\pi \rho(r_*) r_*$$

$$\left.\frac{\mathrm{d}P}{\mathrm{d}r}\right|_{r=r_*} = -G_{\mathrm{N}} \rho(r_*) \left(\frac{\mathcal{M}}{r^2} - \epsilon \frac{\mathrm{d}^2 \mathcal{M}}{\mathrm{d}r^2} \right) \Bigg|_{r=r_*}$$

$$> -\frac{4\pi G_{\mathrm{N}}\, \rho^2 r_*}{3} (1 - 6\epsilon) > 0$$

This contradicts the assumption $(\mathrm{d}\rho/\mathrm{d}r)(r_*) = 0$

⇨ The density continues to increase for higher radii

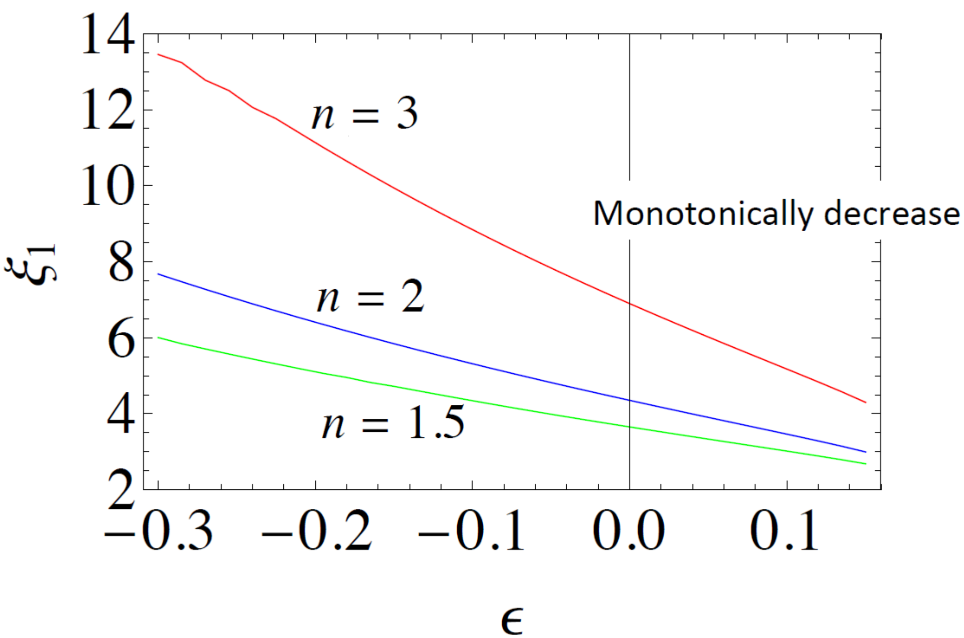
No physically sensible profile for any reasonable EoS for $\epsilon > 1/6$

Modifications in Radius and Mass for $\epsilon < 1/6$

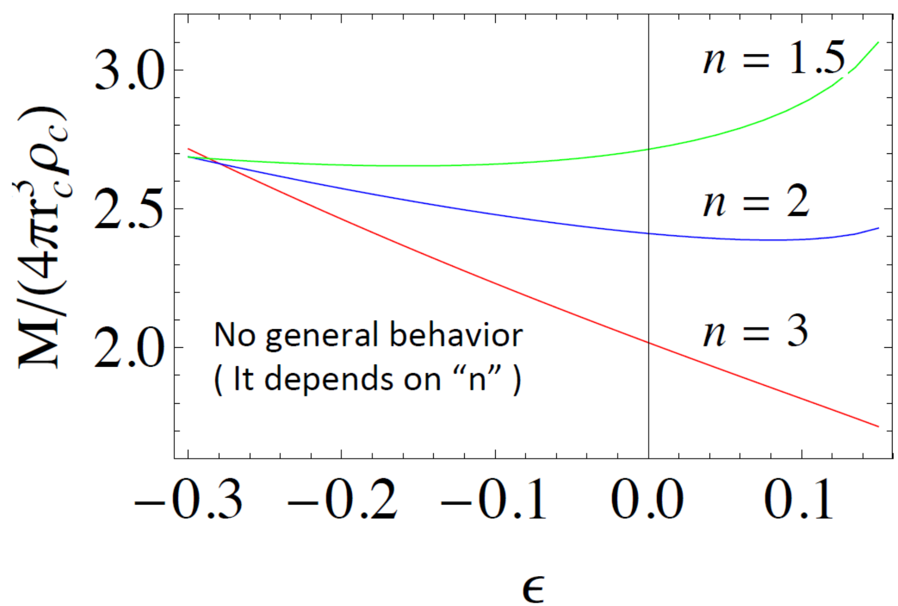
Once the solution is computed, the radius and mass of the object are determined by first zero ξ_1 of the function χ

$$R = r_c \xi_1 \qquad M = 4\pi r_c^3 \rho_c \int_0^{\xi_1} \xi^2 [\chi(\xi)]^n \mathrm{d}\xi$$

Radius



Mass



More sensitive to ϵ for larger polytropic index

4. Summary and discussions

Summary

- Behavior of the scalar d.o.f. in Horndeski theory and beyond Horndeski was studied based on effective theory.

- In beyond Horndeski, a new term partially breaks Vainshtein mechanism is even inside the Vainshtein radius.

- We show that the stellar radial density profile is modified by this effect in the nonrelativistic limit.

- We obtain a universal bound on the amplitude of the modification, independently of the details of the EoS.

Discussions

- Calculations in more realistic situations

Realistic EoS, dynamics, rotation, relativistic corrections

Koyama, Sakstein, `15

- Other interesting effects from beyond Horndeski

Structure formation, Local gravity test

Tsujikawa `15 Kase's talk

- The disformal coupling plays an important role

$$\mathcal{L} \supset \Gamma(X)\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$$

Appendix

Spherical overdensity in beyond Horndeski

- Basic equations

$$2\xi_2 x + \mathcal{G}y - \mathcal{F}z + \alpha_2 x^2 + 2\alpha_* x(rx' + x) - \frac{2}{\mathcal{M}a^3}\partial_t(a^3\xi_t x) = 0,$$

$$\mathcal{G}z - \xi_1 x - \alpha_1 x^2 = A,$$

$$\eta x - 2\xi_1 y + 4\xi_2 z + 2\mu x^2 + 2\nu x^3 - 4\alpha_1 xy + 4\alpha_2 xz - 4\alpha_*(rxz' + 3xz) + \frac{4\xi_t}{\mathcal{M}a^2}\partial_t(a^2 z) = 0,$$

with

$$x(t,r) := \frac{1}{\Lambda^3}\frac{\pi'}{a^2 r}, \qquad y(t,r) := \frac{1}{\Lambda^3}\frac{\Phi'}{a^2 r}, \qquad z(t,r) := \frac{1}{\Lambda^3}\frac{\Psi'}{a^2 r}, \qquad A(t,r) := \frac{1}{\tilde{M}_{\rm Pl}\Lambda^3}\frac{M(t,r)}{8\pi r^3},$$

⇓

$$\left[(\mathcal{F}\xi_1 - 2\mathcal{G}\xi_2)A - \frac{2\mathcal{G}^2\xi_t}{\mathcal{M}a^2}\partial_t\left(\frac{a^2}{\mathcal{G}}A\right)\right] + 2[\kappa_1 + (\mathcal{F}\alpha_1 - \mathcal{G}\alpha_2 + 3\mathcal{G}\alpha_*)A + \mathcal{G}\alpha_* rA']x + \kappa_2 x^2 - \Xi x^3 = 0$$

Expression of the ϵ parameters

- Concrete model (quartic Lagrangian of beyond Horndeski)

$$L_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] - \frac{F_4(\phi, X)}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha'\beta}\phi^{\mu'}\phi_{\mu}\phi_{\nu}{}^{\nu'}\phi_{\alpha}{}^{\alpha'}$$

$$\Rightarrow \quad \epsilon = \frac{\alpha_*^2}{4\pi\widetilde{M}_{\rm Pl}^2 G_{\rm N}\,\Xi}$$

with $\frac{\widetilde{M}_{\rm Pl}}{\Lambda^3}\alpha_* \equiv XF_4$ $\quad (8\pi G_{\rm N})^{-1} \equiv 2G_4 - 8X(G_{4X} + XG_{4XX}) - 4X^2(5F_4 + 2XF_{4X})$

$\Xi \equiv \mathcal{G}\left(4\alpha_1\alpha_2 - 2\alpha_1\alpha_* + \mathcal{G}\nu\right) - 2\mathcal{F}\alpha_1^2$ $\quad \widetilde{M}_{\rm Pl}^2\mathcal{G} \equiv 2\left(G_4 - 2XG_{4X}\right)$ $\quad \widetilde{M}_{\rm Pl}^2\mathcal{F} \equiv 2G_4$

$\frac{\widetilde{M}_{\rm Pl}}{\Lambda^3}\alpha_1 \equiv G_{4X} + 2XG_{4XX} + X(5F_4 + 2XF_{4X})$ $\quad \frac{\widetilde{M}_{\rm Pl}}{\Lambda^3}\alpha_2 \equiv G_{4X} + XF_4$ $\quad \frac{\nu}{\Lambda^6} \equiv G_{4XX} + 2F_4 + XF_{4X}$

When $G_4 = M_{\rm Pl}^2/2$ and $F_4 = {\rm const}$, ϵ is simplified as

$$\epsilon = \frac{X^2 F_4}{M_{\rm Pl}^2}$$

F_4 cannot be large with positive value