Massive gravity and cosmology

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Based on collaboration with
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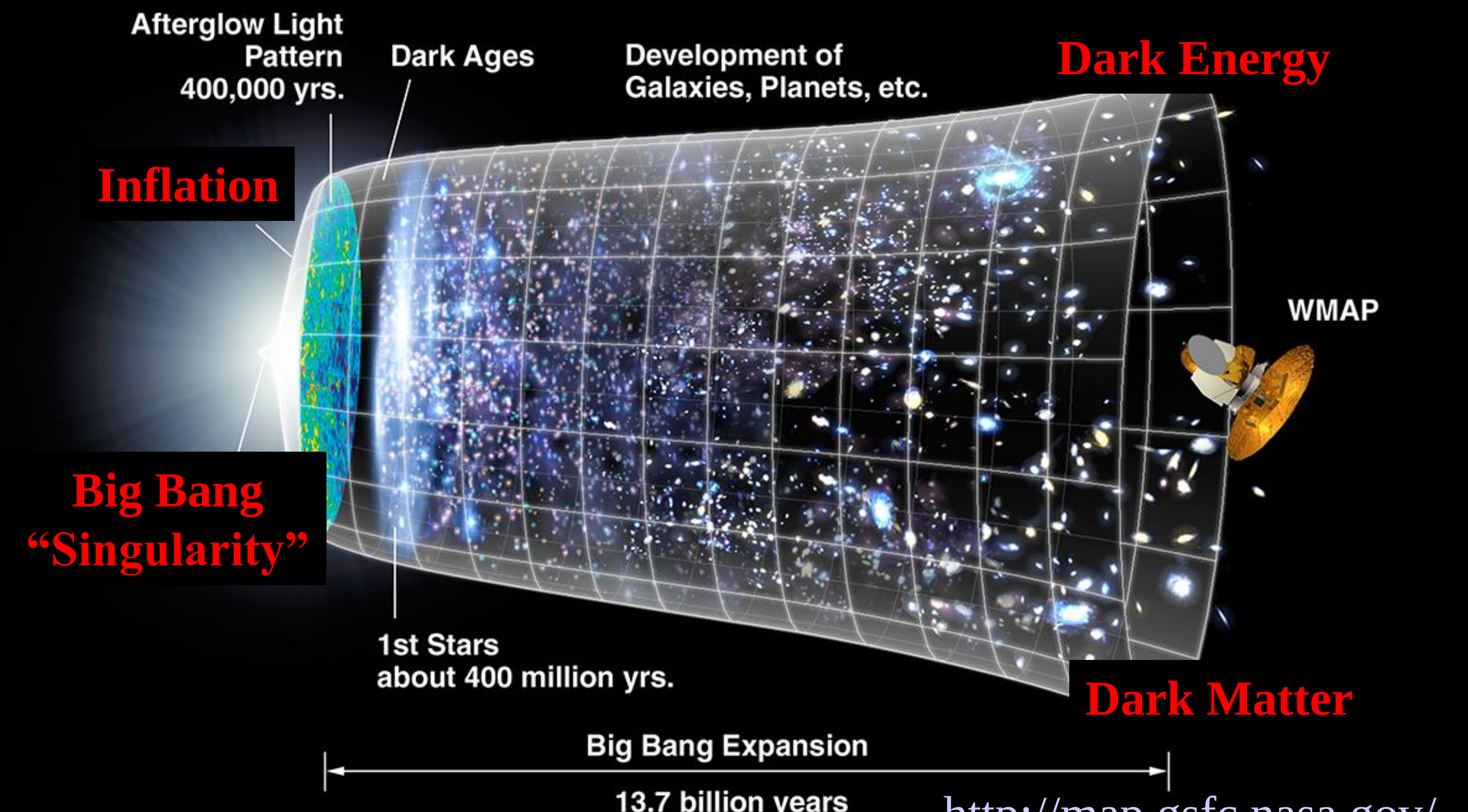
Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter

II. Nonlinear massive gravity
IR modification of gravity
motivation: “Can graviton have mass?”

III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

A motivation for IR modification

- Gravity at long distances

  Flattening galaxy rotation curves

  extra gravity

  Dimming supernovae

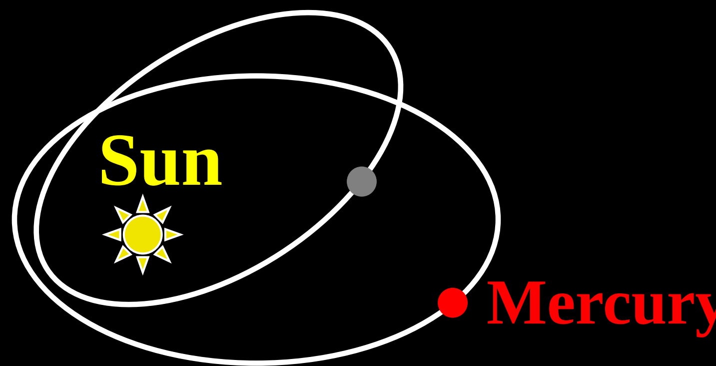
  accelerating universe

- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…

which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

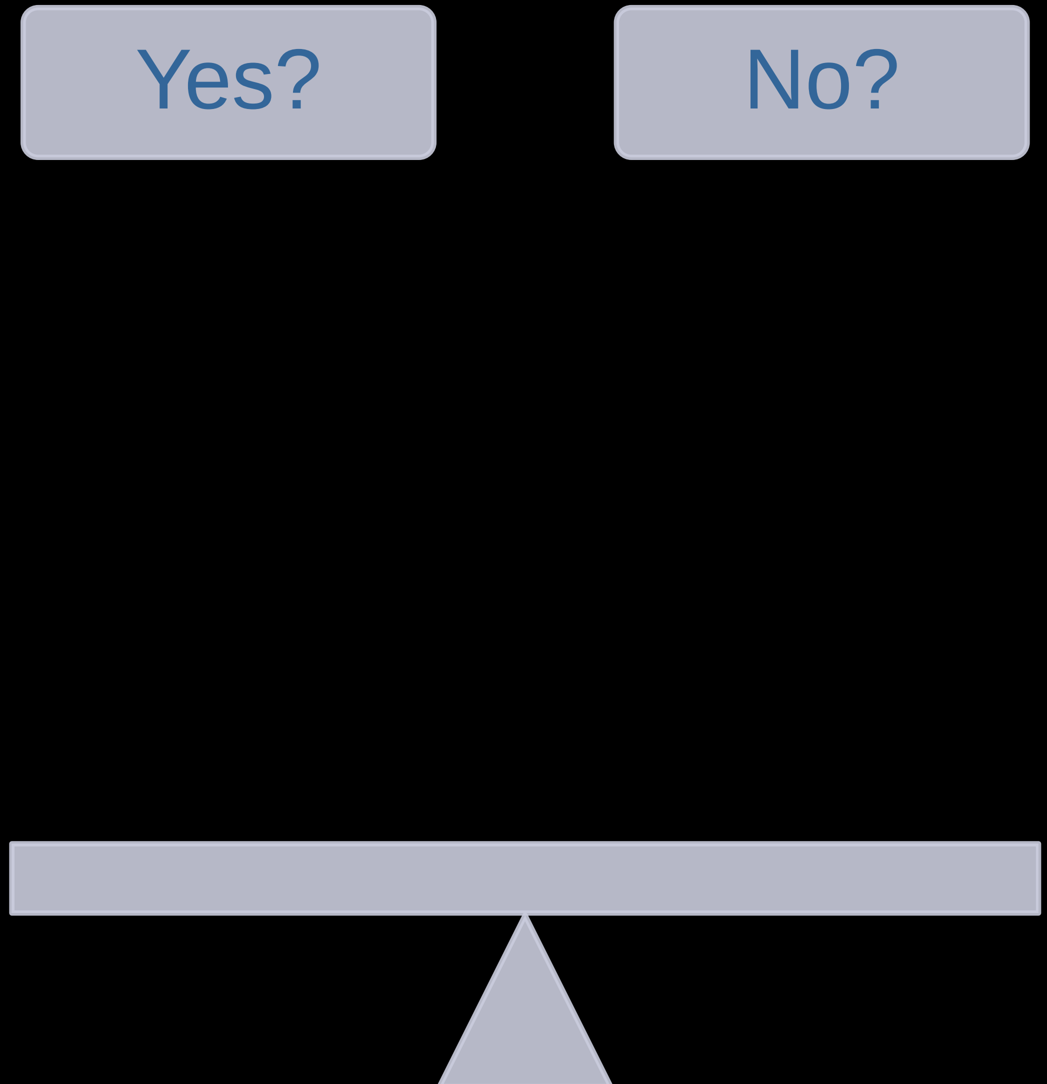
- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



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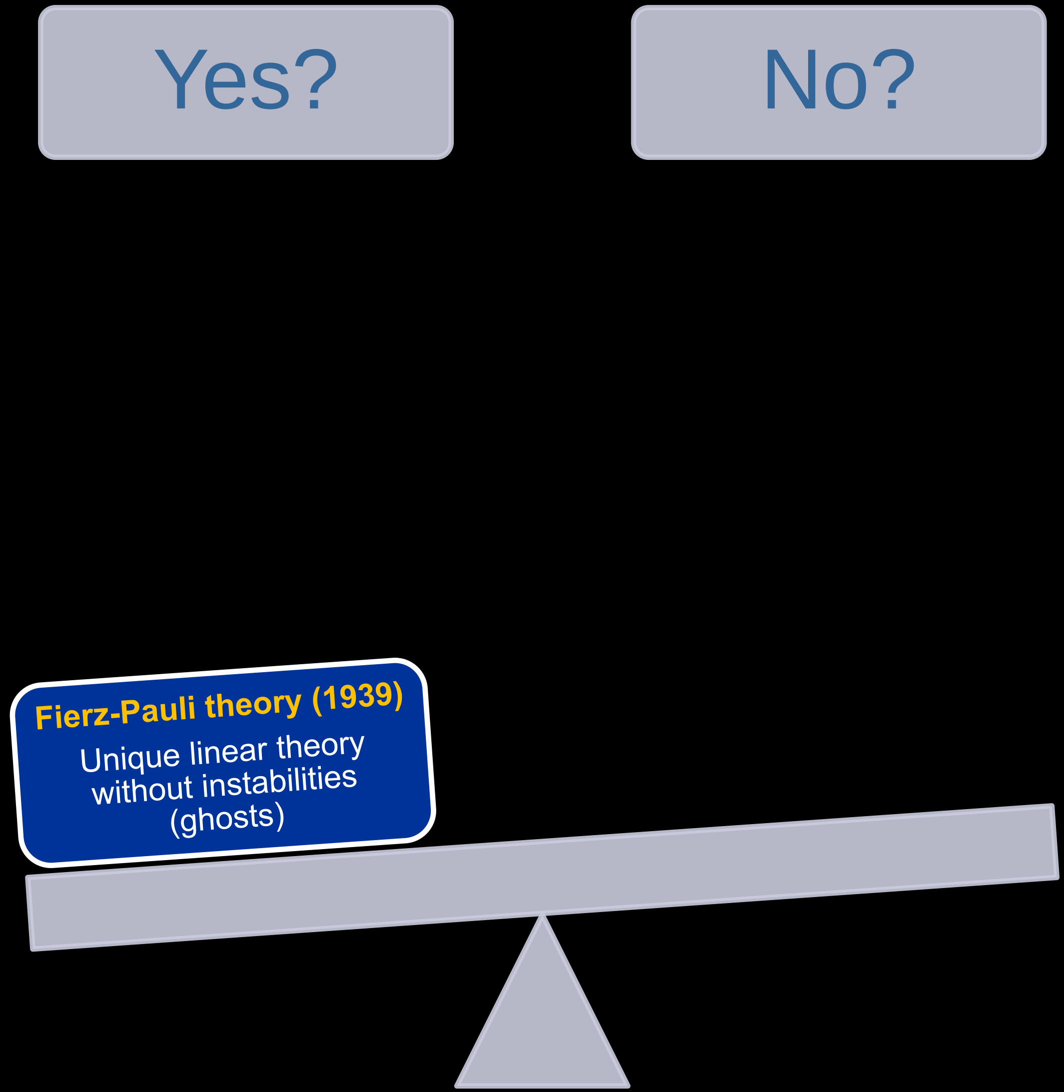

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Fierz-Pauli theory (1939)

Unique linear theory
without instabilities
(ghosts)

van Dam-Veltman-
Zhakharov discontinuity
(1970)

Massless limit ≠
General Relativity

Massive gravity: history

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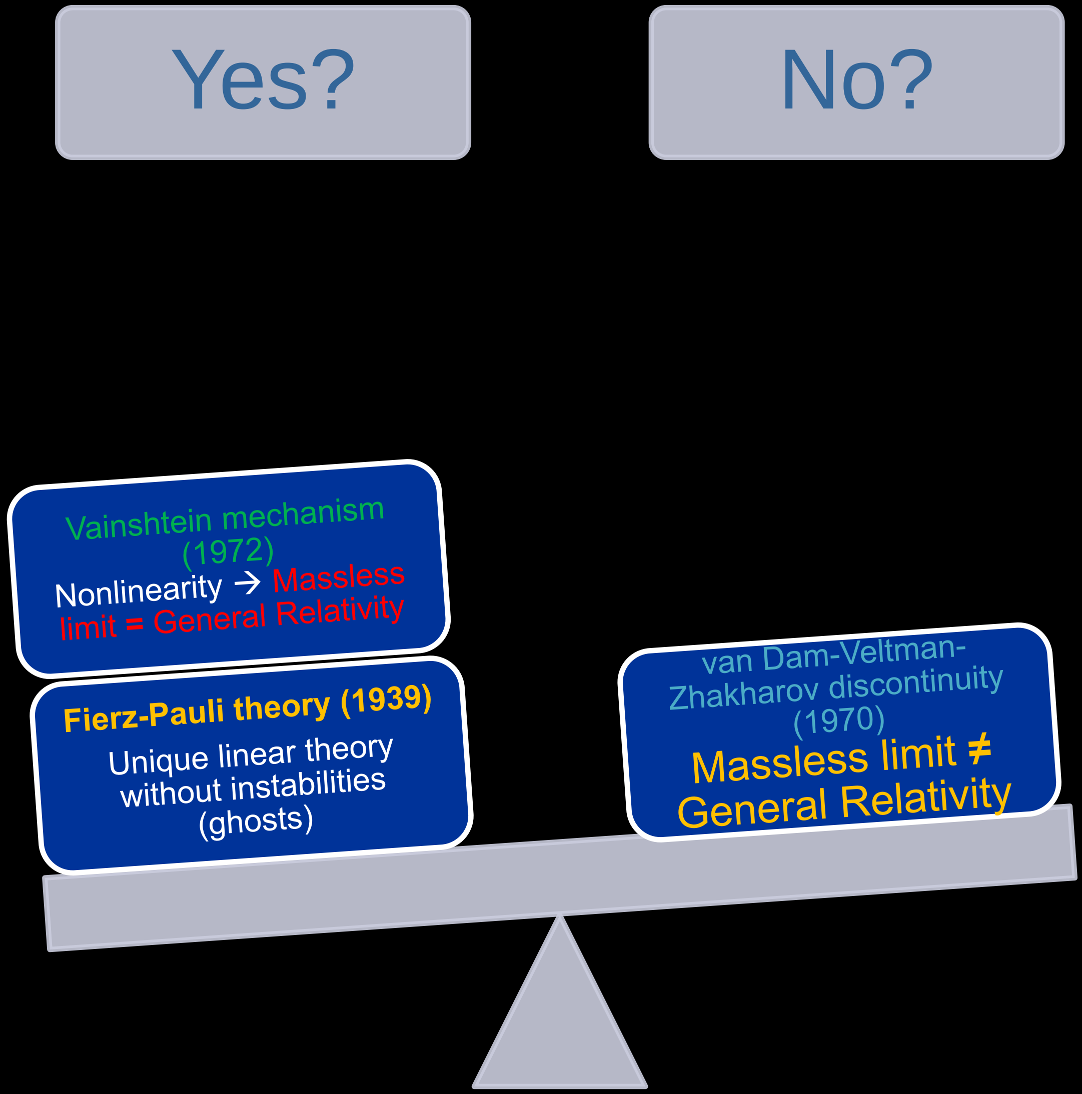

Massive gravity: history

Simple question: Can graviton have mass?

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Vainshtein mechanism (1972)

Nonlinearity → Massless limit = General Relativity

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Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)

6^{th} d.o.f.@Nonlinear level → Instability (ghost)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010
de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

→ $$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

fiducial metric

Pullback of Minkowski metric in field space to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g} \left(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4\right)$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f}\right)^\mu_{\ \nu}$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

$$[\mathcal{A}] \equiv Tr\mathcal{A}$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \quad \Rightarrow \quad \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

Simple question: Can graviton have mass?

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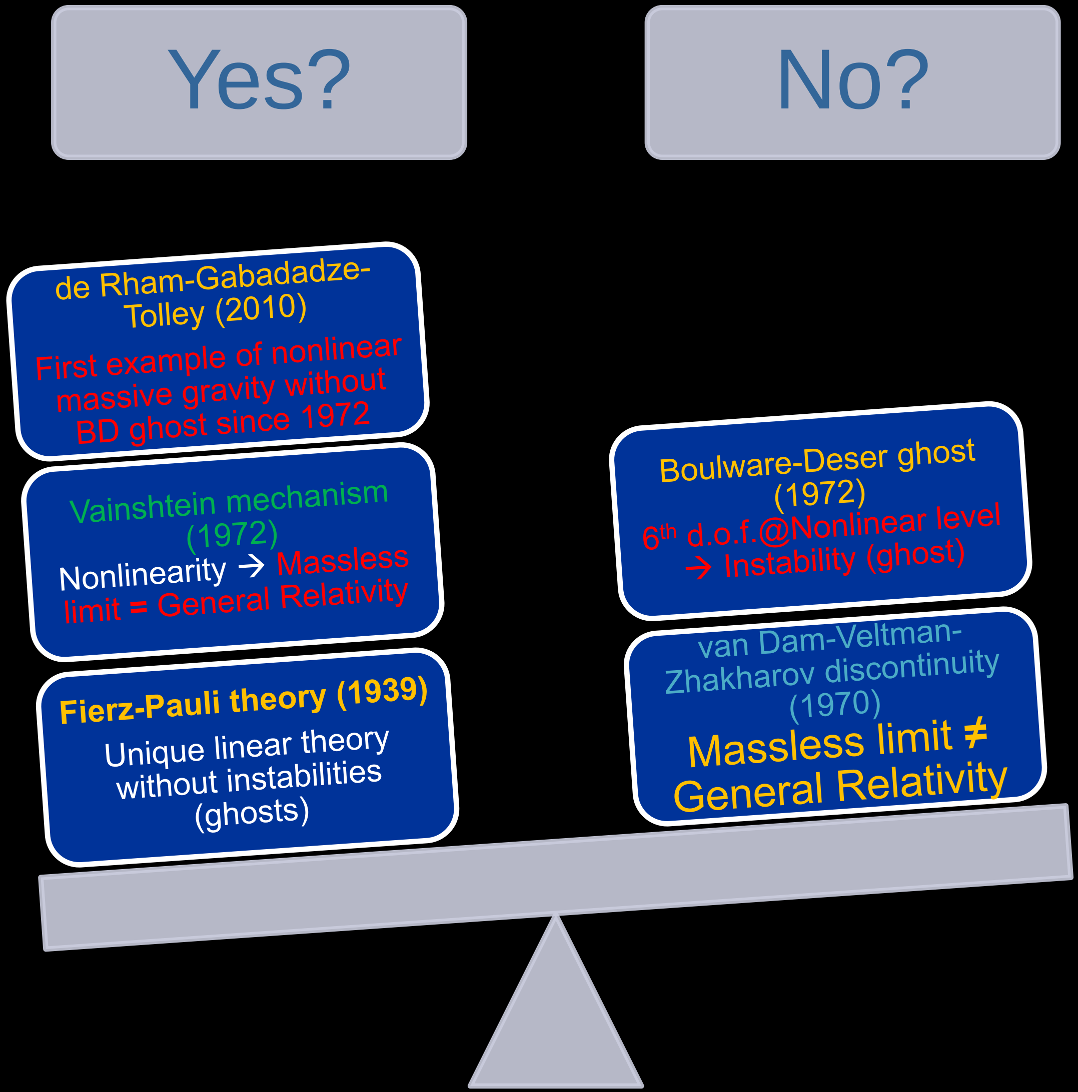

No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in "Unitary gauge"

  $g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2)$

  $\phi^a = x^a \Rightarrow f_{\mu\nu} = \eta_{\mu\nu}$

- Bianchi "identity" → a(t) = const.

  c.f. $$\nabla^\mu \left(\frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \right) = \frac{1}{\sqrt{-g}} \frac{\delta I_g}{\delta \phi^a} \partial_\nu \phi^a$$

  → no non-trivial flat FLRW cosmology

- "Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space"

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes? | No?

Consistent Theory found in 2010 but

No Viable Cosmology?

de Rham-Gabadadze-Tolley (2010)
First example of nonlinear massive gravity without BD ghost since 1972

D'Amico et al. (2011)
Non-existence of flat FRW (homogeneous isotropic) universe!

Vainshtein mechanism (1972)
Nonlinearity → Massless limit = General Relativity

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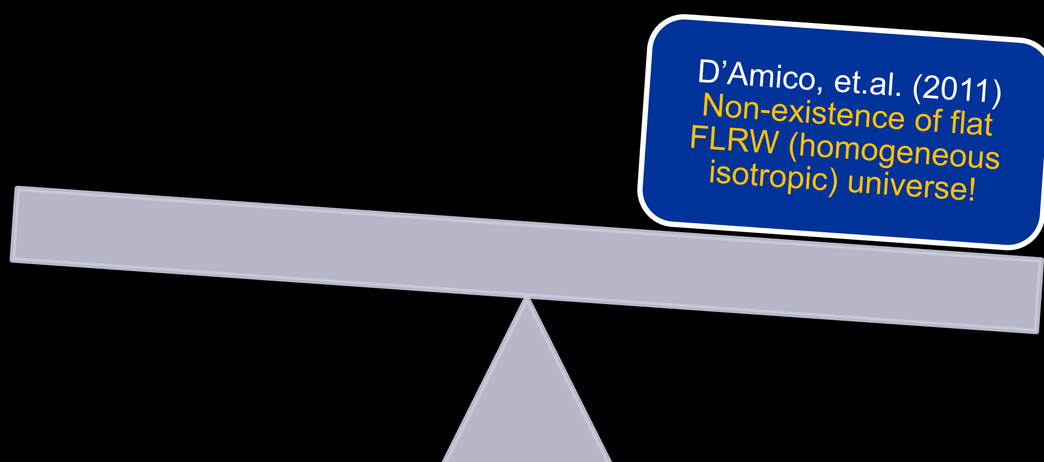
Massless limit ≠ General Relativity

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?



Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$\begin{aligned}\phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)},\\ \phi^1 &= \sqrt{|K|}f(t)x,\\ \phi^2 &= \sqrt{|K|}f(t)y,\\ \phi^3 &= \sqrt{|K|}f(t)z.\end{aligned}$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\,dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$\begin{aligned}g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^i dx^j\,,\\ \Omega_{ij}dx^i dx^j &= dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}\,,\end{aligned}$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a}-\sqrt{|K|}N)\left[\left(3-\frac{2\sqrt{|K|}f}{a}\right)+\alpha_3\left(3-\frac{\sqrt{|K|}f}{a}\right)\left(1-\frac{\sqrt{|K|}f}{a}\right)+\alpha_4\left(1-\frac{\sqrt{|K|}f}{a}\right)^2\right]=0$$

- The first sol $\dot{a}=\sqrt{|K|}N$ implies $g_{\mu\upsilon}$ is Minkowski

  → we consider other solutions

$$f=\frac{a}{\sqrt{|K|}}X_\pm,\quad X_\pm\equiv\frac{1+2\alpha_3+\alpha_4\pm\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$

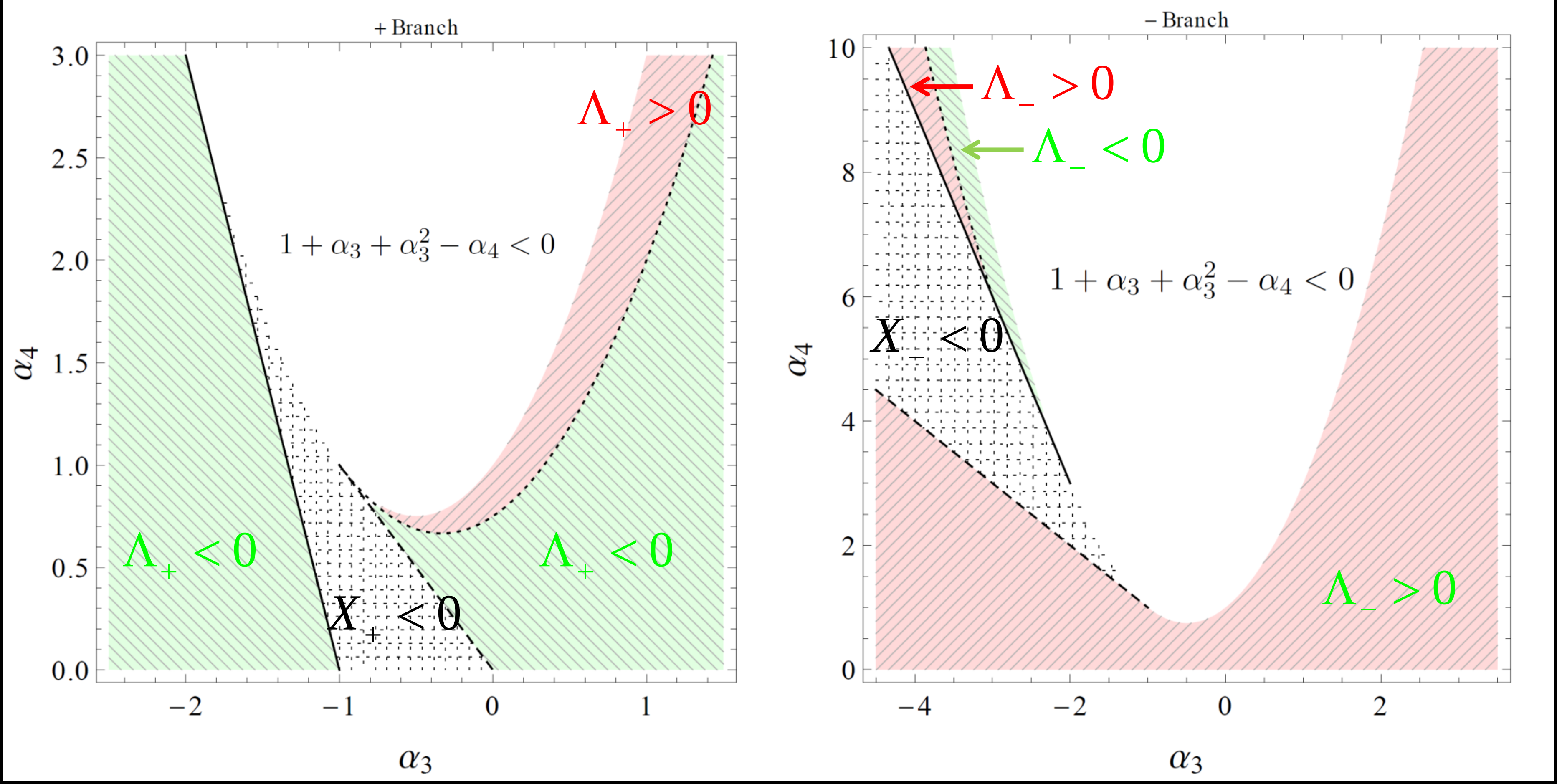
- Latter solutions do not exist if K=0

- Metric EOM → self-acceleration

$$3H^2+\frac{3K}{a^2}=\Lambda_\pm+\frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm\equiv-\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right)\pm2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?

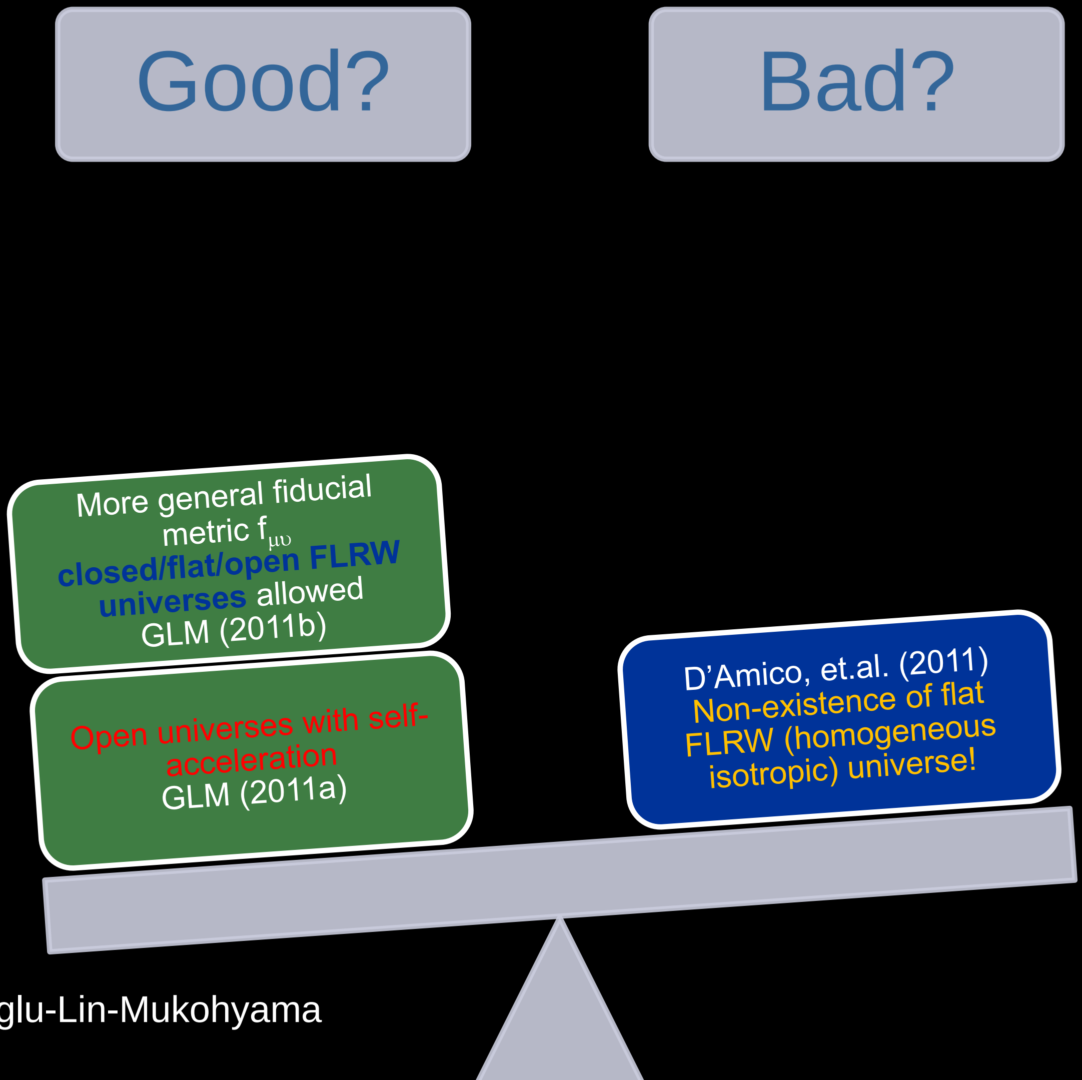
Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Our recent contributions

Cosmological solutions of nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama

Summary so far + α

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

Bad?

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

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Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.
→ Anisotropic FLRW universe!
statistical anisotropy expected
(suppressed by small m_g^2)

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

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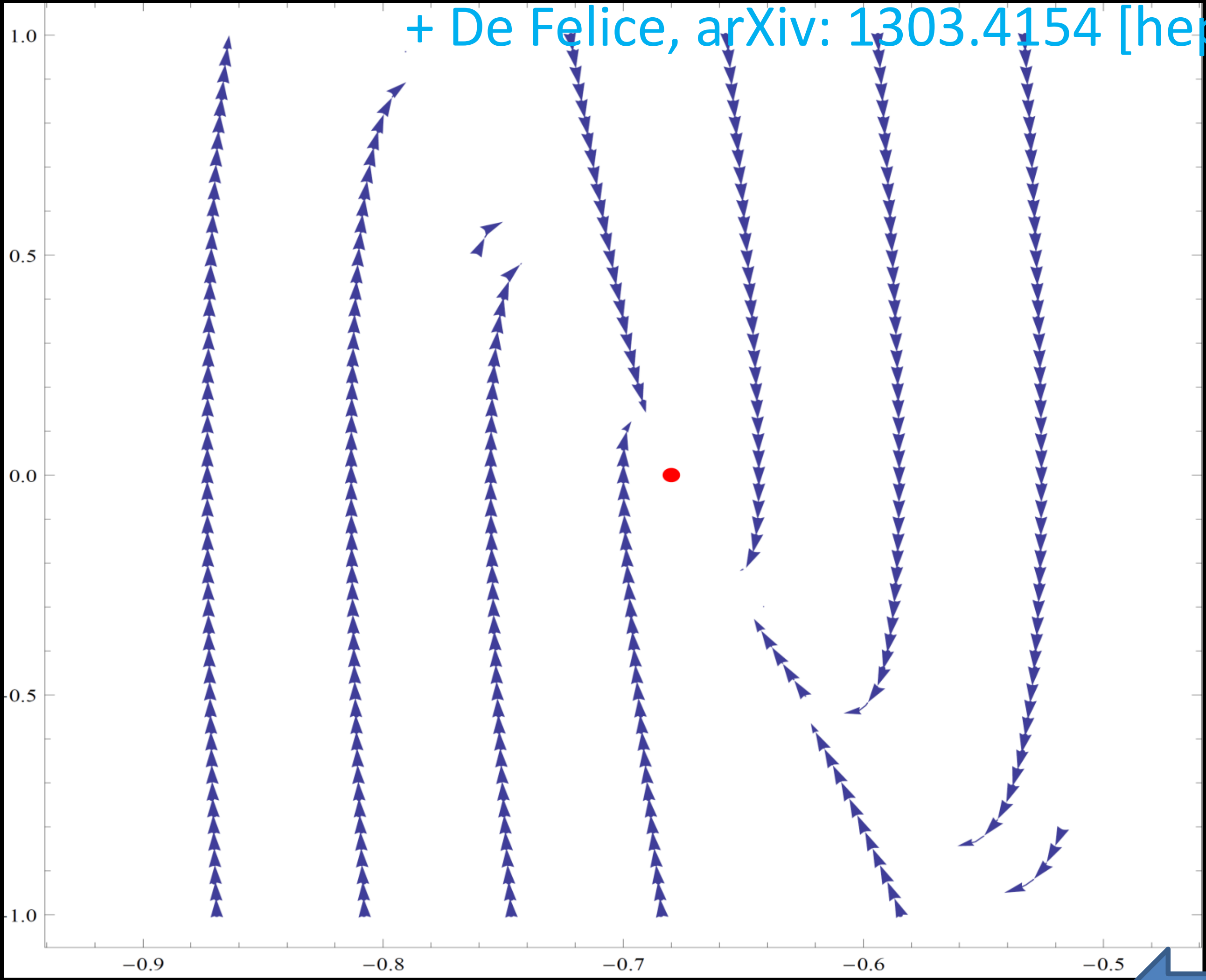


Anisotropy in Expansion

Anisotropy in fiducial metric

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

- **NEW Class of Solutions**
  Anisotropic FLRW universe
  GLM (2012)
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  GLM (2011b)
- Open universes with self-acceleration
  GLM (2011a)

Bad?

- **NEW**
  Nonlinear instability of FLRW solutions
  DGM (2012)
- D'Amico, et.al. (2011)
  Non-existence of flat FLRW (homogeneous isotropic) universe!

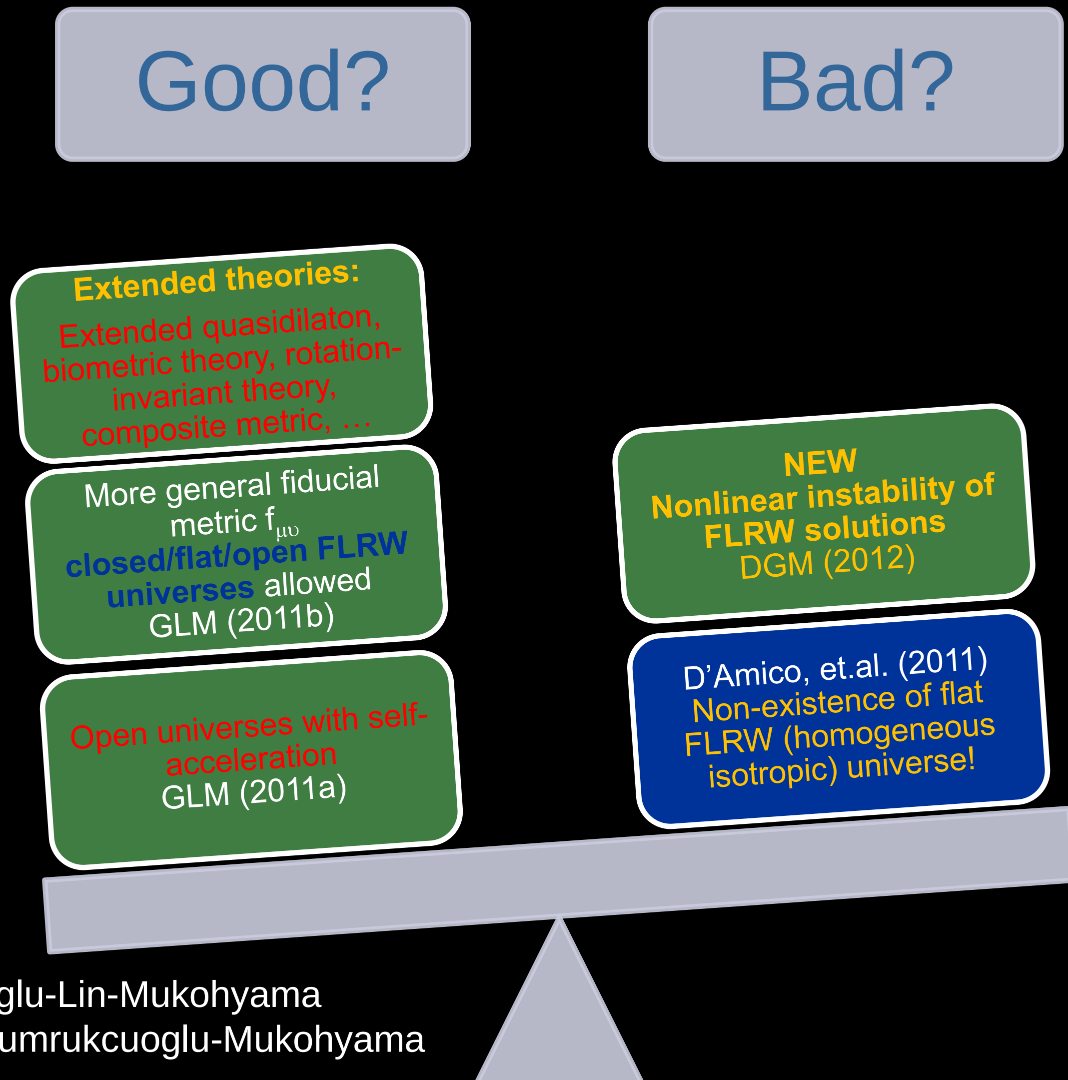
GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (Gumrukcuoglu, Heisenberg, Mukohyama 2014), New quasidilaton (Mukohyama 2014), ...
- They provide stable cosmology.

Our recent contributions

Cosmological solutions of nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

More recent development

Minimal Theory of Massive Gravity

De Felice & Mukohyama, arXiv: 1506.01594

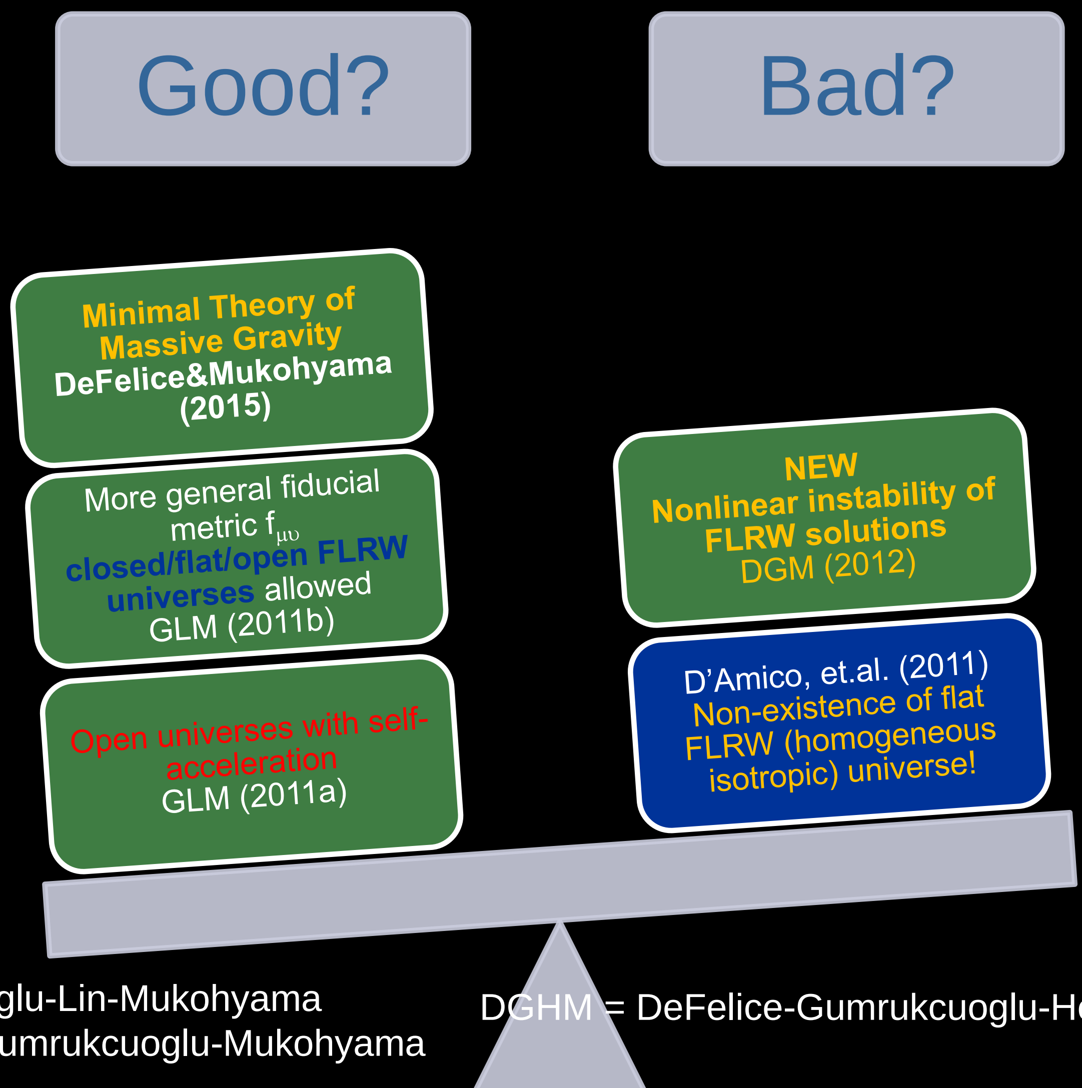
- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

Our recent contributions

Cosmological solutions of nonlinear massive gravity



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DGM = DeFelice-Gumrukcuoglu-Mukohyama

DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

Summary

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- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
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  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have ghost
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof results in stable self-accelerating cosmology