Nonlocal Gravity and Structure in the Universe

Constructing a modified gravity model which accounts for expansion and growth history simultaneously is very hard and I will tell you why it is

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Outline

- Introduction
- Four issues typically discussed in MG
- A nonlocal gravity model passed 3 out of 4
- Why it’s hard to have acceleration and suppressed growth (weak gravity) simultaneously
- Summary and Discussions

Introduction

- **Current cosmic acceleration is a surprise, not explained by GR**
- **i.e., Einstein eq.** $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ **does not work!**
- **The simplest solution, adding Λ is unsatisfactory.**
- **The two options we have:**

New substance : Dark Energy	**New formulation: Modified Gravity**
$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \Delta T_{\mu\nu})$	$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Four Issues in MG

- **Want a MG model which reproduces the background accelerated expansion;**
- **Whether it recovers GR in small scales like the Solar System;**
- **Whether it is stable, i.e., ghost-free;**
- **Whether it is consistent with the growth of perturbations: observation tells us it's suppressed (weak gravity).**

A nonlocally modified gravity model

- **Deser and Woodard, PRL 99 (2007) 111301, 0706.2151**

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}R\left[1 + f(\frac{1}{\Box}R)\right] \longrightarrow G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}$$

- **Features and theoretical motivation:**

➢ $\Box^{-1}R$ is dimensionless: no new mass parameter is required.

➢ $R \simeq 0$ during rad-dom & $\Box^{-1}R$ grows slowly (logarithmically) during mat-dom: the modification does not affect the expansion history until recently, exactly the type of modification we need for the current epoch of acceleration!

➢ Nonlocal terms might arise from a quantum theory, see for example, Polyakov, PLB 103, 207 (1981)

- **It passes 3 out of the 4 conditions**

➢ **f can be fitted to produce the background accelerating expansion w/o Λ or DE.**

➢ $\Box^{-1}R$ **is small in the Solar System: the model passes the local test of gravity**

➢ **Stable unlike its localized version: no ghost**

(For more detailed discussion on the issues of screening and stability, see Deser and Woodard, JCAP 11 (2013) 036, 1307.6639)

Fit ΛCDM expansion history w/o Λ

$$G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}$$

Specialize the modified field eqn to the FLRW (homogeneous, isotropic, spatially flat) geometry and determine f so as to match with the ΛCDM expansion history, which is given as

$$H(t) = H_0\sqrt{\Omega_\Lambda + \Omega_m / a^3 + \Omega_r / a^4} = \frac{d\ln a}{dt}$$

Note: once H_0 & Ω values are given, $H(t)$ is fixed.

- **Friedmann Eq.**

$$H^2(t) = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}(\rho_\Lambda + \rho_m + \rho_r)$$

To get exactly the same H(t) w/o ρ_{Λ}, make Newton's constant G grow with time:

$$H^2(t) = \frac{8\pi G}{3}\rho = \frac{8\pi G_{\text{eff}}(t)}{3}(\rho_m + \rho_r)$$

At 0th order gravity gets stronger! Let's hope perturbations behave opposite way so as to suppress growth of structure…

Problem:
To mimic ΛCDM expansion history

$$G_{\text{eff}}(t) \quad > \quad G$$

Data says growth is a bit lower than what's expected in the ΛCDM model

$f(\Box^{-1}R)$, $f(R)$, $\cdots$ any modification to GR

Perturbation Eqs. & growth of structure

To see the growth of structure, perturb the metric around the FLRW background;

$$ds^2 = -(1+2\Psi(t,\vec{x}))dt^2 + a^2(t)(1+2\Phi(t,\vec{x}))dx^2$$

- **4 evolution Eqs. for 4 perturbations,** Ψ , Φ , δ , θ

General Relativity	Nonlocal Gravity
$(\Phi+\Psi)=0$	$(\Phi+\Psi) = -(\Phi+\Psi)\left\{f(\bar{X}) + \frac{1}{\Box}\left[\bar{R}f'(\bar{X})\right]\right\} - \left\{f'(\bar{X})\frac{1}{\Box}\delta R + \frac{1}{\Box}\left[f'(\bar{X})\delta R\right]\right\}$
$\frac{k^2}{a^2}\Phi = 4\pi G\bar{\rho}\delta$	$\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}\left[\Phi\left\{f(\bar{X}) + \frac{1}{\Box}\left[\bar{R}f'(\bar{X})\right]\right\} + \frac{1}{2}\left\{f'(\bar{X})\frac{1}{\Box}\delta R + \frac{1}{\Box}\left[f'(\bar{X})\delta R\right]\right\}\right] = 4\pi G\bar{\rho}\delta$
$\dot{\delta} + H\theta = 0$,	→ same
$H\dot{\theta} + \left(\dot{H} + 2H^2\right)\theta - \frac{k^2}{a^2}\Psi = 0$	→ same

Stress-energy conservation

$$\nabla^\mu \Delta G_{\mu\nu} = 0$$

still holds in this nonlocal model

Blue < 0: time only,
fixed by the background

Red > 0: time and space,
purely from perturbation

$$\bar{X} \equiv \Box^{-1} R$$

Parameterization of the deviations from GR

We solve the system of the 4 integro-differential eqs. for $\Psi\ ,\ \Phi\ ,\ \delta\ ,\ \theta$ (numerically)

$$(\Phi+\Psi)=-(\Phi+\Psi)\left\{f(\bar{X})+\frac{1}{\bar{\Box}}\left[\bar{R}f'\left(\bar{X}\right)\right]\right\}-\left\{f'(\bar{X})\frac{1}{\bar{\Box}}\delta R+\frac{1}{\bar{\Box}}\left[f'\left(\bar{X}\right)\delta R\right]\right\}$$

$$\frac{k^2}{a^2}\Phi+\frac{k^2}{a^2}\left[\Phi\left\{f(\bar{X})+\frac{1}{\bar{\Box}}\left[\bar{R}f'\left(\bar{X}\right)\right]\right\}+\frac{1}{2}\left\{f'(\bar{X})\frac{1}{\bar{\Box}}\delta R+\frac{1}{\bar{\Box}}\left[f'\left(\bar{X}\right)\delta R\right]\right\}\right]\equiv\frac{k^2}{a^2}\Phi+\frac{k^2}{a^2}E[\Phi,\Psi]=4\pi G\bar{\rho}\delta$$

$$\dot{\delta}+H\theta=0\,,$$

$$H\dot{\theta}+\left(\dot{H}+2H^2\right)\theta-\frac{k^2}{a^2}\Psi=0$$

and parameterize the deviations from GR as follows:

$$\eta\equiv\frac{\Phi+\Psi}{\Phi}$$

$$\frac{G_{\rm eff}}{G}\equiv\frac{k^2\Phi}{4\pi G\bar{\rho}a^2\delta}=\frac{1}{1+\dfrac{E[\Phi,\Psi]}{\Phi}}$$

or

$$\Psi\equiv(1+\mu)\Psi_{\rm GR}$$

$$\Psi-\Phi\equiv(1+\Sigma)[\Psi_{\rm GR}-\Phi_{\rm GR}]$$

related by $1+\mu=(1-\eta)\dfrac{G_{\rm eff}}{G}$

It turns out

$\eta\ <\ 0\ \because\ \Phi>0\,,\ \Psi<0\,,\ |\Psi|>|\Phi|$ ⇒ $1-\eta\ >\ 1$: bad news

$\dfrac{G_{\rm eff}}{G}\ <\ 1\ \because\ E[\Phi,\Psi]>0$: good news, recall this was greater than 1 at 0th order.

Growth Equation

Combining the 4 evolution eqns. we have an eqn for δ:

General Relativity

$$\frac{d^2\delta}{da^2}+\left[\frac{d\ln(H)}{da}+\frac{3}{a}\right]\frac{d\delta}{da}-\frac{3}{2}\frac{\Omega_m}{h^2(a)a^5}\delta=0$$

Nonlocal Gravity

$$\frac{d^2\delta}{da^2}+\left[\frac{d\ln(H)}{da}+\frac{3}{a}\right]\frac{d\delta}{da}-\frac{3}{2}(1+\mu)\frac{\Omega_m}{h^2(a)a^5}\delta=0$$

$$1+\mu=(1-\eta)\frac{G_{\rm eff}}{G} > 1$$

Growth gets enhanced

$$1+\mu=(1-\eta)\frac{G_{\rm eff}}{G} < 1$$

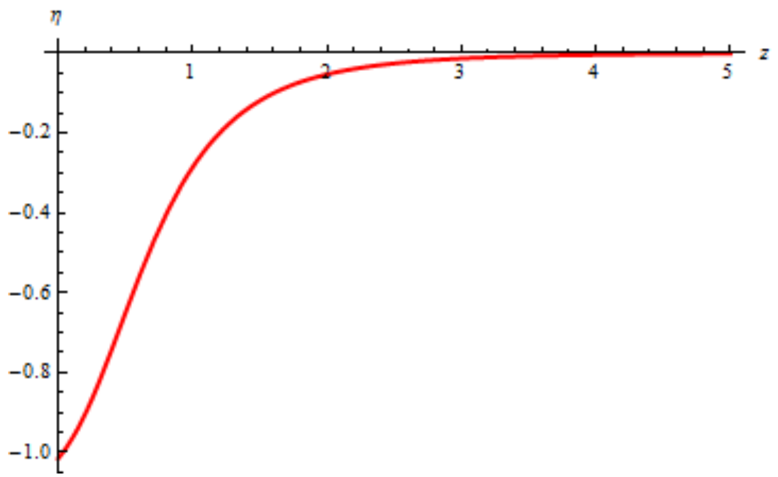
Growth gets suppressed

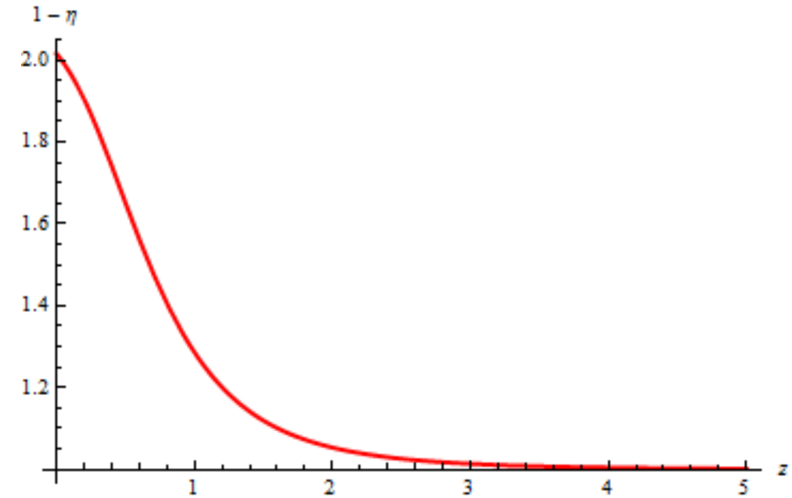
Who wins?

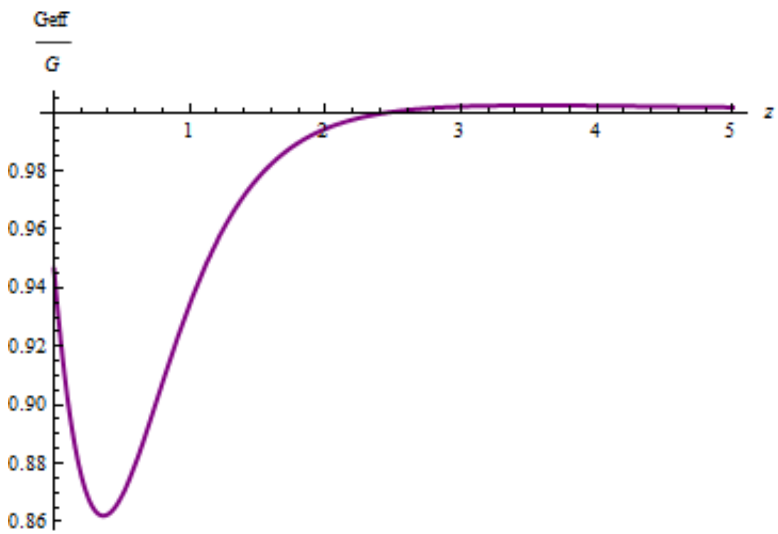
$\frac{G_{\rm eff}}{G} < 1$ or $1-\eta > 1$ (equivalently $|\Psi|>|\Phi|$)

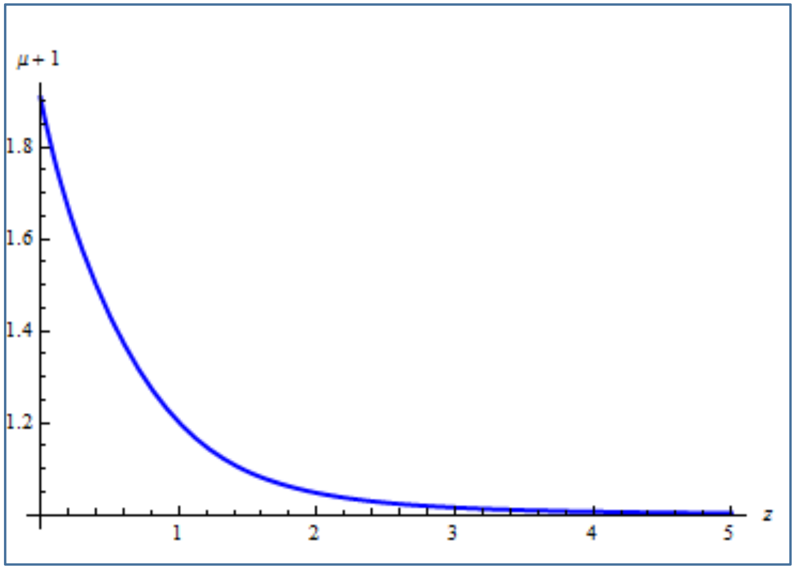
Partially successful: $\frac{G_{\rm eff}}{G} < 1$ Would it be possible to make $|\Psi|<|\Phi|$?

Figures for $1+\mu=(1-\eta)\frac{G_{\mathrm{eff}}}{G} \;\; > \;\; 1$







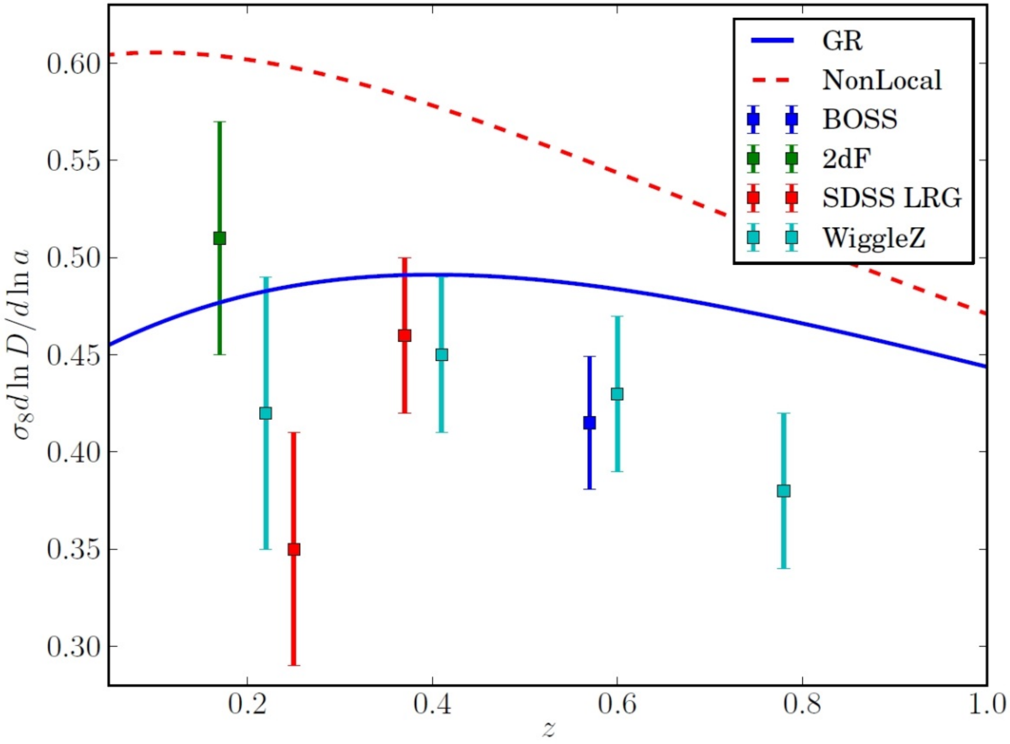


Redshift Space Distortions

Redshift space distortions probe the product of

the growth rate $\beta \equiv d\ln D / d\ln a$ and $\sigma_8(z)$ a measure of the clustering amplitude.

Here the *growth function* $D(a)$ is the solution to the growth eq. with initial condition $D(a) = a$.



Measurement of $\beta\sigma_8$

This is directly measured in spectroscopic surveys capable of probing redshift space distortions.

Data points come from
BOSS,
2dF,
SDSS LRG,
WiggleZ.

7.8-σ preference for
GR over the nonlocal model

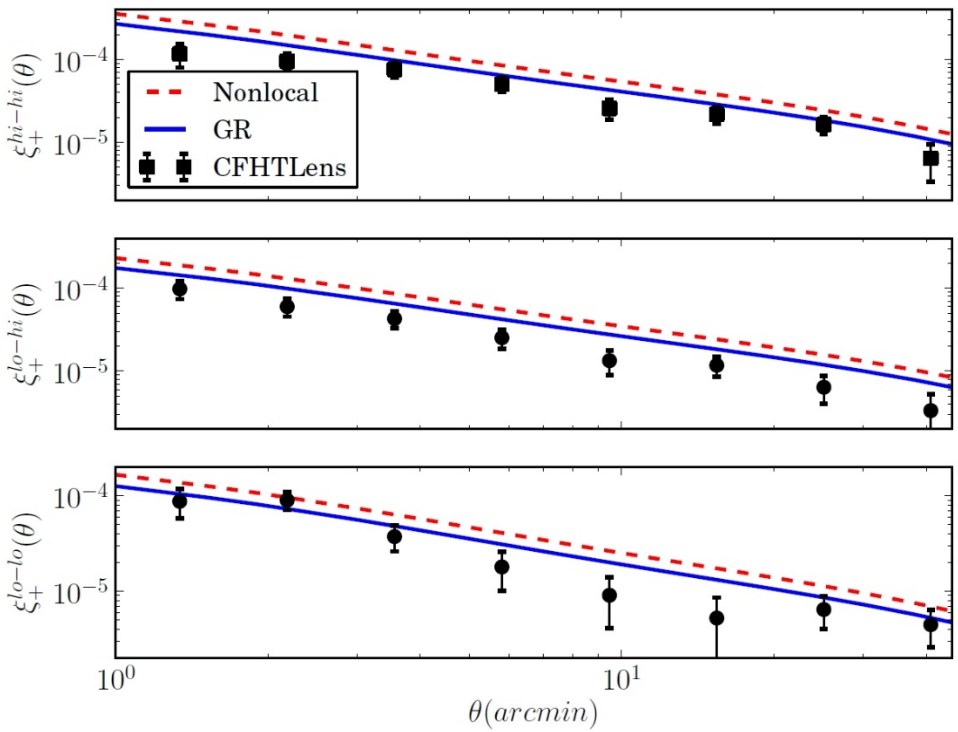
Weak Lensing

Fixing the redshift-distance relation and the initial amplitude of fluctuations, the power spectrum of the convergence of galaxies in two redshift bins is

$$C_l^{ij} = \left(\frac{3\Omega_m H_0^2}{2}\right)^2 \int_0^\infty d\chi \frac{g_i(\chi) g_j(\chi)}{a^2(\chi)} P(l/\chi;\chi)\left[1+\Sigma(\chi)\right]^2$$

where $g_i(\chi) \equiv \int_\chi^\infty d\chi' \frac{dn_i}{d\chi'}\left(1-\frac{\chi}{\chi'}\right)$ the weighting function in each redshift bin

$dn_i / d\chi$ the redshift distribution of source galaxies in bin *i*



ξ in three different redshift bins as measured in **CFHTLenS** (black points with error bars).

Top and bottom panels: correlation function in the high and low redshift bins resp.
Middle panel: the cross spectrum.

Both GR and nonlocal models have the redshift-distance relation corresponding to Planck parameters.

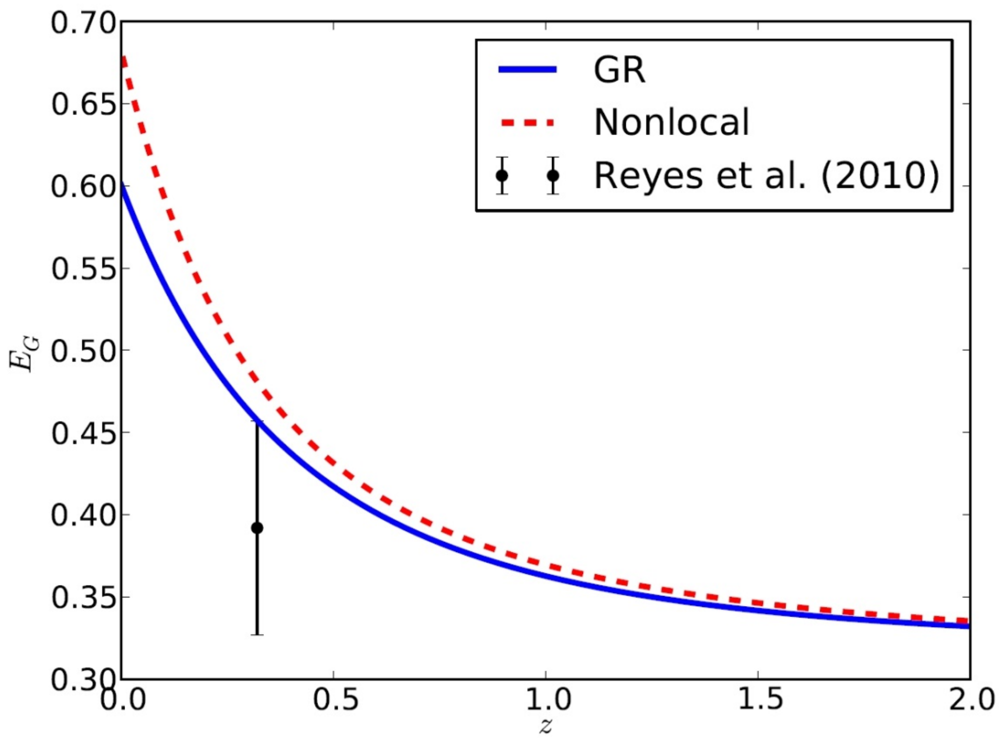
5.9-σ preference for GR over the nonlocal model

Estimator of Gravity E_G

Gravitational lensing is sensitive to the combination, $\Phi - \Psi$ while spectroscopic surveys are sensitive to the velocity field, which is related to $\dot{\delta}$

Combining the two, we have an estimator of gravity E_G
(Zhang, Liguori, Bean, and Dodelson, PRL 99, 141302 (2007), 0704.1932)

$$E_G \equiv \frac{k^2 a(\Phi - \Psi)}{3H_0^2 \beta\delta} = \frac{\Omega_m\left[1+\Sigma\right]}{\beta}$$



Estimator of gravity E_G
as a function of redshift
in GR and the nonlocal model.

The data point is from
Reyes et al. (2010)

The growth rate β is larger in nonlocal gravity, but Σ is positive; an interesting interplay between the two effects. On balance, the Σ enhancement wins, leading to larger values of E_G in the nonlocal model.

Summary and Discussion

- Modified gravity, aimed at reproducing the expansion history, tends to make gravity stronger at 0th order.
- Growth of structure is observed to be a bit lower than expected in the simplest ΛCDM model.
- How can we make perturbations weaken gravity enough to overcome the strengthened gravity in the background level?