

Lambda or Not Lambda

Arman Shafieloo

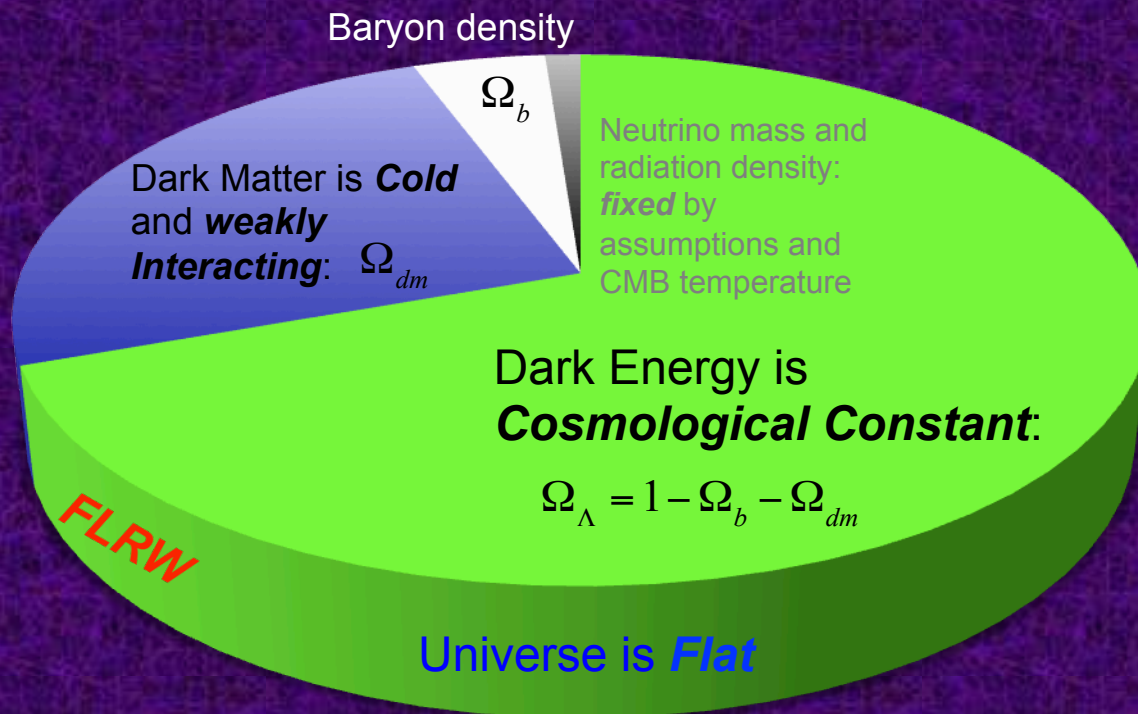
Korea Astronomy and Space Science Institute

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Tokyo University of Science, August 2-5 2015

Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological model.



Initial Conditions:
Form of the Primordial
Spectrum is **Power-law**

$$n_s, A_s$$

Epoch of reionization

$$\tau$$

Hubble Parameter and
the Rate of Expansion

$$H_0$$

Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological model.

Baryon density

Combination of Assumptions

Dark Energy is
Cosmological Constant:

$$\Omega_{\Lambda} = 1 - \Omega_b - \Omega_{dm}$$

Universe is *Flat*

Epoch of reionization

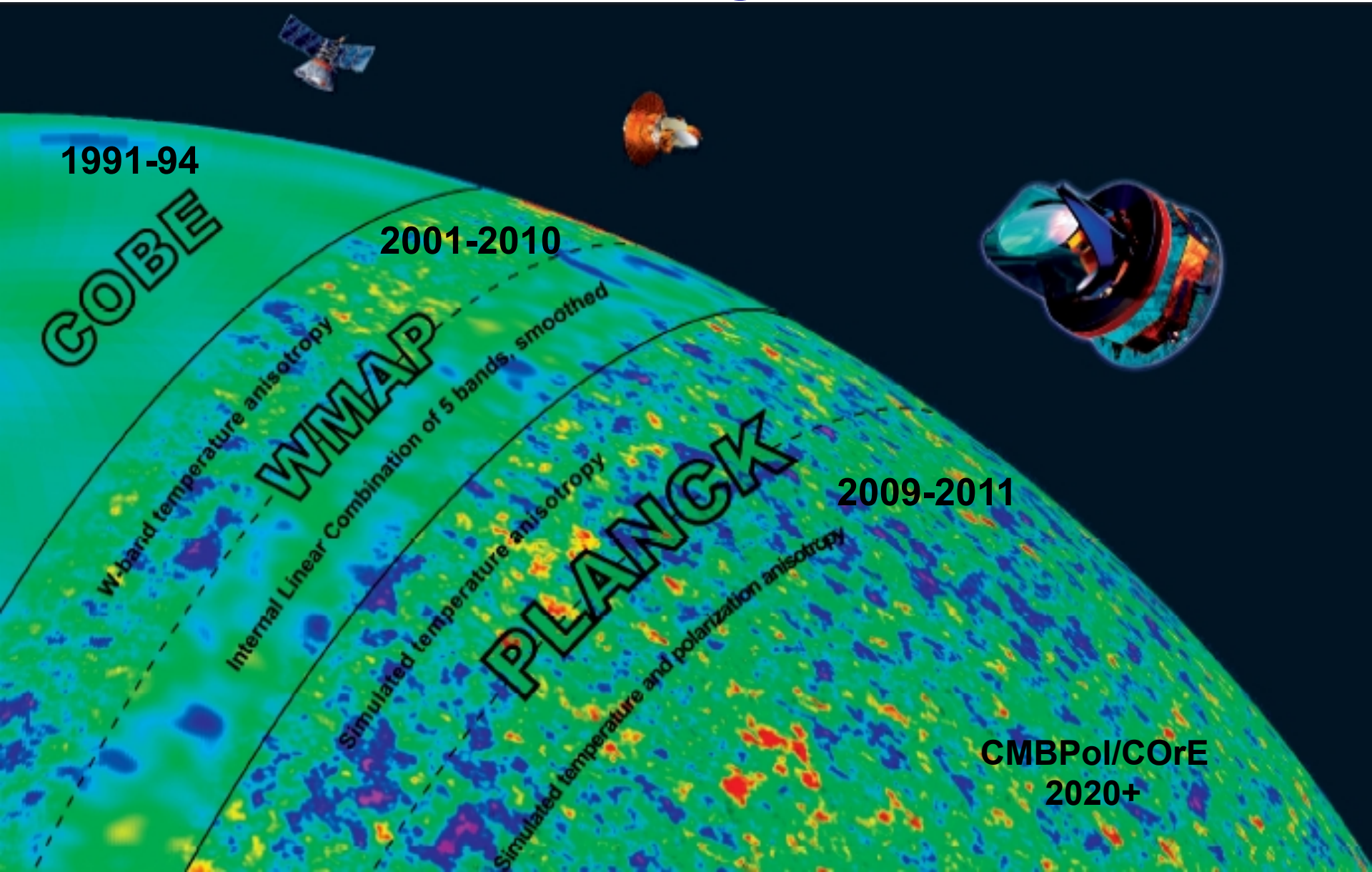
τ

Hubble Parameter and
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H_0

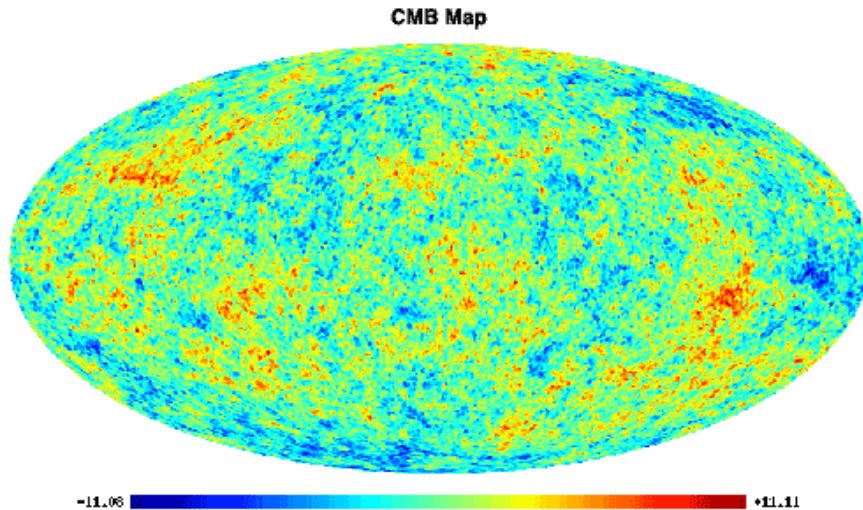
Why such assumptions?

Hints from Cosmological Observations



Statistics of CMB

CMB Anisotropy Sky map \Rightarrow Spherical Harmonic decomposition

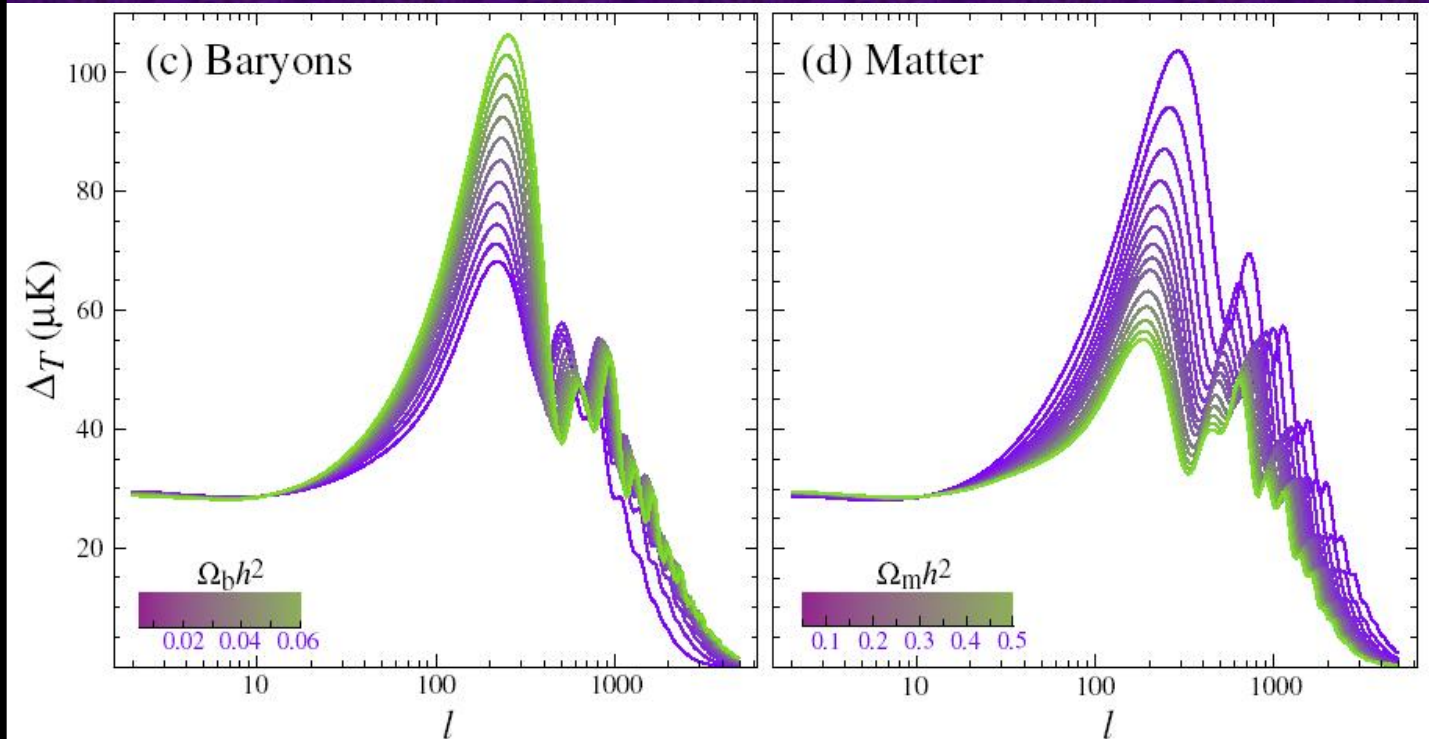
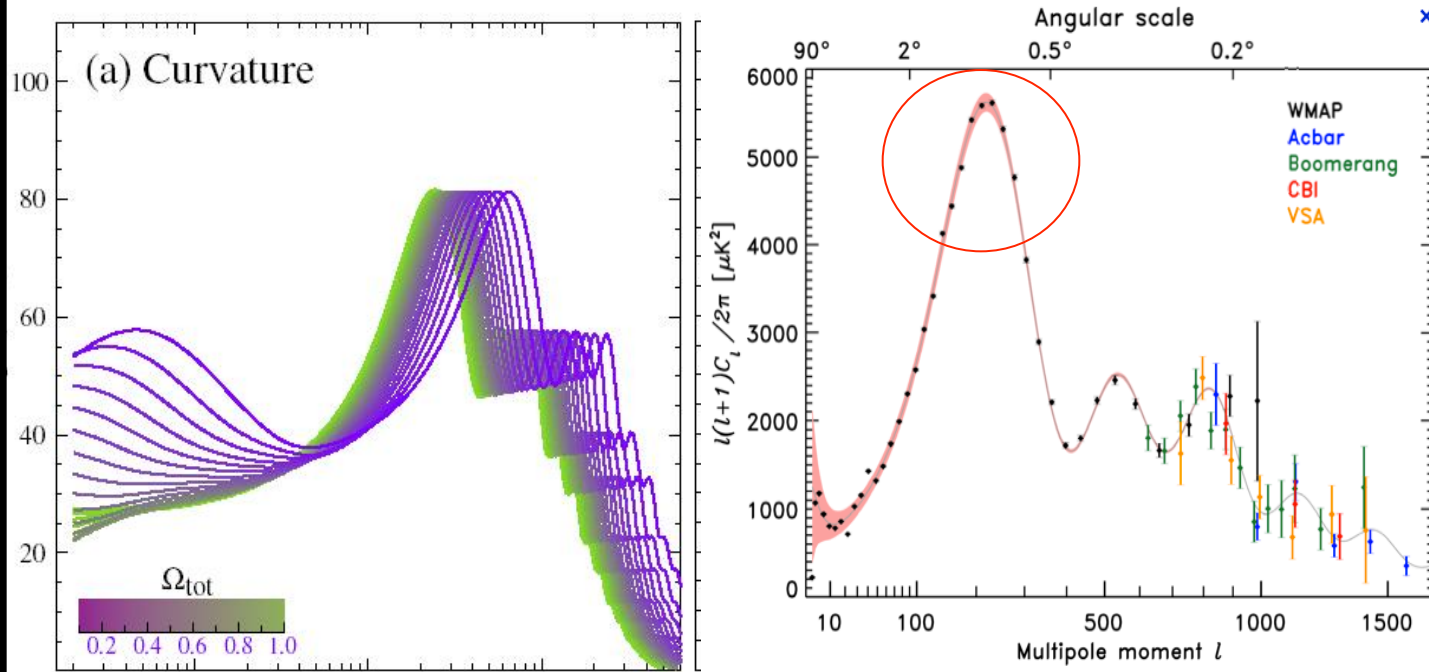


$$\Delta T(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi)$$

$$\langle a_{lm} a_{l'm'}^* \rangle = C_l \delta_{ll'} \delta_{mm'}$$

Gaussian Random field \Rightarrow Completely specified by
angular power spectrum $l(l+1)C_l$:

Power in fluctuations on angular scales of $\sim \pi/l$



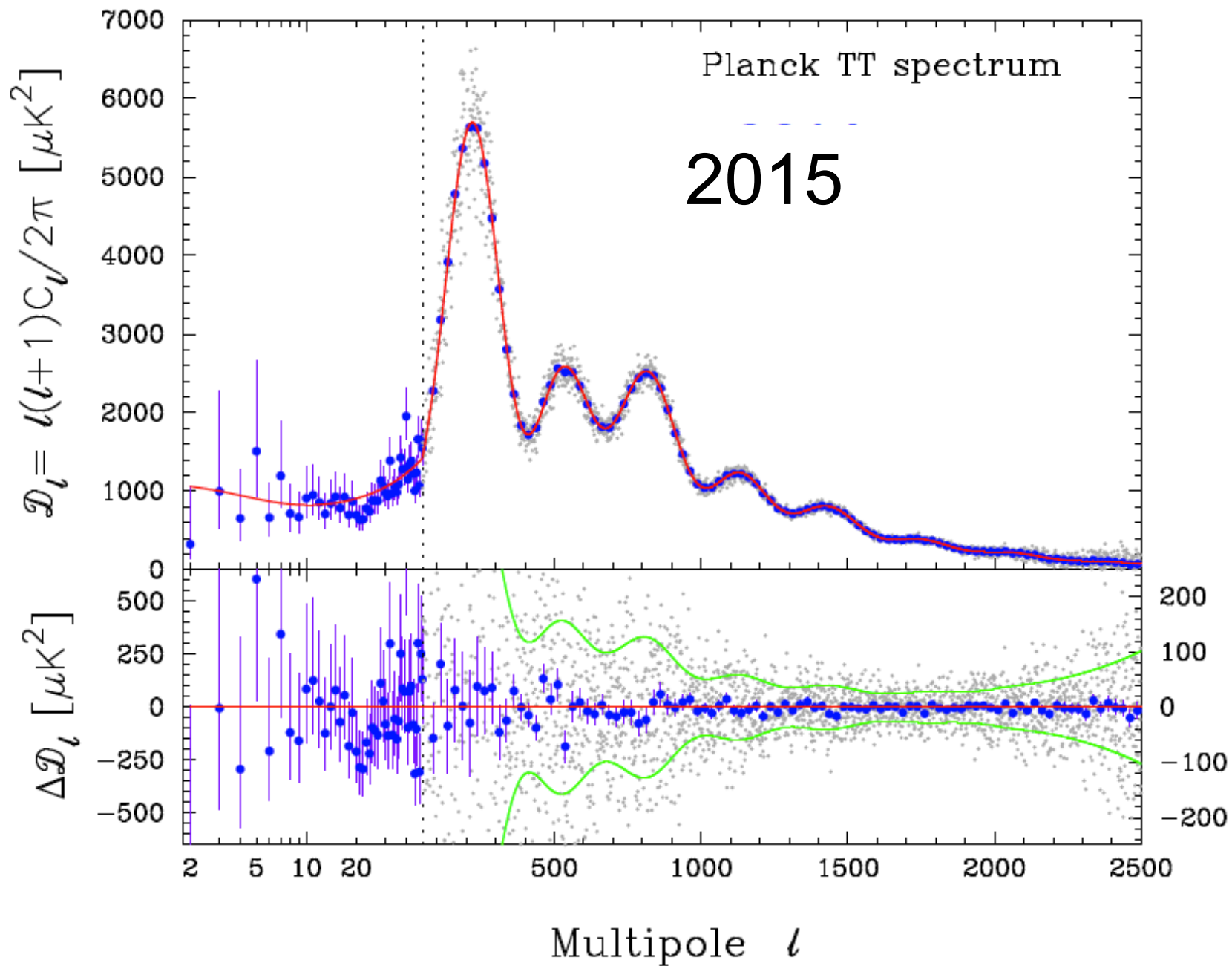
Sensitivity of the CMB acoustic temperature spectrum to four fundamental cosmological parameters.

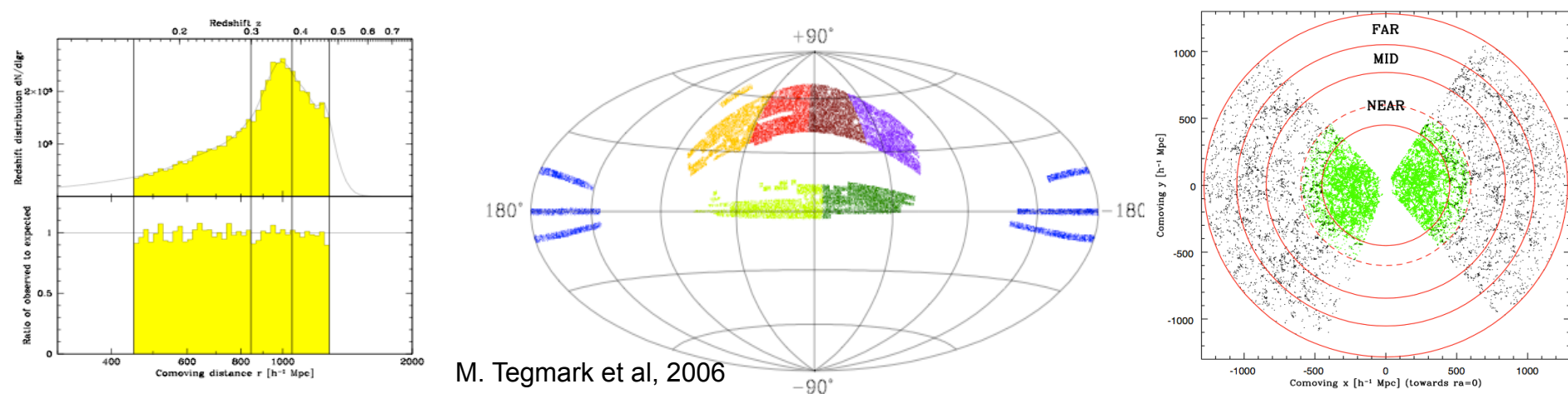
Total density

Dark Energy

Baryon density and

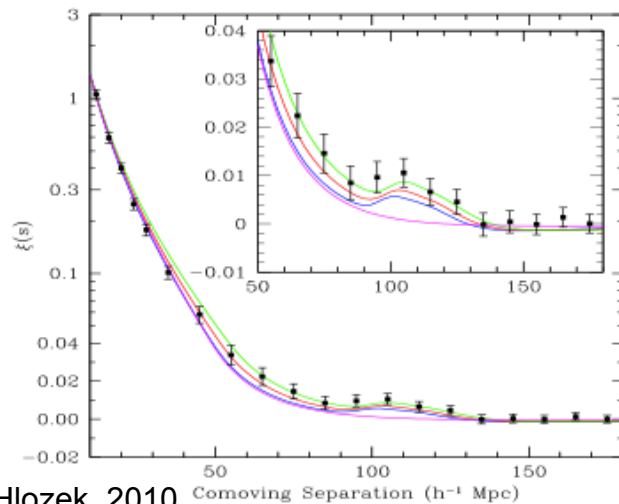
Matter density.





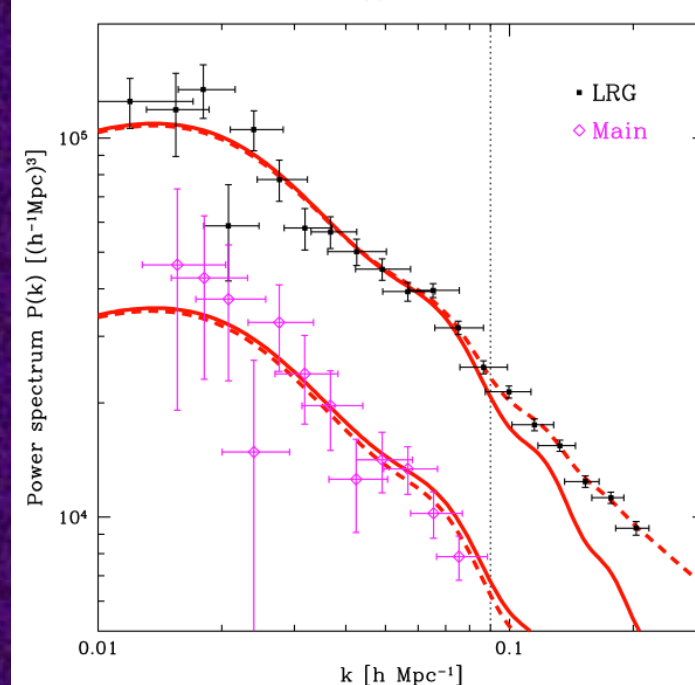
Large Scale Structure Data and Distribution of Galaxies

$$P(k) = \int_{-\infty}^{\infty} \xi(r) \exp(-ikr) r^2 dr.$$



Bassett & Hlozek, 2010

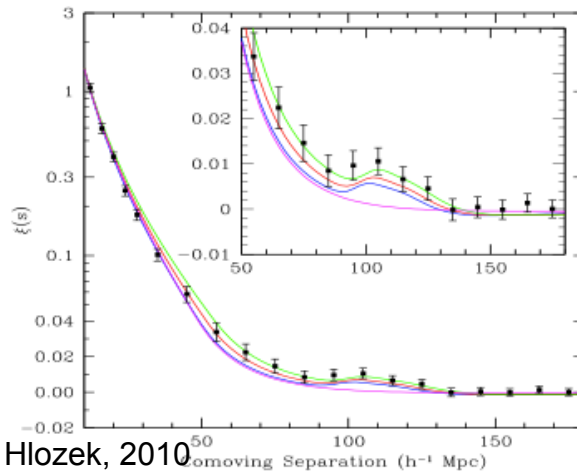
Fig. 1.1. The Baryon Acoustic Peak (BAP) in the correlation function – the BAP is visible in the clustering of the SDSS LRG galaxy sample, and is sensitive to the matter density (shown are models with $\Omega_m h^2 = 0.12$ (top), 0.13 (second) and 0.14 (third), all with $\Omega_b h^2 = 0.024$). The bottom line without a BAP is the correlation function in the pure CDM model, with $\Omega_b = 0$. From Eisenstein *et al.*, 2005 (52).



Large Scale Structure Data and Distribution of Galaxies

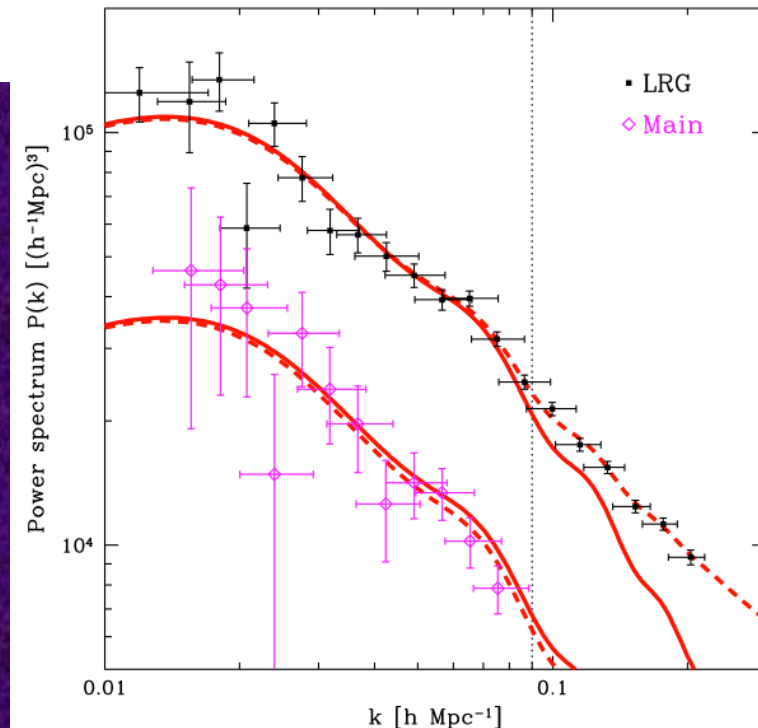
$$P(k) = \int_{-\infty}^{\infty} \xi(r) \exp(-ikr) r^2 dr.$$

Fig. 1.5. Rings of power superposed. Schematic galaxy distribution formed by placing the galaxies on rings of the same characteristic radius L . The preferred radial scale is clearly visible in the left hand panel with many galaxies per ring. The right hand panel shows a more realistic scenario - with many rings and relatively few galaxies per ring, implying that the preferred scale can only be recovered statistically.



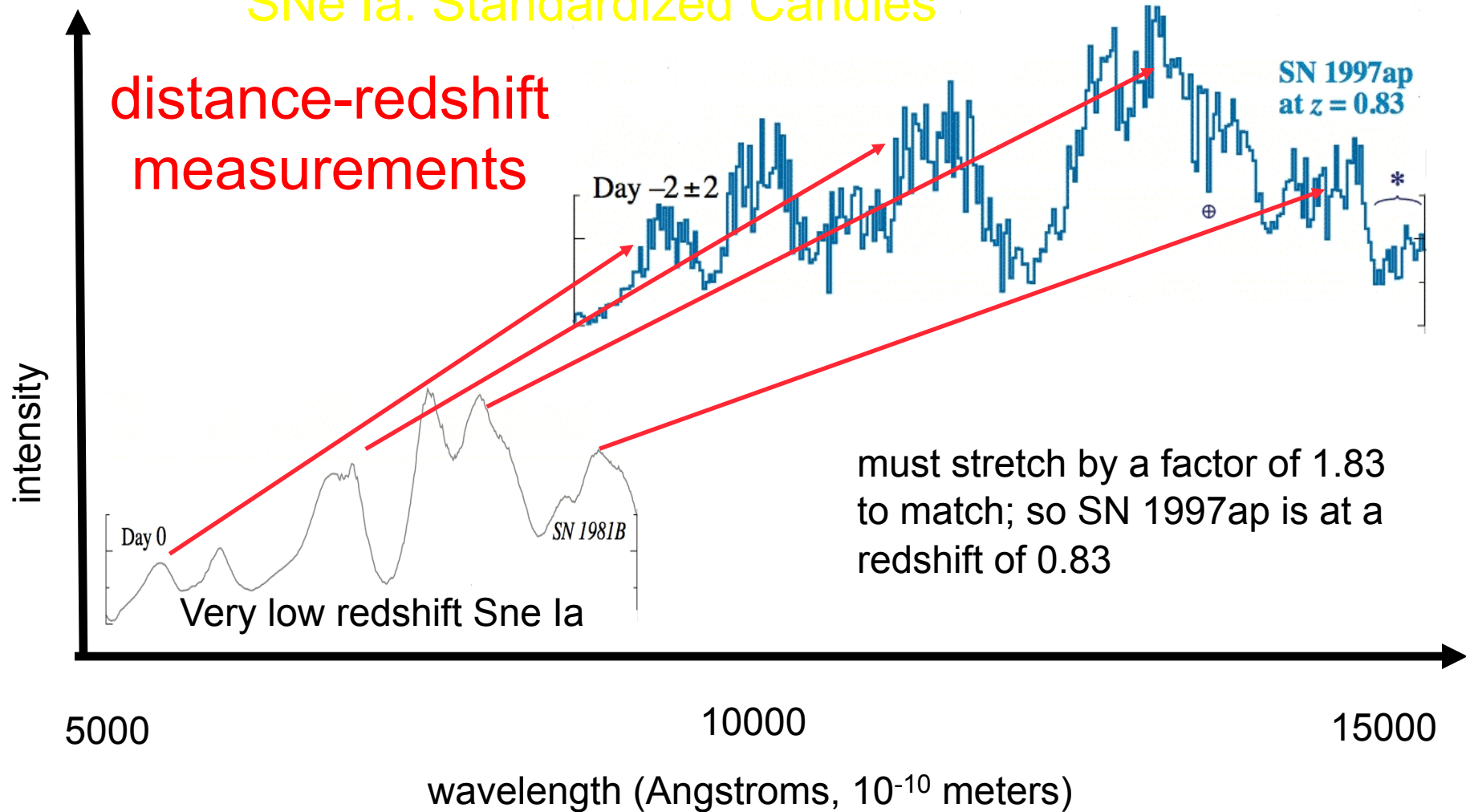
Bassett & Hlozek, 2010

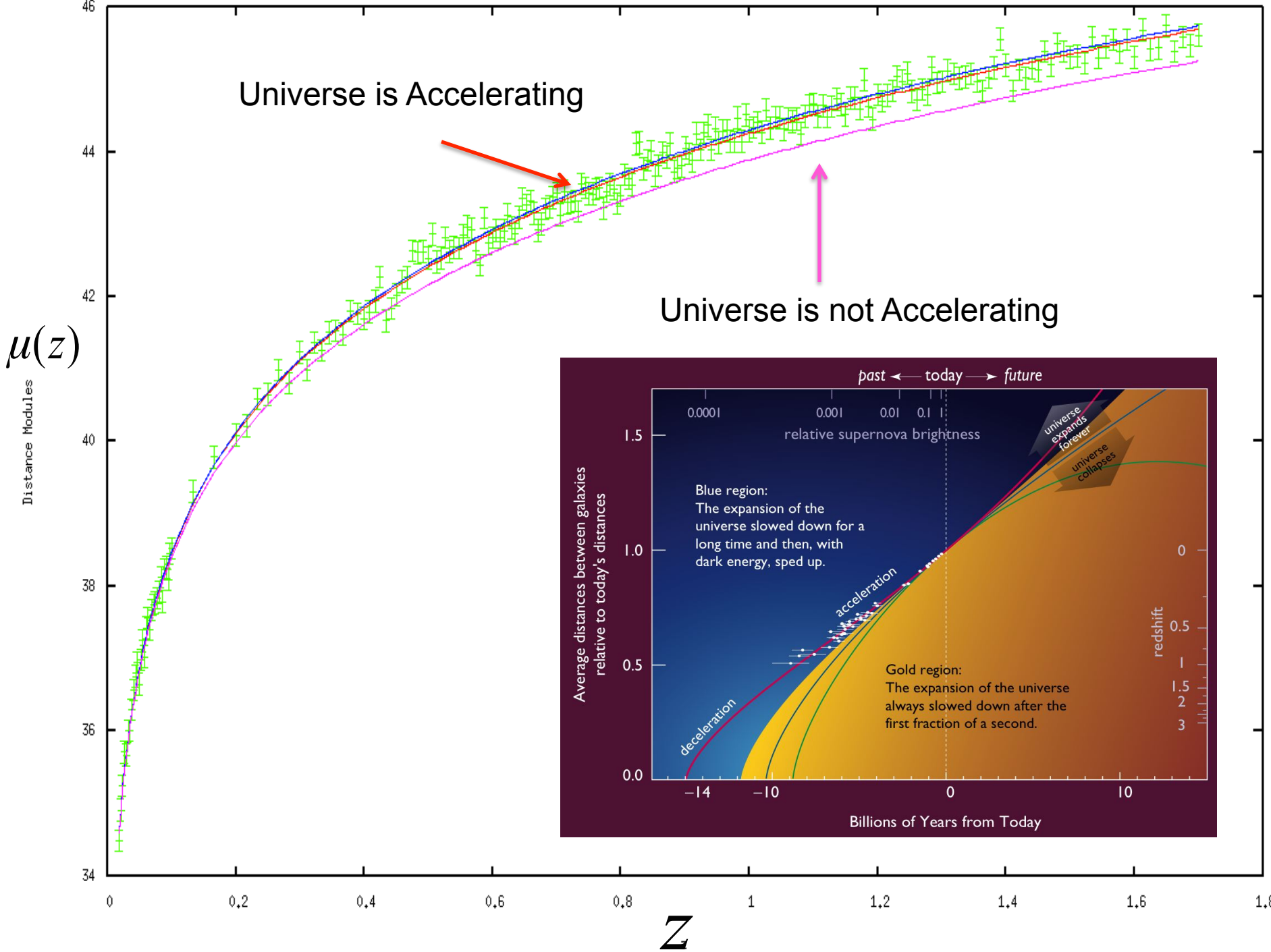
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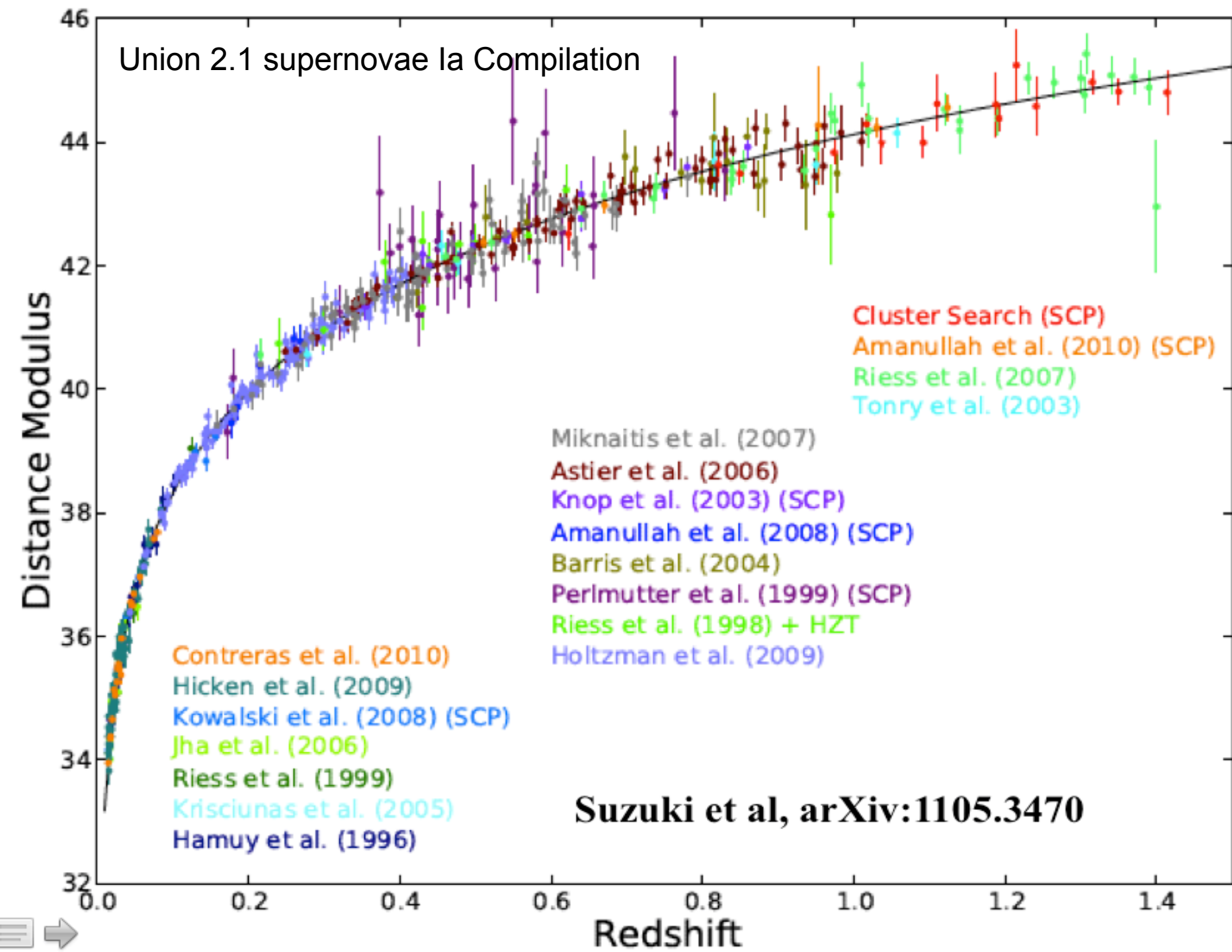
Measuring Distances in Astronomy

SNe Ia: Standardized Candles



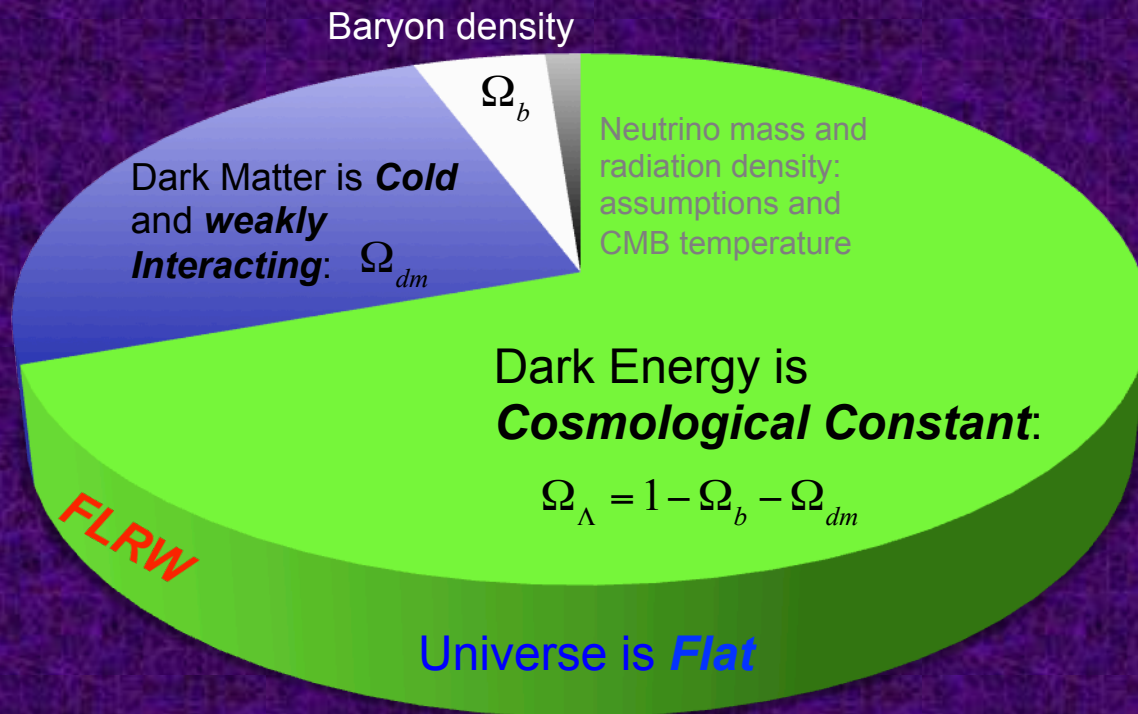


Union 2.1 supernovae Ia Compilation



Standard Model of Cosmology

combination of *reasonable* assumptions, but.....



Initial Conditions:
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Beyond the Standard Model of Cosmology

- The universe might be more complicated than its current standard model (Vanilla Model).
- There might be some extensions to the standard model in defining the cosmological quantities.
- This needs proper investigation, using advanced statistical methods, high performance computational facilities and high quality observational data.

(Present)_t

Standard Model of Cosmology

Universe is Flat

Universe is Isotropic

Universe is Homogeneous (large scales)

Dark Energy is Lambda ($w=-1$)

Power-Law primordial spectrum ($n_s=\text{const}$)

Dark Matter is cold

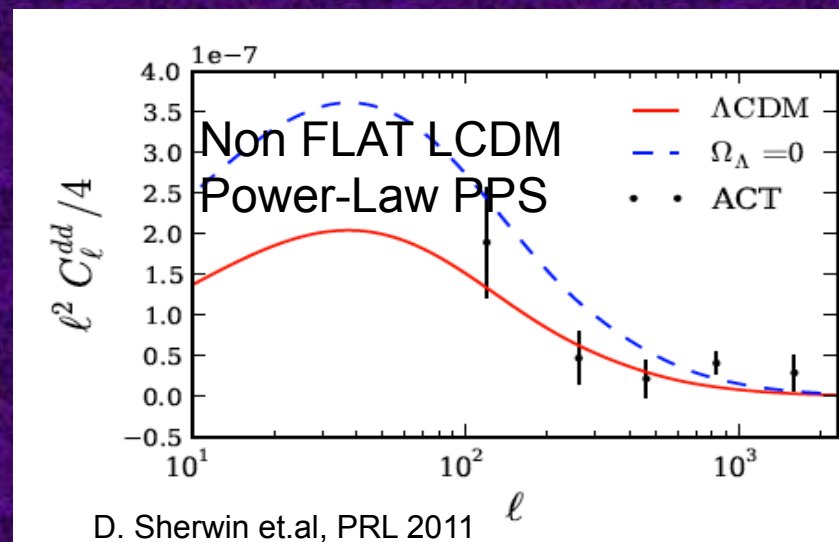
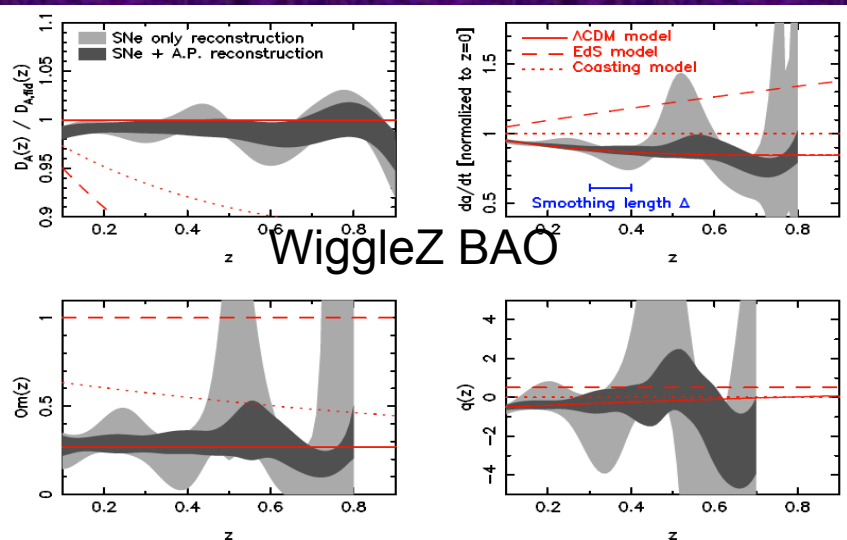
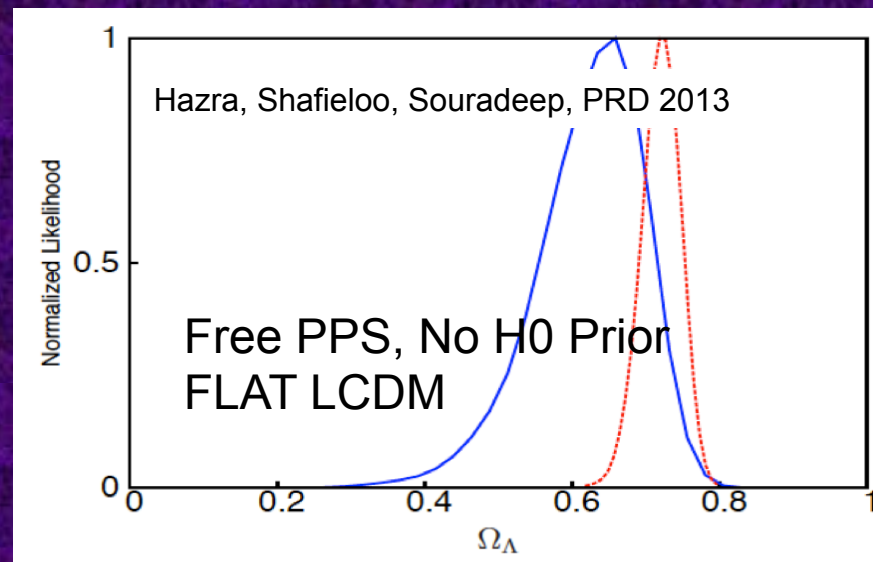
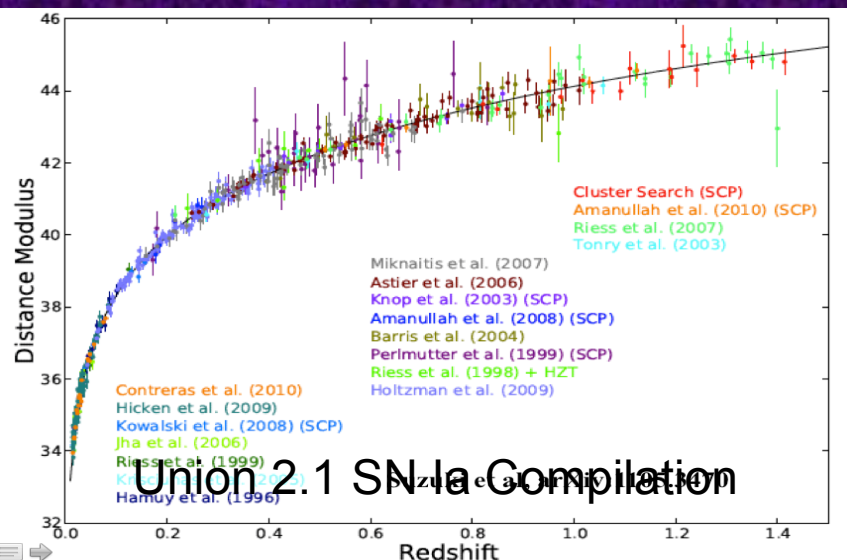
All within framework of FLRW

Era of Accelerating Universe

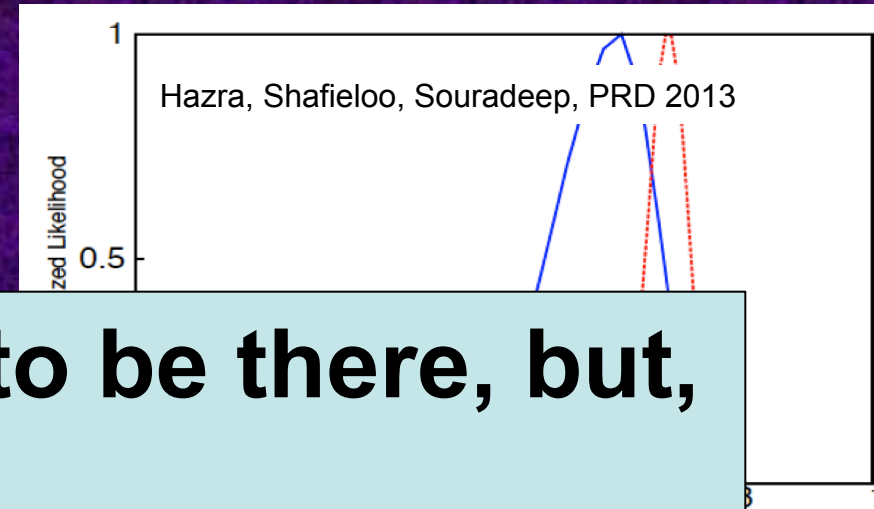
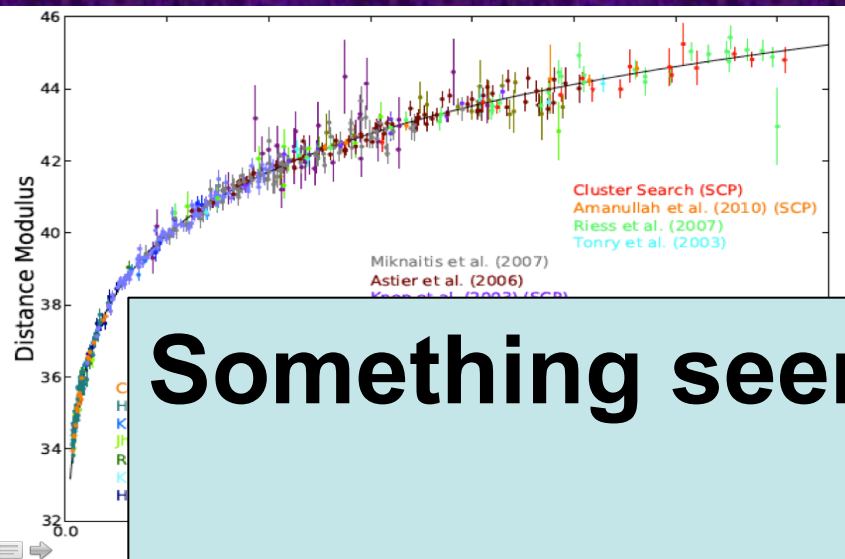
- Mid 90's: Indirect evidences were seen in the distribution of the galaxies where Λ CDM could not explain the excess of power at large scales.
- 1998: Direct evidence came by Supernovae Type Ia Observations. *Going to higher redshifts, supernovae are fainter than expected. One can explain this only (?!=Nobel Prize) by considering an accelerating universe.*

Accelerating Universe, Now-2015

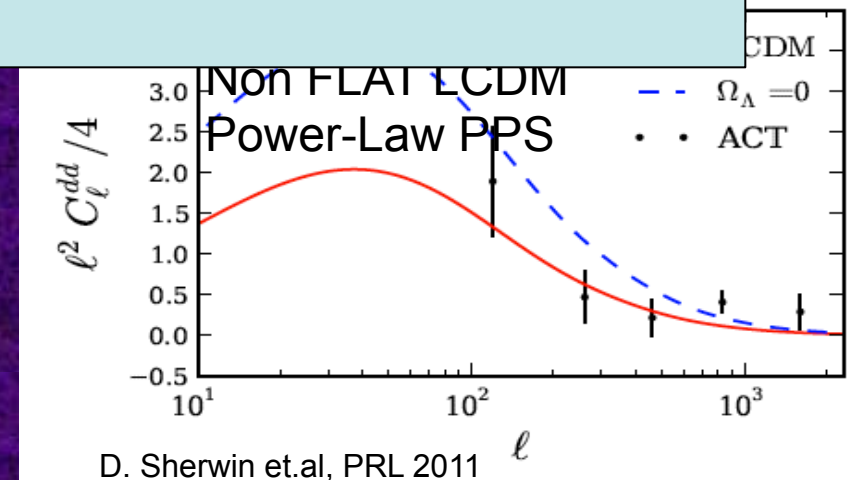
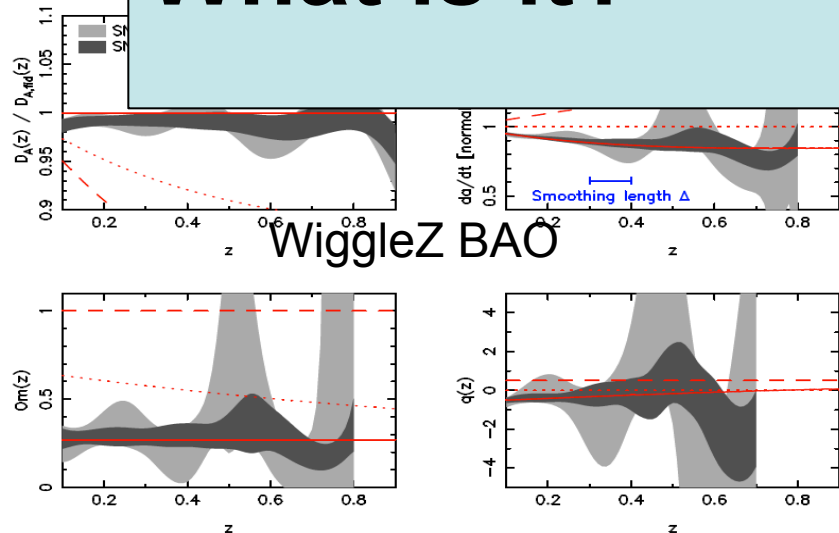
Or better to say, ruling out zero-Lambda Universe



Accelerating Universe, Now



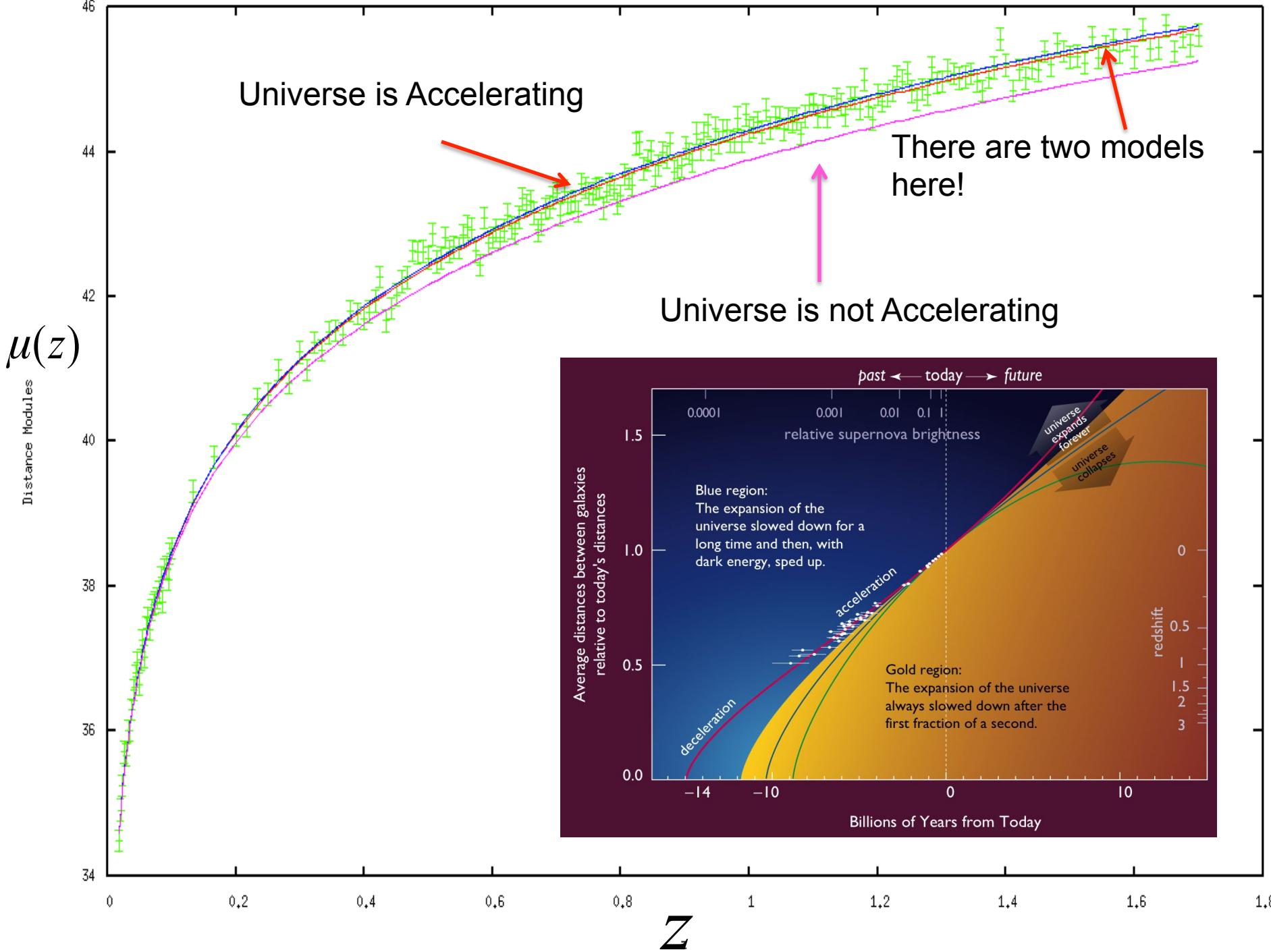
**Something seems to be there, but,
What is it?**



Dark Energy Models

- Cosmological Constant
- Quintessence and k-essence (scalar fields)
- Exotic matter (Chaplygin gas, phantom, etc.)
- Braneworlds (higher-dimensional theories)
- Modified Gravity
-

But which one is really responsible for the acceleration of the expanding universe?!



Reconstructing Dark Energy

To find cosmological quantities and parameters there are two general approaches:

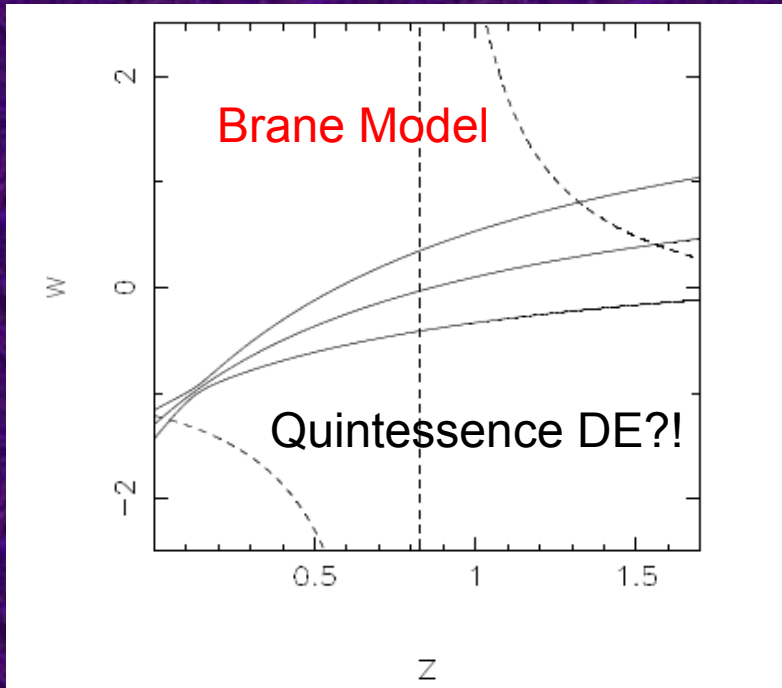
1. Parametric methods

Easy to confront with cosmological observations to put constraints on the parameters, but the results are highly biased by the assumed models and parametric forms.

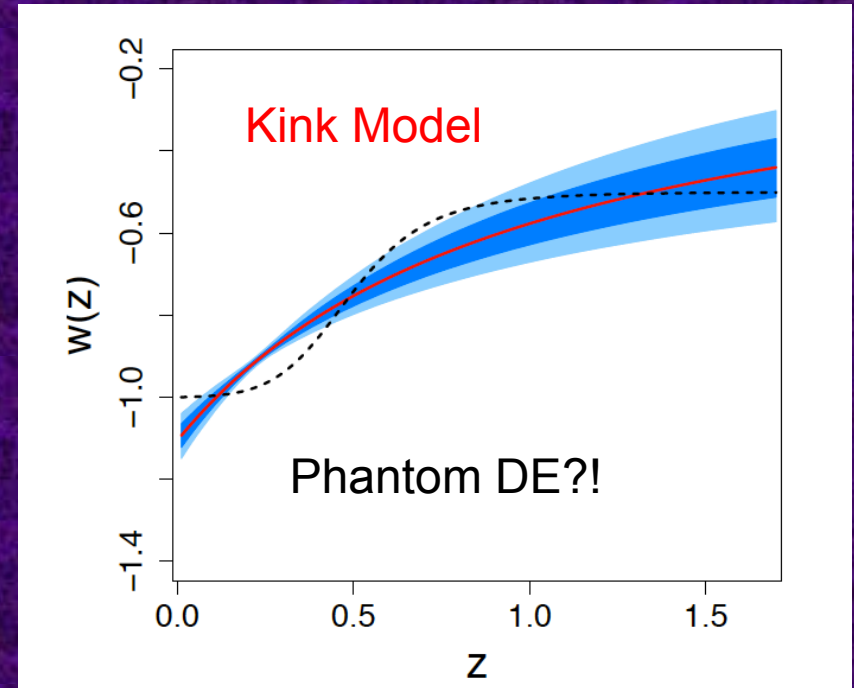
2. Non Parametric methods

Difficult to apply ***properly*** on the raw data, but the results will be less biased and more reliable and independent of theoretical models or parametric forms.

Problems of Dark Energy Parameterizations (model fitting)



Shafieloo, Alam, Sahni &
Starobinsky, MNRAS 2006



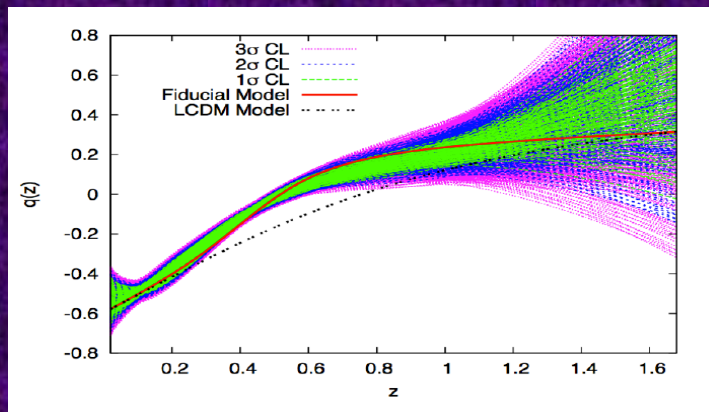
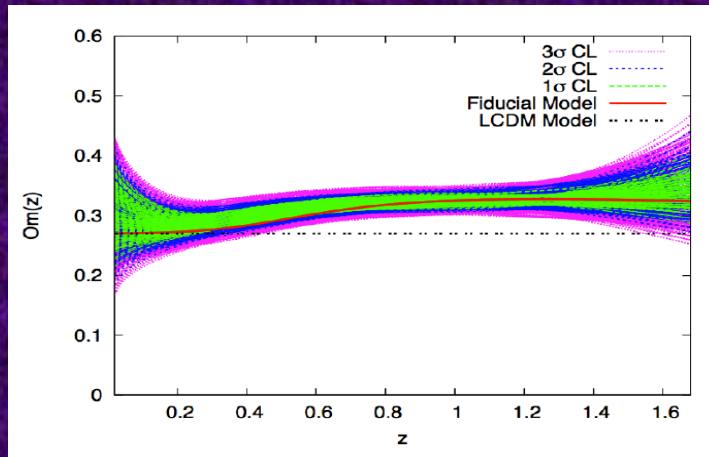
Holsclaw et al, PRD 2011

$$w(z) = w_0 - w_a \frac{z}{1+z}.$$

Chevallier-Polarski-Linder ansatz (CPL).

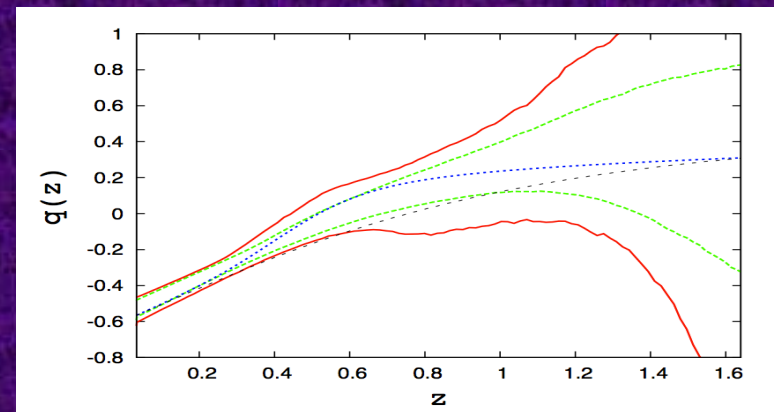
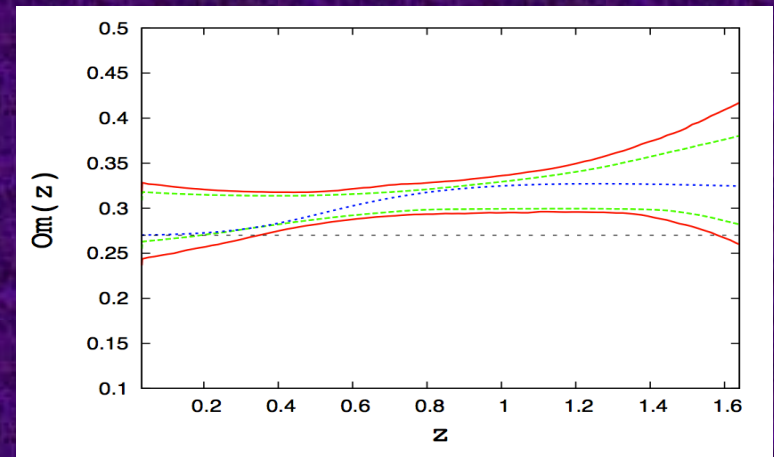
Model independent reconstruction of the expansion history

Crossing Statistic + Smoothing



Shafieloo, JCAP (b) 2012

Gaussian Processes



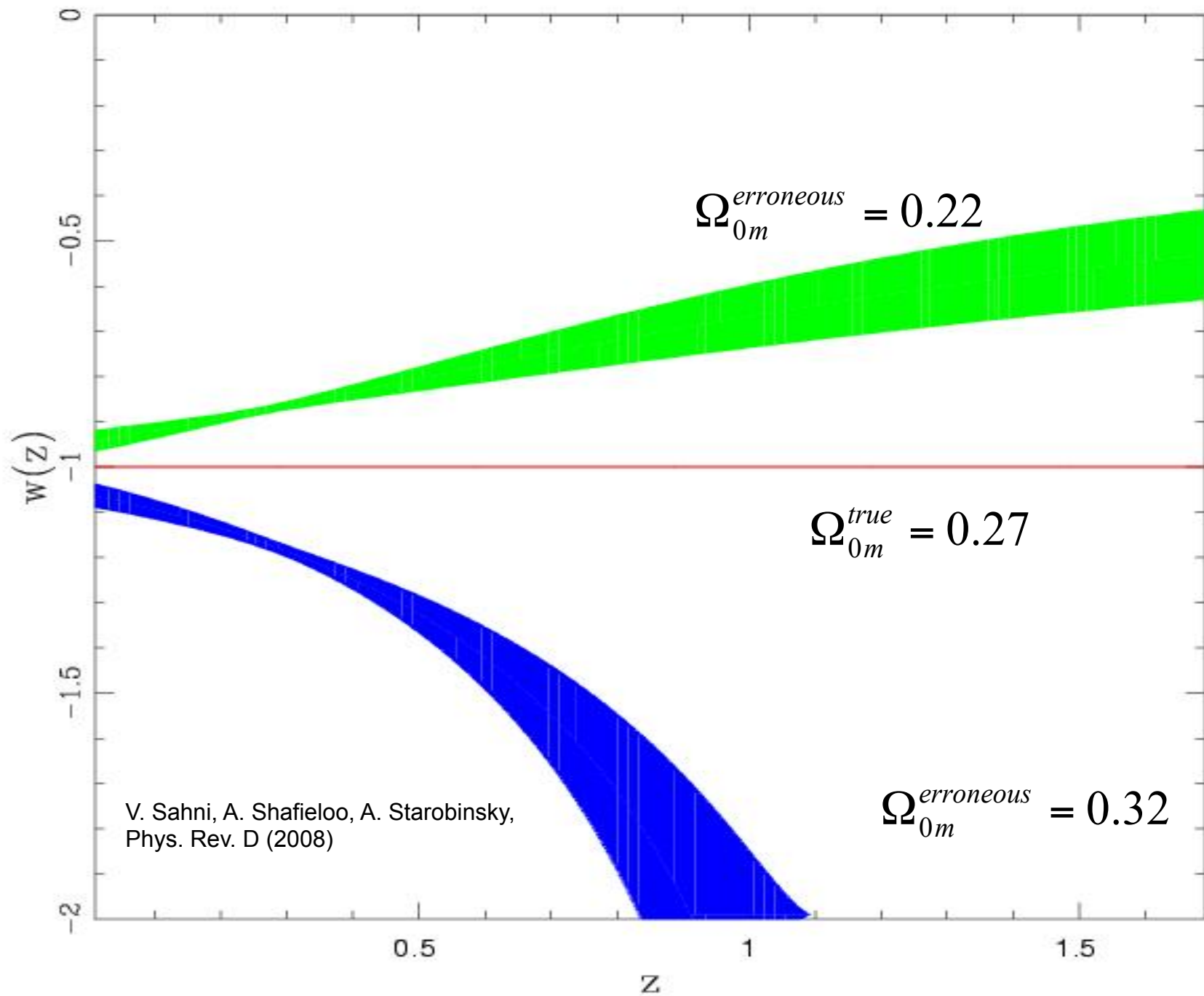
Shafieloo, Kim & Linder, PRD 2012

Dealing with observational uncertainties in matter density (and curvature)

- Small uncertainties in the value of matter density affects the reconstruction exercise quite dramatically.
- Uncertainties in matter density is in particular bound to affect the reconstructed $w(z)$.

$$H(z) = \left[\frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right]^{-1}$$

$$\omega_{DE} = \frac{\left(\frac{2(1+z)}{3} \frac{H'}{H} \right) - 1}{1 - \left(\frac{H_0}{H} \right)^2 \Omega_{0M} (1+z)^3}$$



Full theoretical picture:

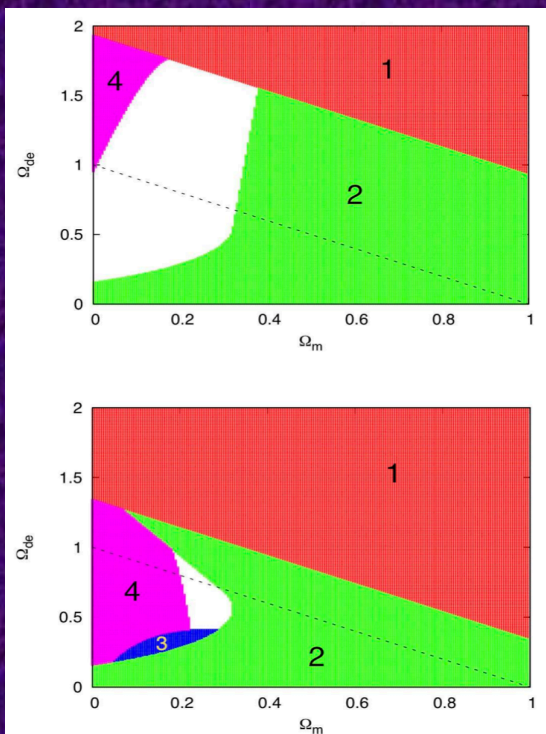
Cosmographic Degeneracy

$$d_l(z) = \frac{1+z}{\sqrt{1 - \Omega_m - \Omega_{de}}} \sinh \left(\sqrt{1 - \Omega_m - \Omega_{de}} \int_0^z \frac{dz'}{h(z')} \right)$$

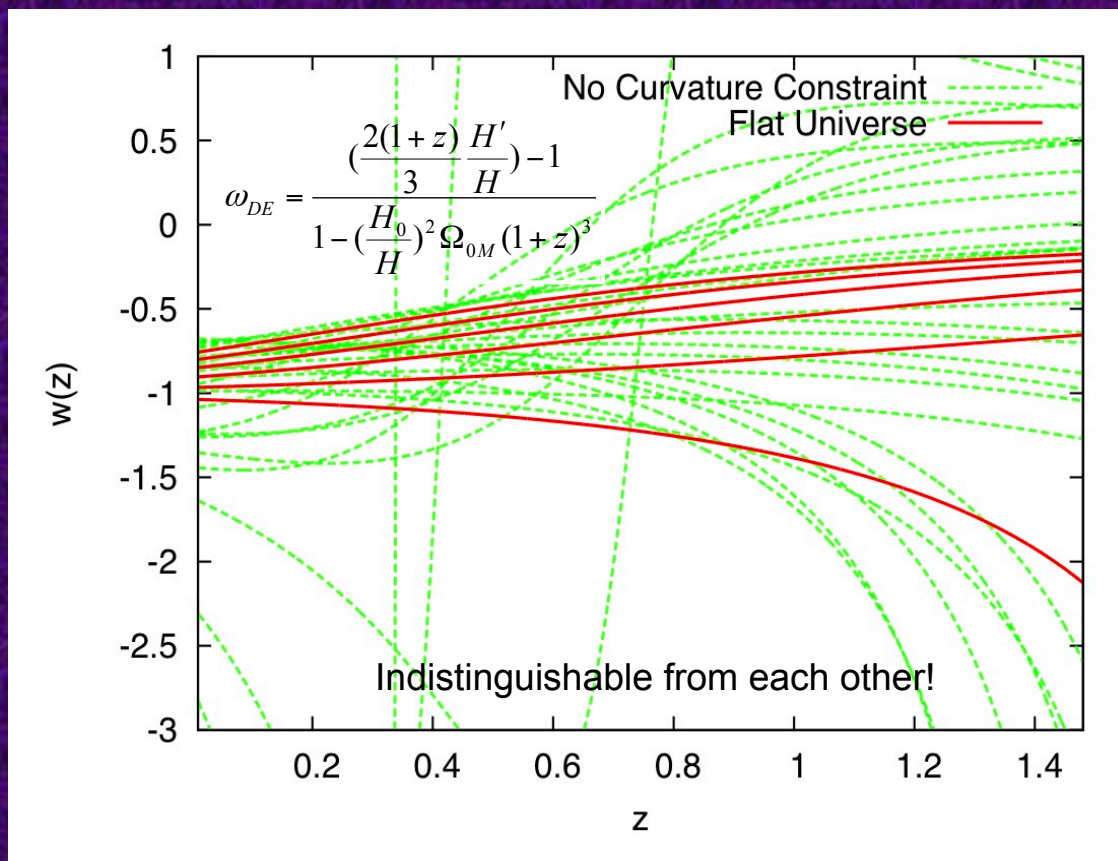
$$\begin{aligned} h(z)^2 &\equiv [H(z)/H_0]^2 \equiv (\dot{a}/a)^2 \\ &= \Omega_m (1+z)^3 + (1 - \Omega_m - \Omega_{de})(1+z)^2 \\ &\quad + \Omega_{de} \exp \left[3 \int_0^z \frac{dz'}{1+z'} [1 + w(z')] \right], \end{aligned}$$

Cosmographic Degeneracy

- **Cosmographic Degeneracies** would make it so hard to pin down the actual model of dark energy even in the near future.



Shafieloo & Linder, PRD 2011



Reconstruction & *Falsification*

Considering (low) quality of the data and cosmographic degeneracies we should consider a new strategy sideways to reconstruction: **Falsification.**

Yes-No to a hypothesis is easier than characterizing a phenomena.

We should look for special characteristics of the standard model and relate them to observables.

But, How?

Falsification of Cosmological Constant

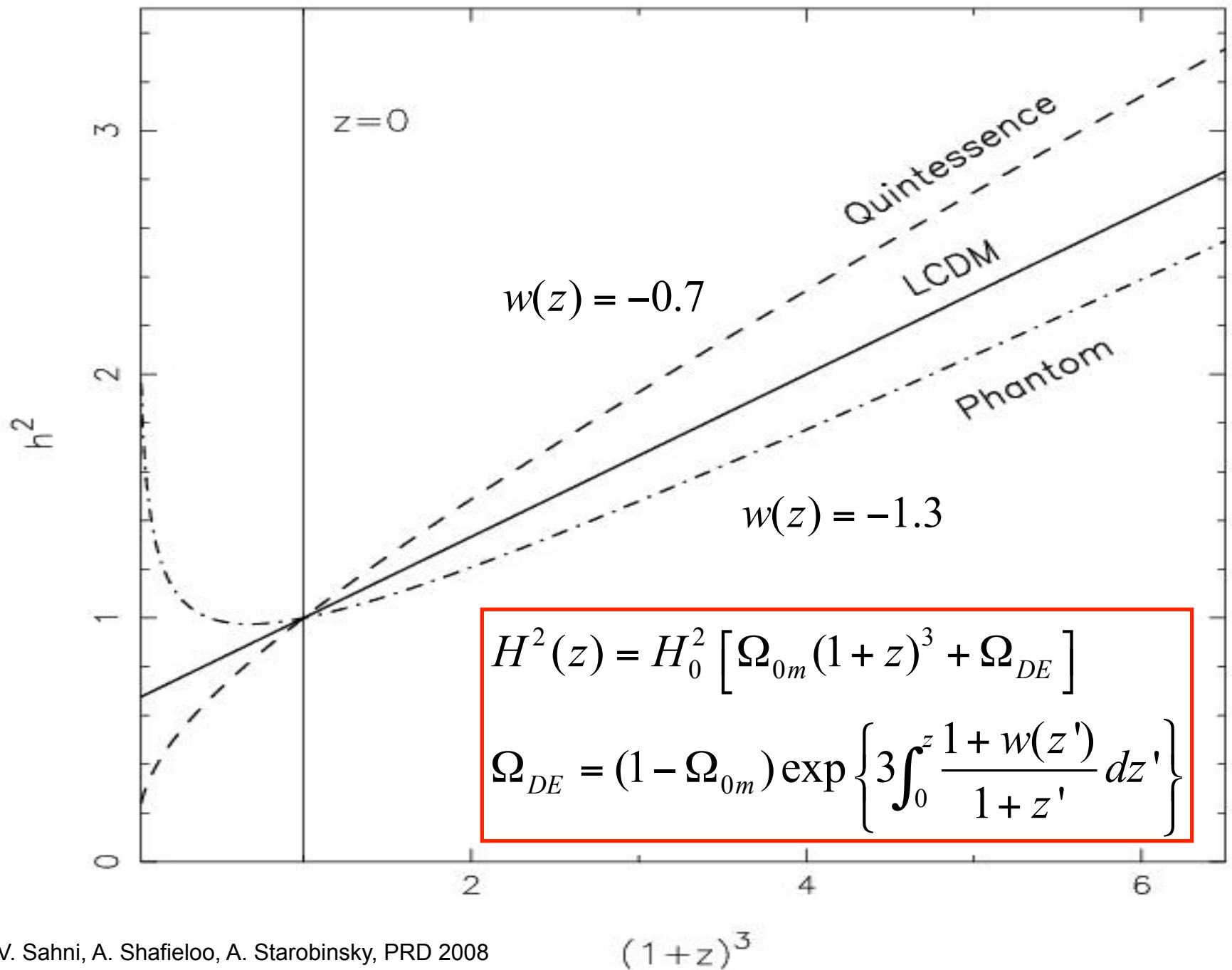
- Instead of looking for $w(z)$ and exact properties of dark energy at the current status of data, we can concentrate on a more reasonable problem:



OR NOT



Yes-No to a hypothesis is easier than characterizing a phenomena



Falsification: Null Test of Lambda

Om diagnostic

$$Om(z) = \frac{h^2(z) - 1}{(1+z)^3 - 1}$$

Om(z) is constant only
for FLAT LCDM model

We Only Need h(z)

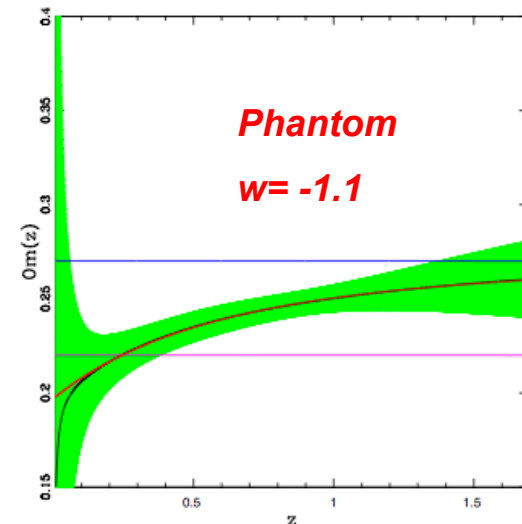
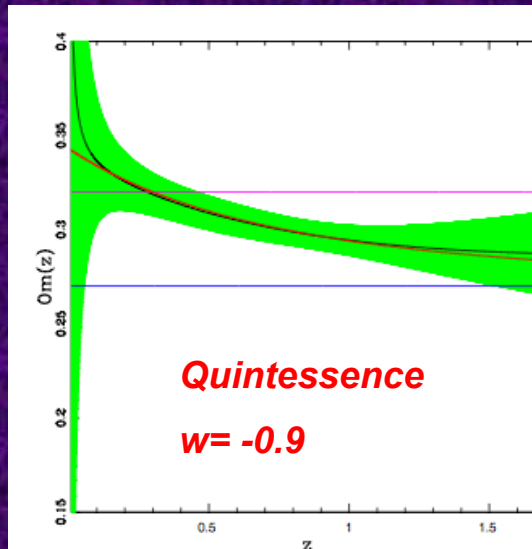
$$h(z) = H(z)/H_0$$

V. Sahni, A. Shafieloo, A. Starobinsky,
PRD 2008

$$w = -1 \rightarrow Om(z) = \Omega_{0m}$$

$$w < -1 \rightarrow Om(z) < \Omega_{0m}$$

$$w > -1 \rightarrow Om(z) > \Omega_{0m}$$



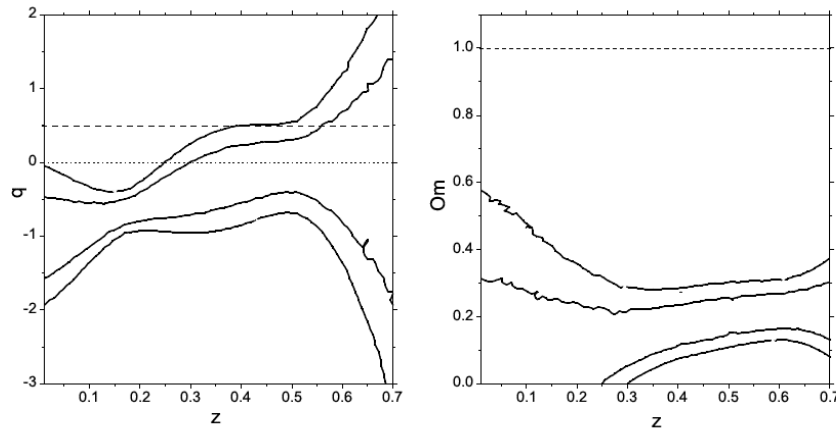


Figure 12. Confidence levels (1σ and 2σ) for the deceleration parameter as a function of redshift and $Om(z)$ reconstructed from the compilation of geometric measurements in tables 2 and 3. H_0 is marginalised over with an HST prior. The dotted line in the left panel demarcates accelerating expansion (below the line) from decelerated expansion (above the line). The dashed line in both panels shows the expectation for an EdS model.

Om diagnostic is very well established

SDSS III / BOSS collaboration
L. Samushia et al, MNRAS 2013

WiggleZ collaboration
C. Blake et al, MNRAS 2011
(Alcock-Paczynski measurement)

10 Blake et al.

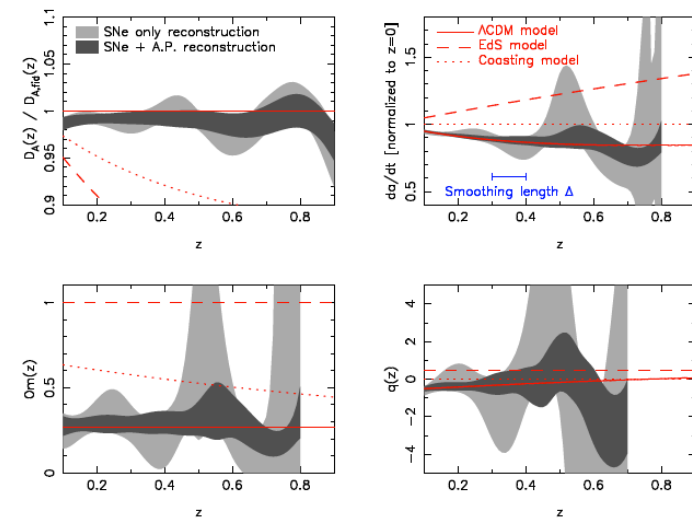


Figure 6. This Figure shows our non-parametric reconstruction of the cosmic expansion history using Alcock-Paczynski and supernovae data. The four panels of this figure display our reconstructions of the distance-redshift relation $D_A(z)$, the expansion rate \dot{a}/H_0 , the $Om(z)$ statistic and the deceleration parameter $q(z)$ using our adaptation of the iterative method of Shafieloo et al. (2006) and Shafieloo & Clarkson (2010). The distance-redshift relation in the upper left-hand panel is divided by a fiducial model for clarity, where the model corresponds to a flat Λ CDM cosmology with $\Omega_m = 0.27$. This fiducial model is shown as the solid line in all panels; Einstein de-Sitter and coasting models are also shown as defined in Figure 5. The shaded regions illustrate the 68% confidence range of the reconstructions of each quantity obtained using bootstrap resamples of the data. The dark-grey regions illustrate a combination of the Alcock-Paczynski and supernovae data and the light-grey regions are based on the supernovae data alone. The redshift smoothing scale $\Delta = 0.1$ is also illustrated. The reconstructions in each case are terminated when the SNe-only results become very noisy; this maximum redshift reduces with each subsequent derivative of the distance-redshift relation [i.e. is lowest for $q(z)$].

Om3

A null diagnostic customized for reconstructing the properties of dark energy directly from BAO data

$$Om3(z_1, z_2, z_3) = \frac{Om(z_2, z_1)}{Om(z_3, z_1)} = \frac{\frac{h^2(z_2) - h^2(z_1)}{(1+z_2)^3 - (1+z_1)^3}}{\frac{h^2(z_3) - h^2(z_1)}{(1+z_3)^3 - (1+z_1)^3}} = \frac{\frac{\frac{h^2(z_2)}{h^2(z_1)} - 1}{(1+z_2)^3 - (1+z_1)^3}}{\frac{\frac{h^2(z_3)}{h^2(z_1)} - 1}{(1+z_3)^3 - (1+z_1)^3}} = \frac{\frac{\frac{\frac{H^2(z_2)}{H_0^2} - 1}{\frac{H^2(z_2)}{H^2(z_1)} - 1}}{(1+z_2)^3 - (1+z_1)^3}}{\frac{\frac{\frac{H^2(z_3)}{H_0^2} - 1}{\frac{H^2(z_3)}{H^2(z_1)} - 1}}{(1+z_3)^3 - (1+z_1)^3}} = \frac{\frac{H^2(z_2)}{H^2(z_1)} - 1}{\frac{H^2(z_3)}{H^2(z_1)} - 1} \frac{(1+z_3)^3 - (1+z_1)^3}{(1+z_2)^3 - (1+z_1)^3}$$

$$d(z) = \frac{r_s(z_{\text{CMB}})}{D_V(z)}$$

Observables

Shafieloo, Sahni, Starobinsky, PRD 2013

$$H(z_i; z_j) := \frac{H(z_i)}{H(z_j)} = \frac{z_i}{z_j} \left[\frac{D(z_i)}{D(z_j)} \right]^2 \left[\frac{D_V(z_j)}{D_V(z_i)} \right]^3 = \frac{z_i}{z_j} \left[\frac{D(z_i)}{D(z_j)} \right]^2 \left[\frac{d(z_i)}{d(z_j)} \right]^3,$$

Characteristics of Om3

Om is constant only for Flat LCDM model

Om3 is equal to one for Flat LCDM model

$$Om3(z_1; z_2; z_3) = \frac{H(z_2; z_1)^2 - 1}{x_2^3 - x_1^3} \bigg/ \frac{H(z_3; z_1)^2 - 1}{x_3^3 - x_1^3}, \quad \text{where } x = 1 + z,$$

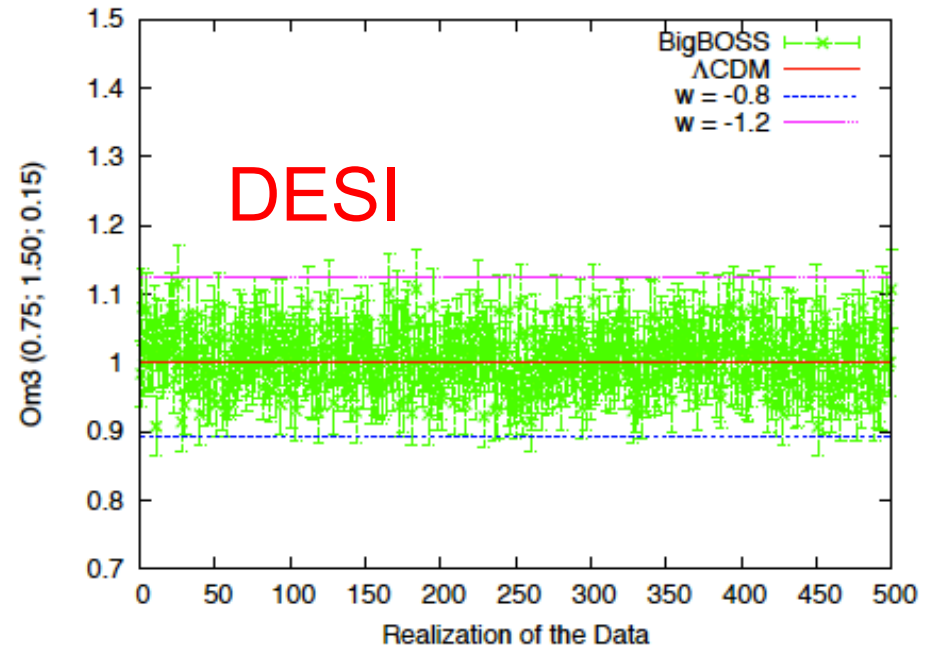
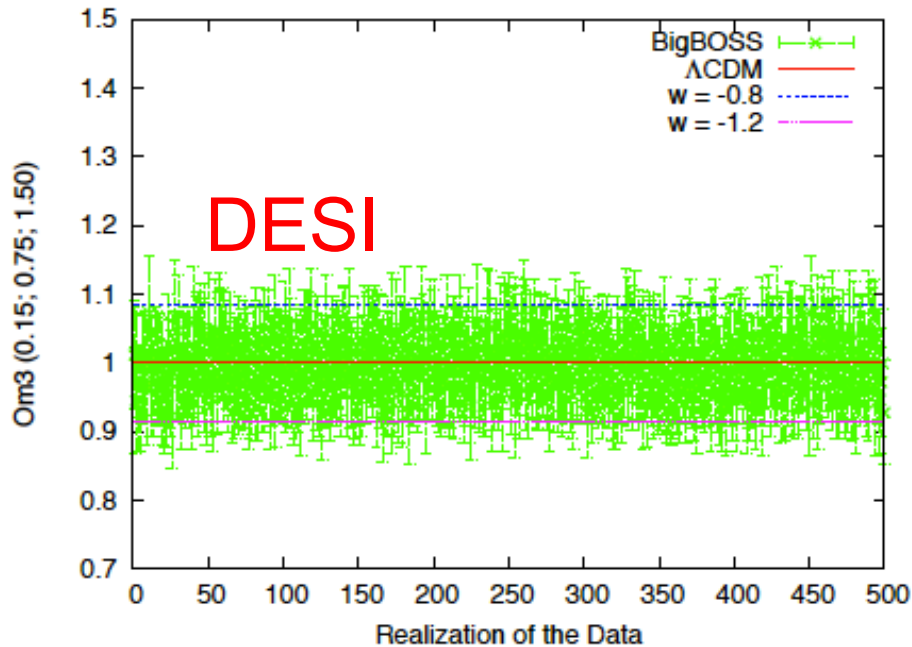
$$H(z_i; z_j) = \left(\frac{z_j}{z_i} \right)^2 \left[\frac{D(z_i)}{D(z_j)} \right]^2 \left[\frac{A(z_j)}{A(z_i)} \right]^3 = \frac{z_i}{z_j} \left[\frac{D(z_i)}{D(z_j)} \right]^2 \left[\frac{d(z_i)}{d(z_j)} \right]^3,$$

Om3 is independent of H0 and the distance to the last scattering surface and can be derived directly using BAO observables.

Characteristics of Om3

Om is constant only for Flat LCDM model

Om3 is equal to one for Flat LCDM model



$$Om3(z_1; z_2; z_3) = \frac{H(z_2; z_1)^2 - 1}{x_2^3 - x_1^3} \bigg/ \frac{H(z_3; z_1)^2 - 1}{x_3^3 - x_1^3}, \quad \text{where } x = 1 + z,$$

Om h^2

A very recent result.

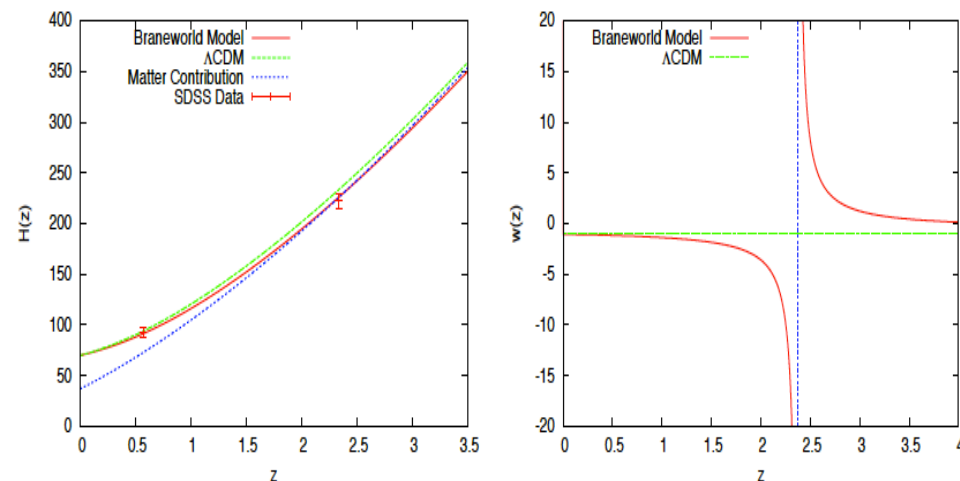
Important discovery if no systematic in the SDSS Quasar BAO data

Model Independent Evidence for Dark Energy Evolution from Baryon Acoustic Oscillation

$$Om h^2(z_1, z_2) = \frac{H^2(z_2) - H^2(z_1)}{(1+z_2)^3 - (1+z_1)^3} = \Omega_{0m} H_0^2$$

Only for LCDM

Sahni, Shafieloo, Starobinsky, ApJ Lett 2014



$$Om h^2 = 0.1426 \pm 0.0025$$

LCDM
+Planck+WP

$$Om h^2(z_1; z_2) = 0.124 \pm 0.045$$

$$Om h^2(z_1; z_3) = 0.122 \pm 0.010$$

$$Om h^2(z_2; z_3) = 0.122 \pm 0.012$$

BAO+H0

$$H(z = 0.00) = 70.6 \pm 3.3 \text{ km/sec/Mpc}$$

$$H(z = 0.57) = 92.4 \pm 4.5 \text{ km/sec/Mpc}$$

$$H(z = 2.34) = 222.0 \pm 7.0 \text{ km/sec/Mpc}$$

Testing deviations from an assumed model (without comparing different models)

Gaussian Processes:

Modeling of the data around a mean function searching for likely features by looking at the the likelihood space of the hyperparameters.

Bayesian Interpretation of Crossing Statistic:

Comparing a model with its own possible variations.

REACT:

Risk Estimation and Adaptation after Coordinate Transformation

Gaussian Process

- Efficient in statistical modeling of stochastic variables
- Derivatives of Gaussian Processes are Gaussian Processes
- Provides us with all covariance matrices

Shafieloo, Kim & Linder, PRD 2012
Shafieloo, Kim & Linder, PRD 2013

Data

Mean Function

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z}_1) \\ \mathbf{m}'(\mathbf{Z}_1) \\ \mathbf{m}''(\mathbf{Z}_1) \end{bmatrix}, \begin{bmatrix} \Sigma_{00}(\mathbf{Z}, \mathbf{Z}) & \Sigma_{00}(\mathbf{Z}, \mathbf{Z}_1) & \Sigma_{01}(\mathbf{Z}, \mathbf{Z}_1) & \Sigma_{02}(\mathbf{Z}, \mathbf{Z}_1) \\ \Sigma_{00}(\mathbf{Z}_1, \mathbf{Z}) & \Sigma_{00}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{01}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{02}(\mathbf{Z}_1, \mathbf{Z}_1) \\ \Sigma_{10}(\mathbf{Z}_1, \mathbf{Z}) & \Sigma_{10}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{11}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{12}(\mathbf{Z}_1, \mathbf{Z}_1) \\ \Sigma_{20}(\mathbf{Z}_1, \mathbf{Z}) & \Sigma_{20}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{21}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{22}(\mathbf{Z}_1, \mathbf{Z}_1) \end{bmatrix} \right),$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)} K}{dz_i^\alpha dz_j^\beta},$$

$$\begin{bmatrix} \bar{\mathbf{f}} \\ \bar{\mathbf{f}}' \\ \bar{\mathbf{f}}'' \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z}_1) \\ \mathbf{m}'(\mathbf{Z}_1) \\ \mathbf{m}''(\mathbf{Z}_1) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(\mathbf{Z}_1, \mathbf{Z}) \\ \Sigma_{10}(\mathbf{Z}_1, \mathbf{Z}) \\ \Sigma_{20}(\mathbf{Z}_1, \mathbf{Z}) \end{bmatrix} \Sigma_{00}^{-1}(\mathbf{Z}, \mathbf{Z}) \mathbf{y}$$

Kernel

$$k(z, z') = \sigma_f^2 \exp \left(-\frac{|z - z'|^2}{2l^2} \right),$$

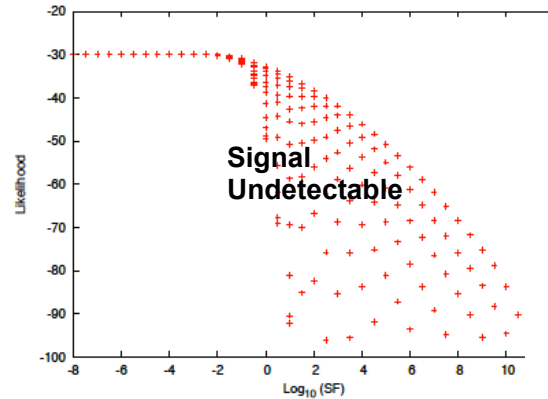
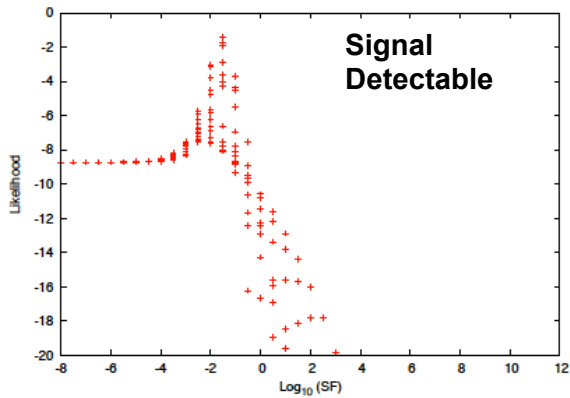
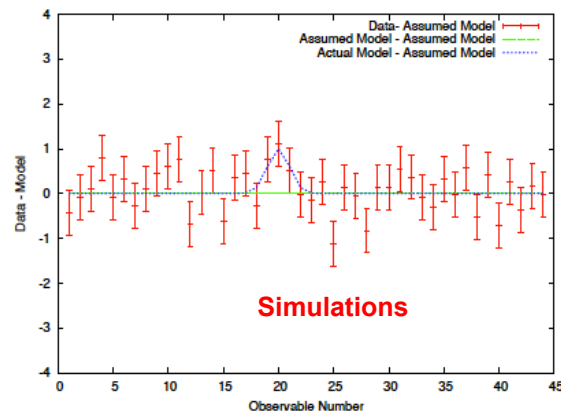
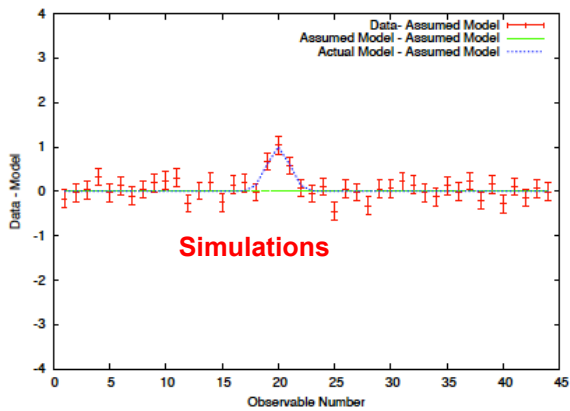
GP Hyper-parameters

$$\text{Cov} \left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix} \right) = \begin{bmatrix} \Sigma_{00}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{01}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{02}(\mathbf{Z}_1, \mathbf{Z}_1) \\ \Sigma_{10}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{11}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{12}(\mathbf{Z}_1, \mathbf{Z}_1) \\ \Sigma_{20}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{21}(\mathbf{Z}_1, \mathbf{Z}_1) & \Sigma_{22}(\mathbf{Z}_1, \mathbf{Z}_1) \end{bmatrix} - \begin{bmatrix} \Sigma_{00}(\mathbf{Z}_1, \mathbf{Z}) \\ \Sigma_{10}(\mathbf{Z}_1, \mathbf{Z}) \\ \Sigma_{20}(\mathbf{Z}_1, \mathbf{Z}) \end{bmatrix} \Sigma_{00}^{-1}(\mathbf{Z}, \mathbf{Z}) [\Sigma_{00}(\mathbf{Z}, \mathbf{Z}_1), \Sigma_{01}(\mathbf{Z}, \mathbf{Z}_1), \Sigma_{02}(\mathbf{Z}, \mathbf{Z}_1)].$$

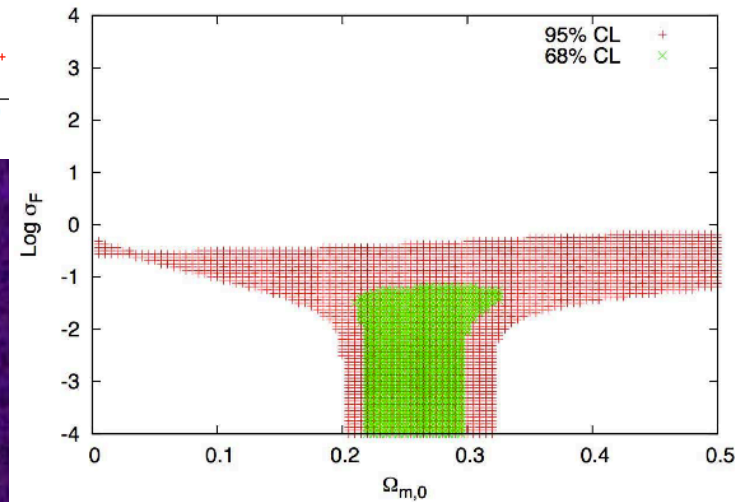
$$2 \ln p(\mathbf{y}|\mathbf{f}) = -\mathbf{y}^T \Sigma_{00}(\mathbf{Z}, \mathbf{Z})^{-1} \mathbf{y} - \ln \det \Sigma_{00}(\mathbf{Z}, \mathbf{Z}) - n \ln(2\pi),$$

GP Likelihood

Detection of the features in the residuals



GP to test GR
Shafieloo, Kim, Linder, PRD 2013



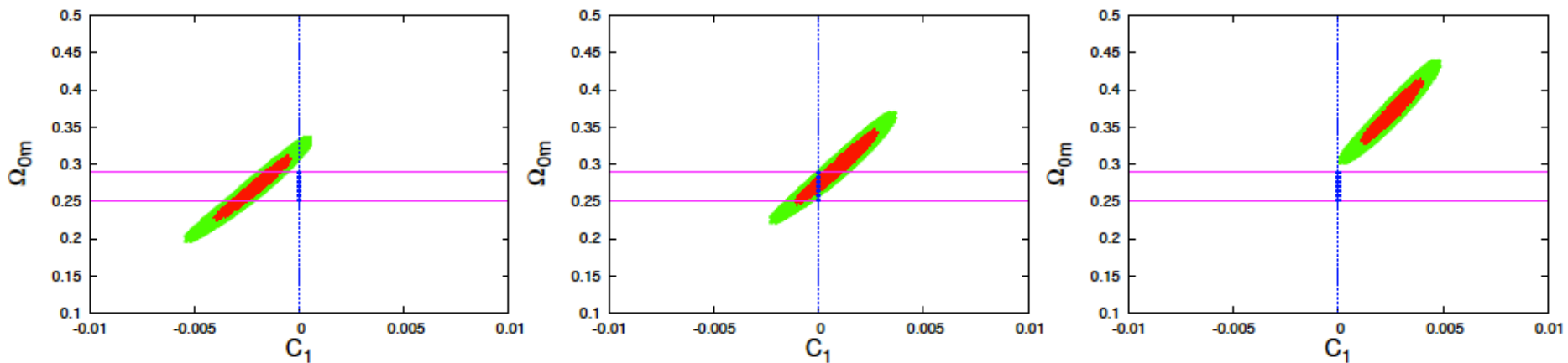
Crossing Statistic (Bayesian Interpretation)

Theoretical model

Crossing function

Comparing a model
with its own variations

$$\mu_M^{T_N}(z) = \mu_M(p_i, z) \times T_N(C_1, \dots, C_N, z)$$



$$T_I(C_1, z) = 1 + C_1 \left(\frac{z}{z_{max}} \right)$$

Chebyshev Polynomials
as Crossing Functions

$$T_{II}(C_1, C_2, z) = 1 + C_1 \left(\frac{z}{z_{max}} \right) + C_2 \left[2 \left(\frac{z}{z_{max}} \right)^2 - 1 \right],$$

Shafieloo, JCAP 2012 (a)

Shafieloo, JCAP 2012 (b)

Crossing Statistic (Bayesian Interpretation)

Theoretical model

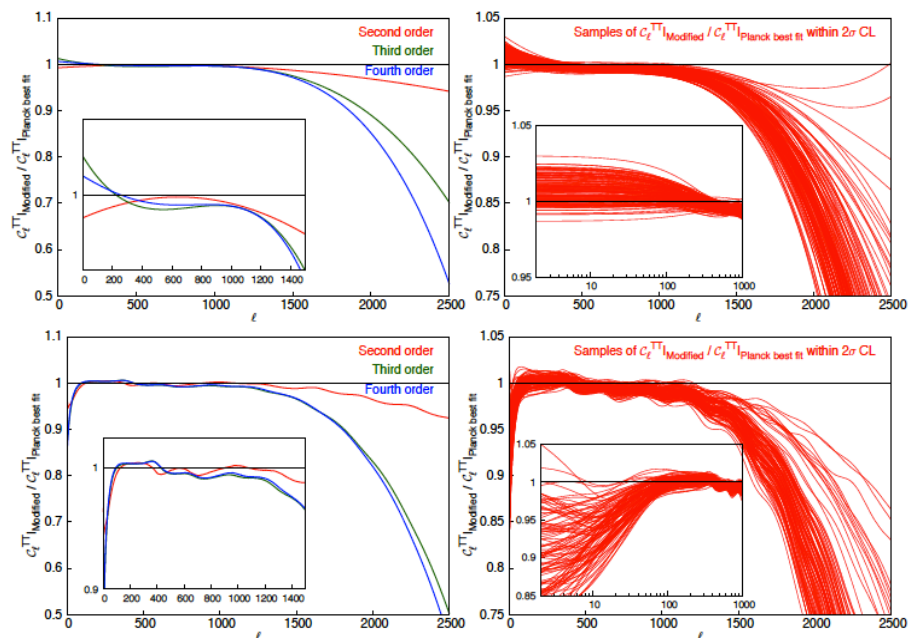
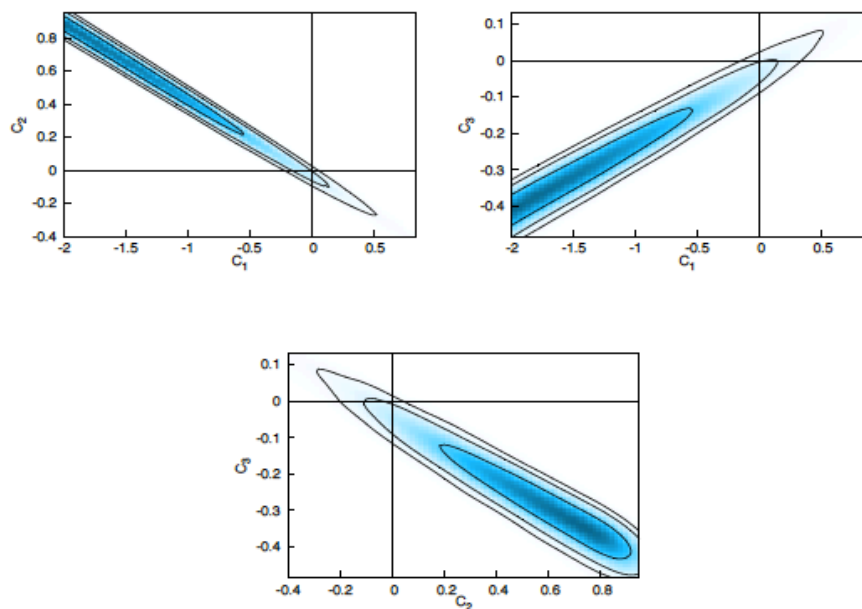
Crossing function

$$C_{\ell}^{\text{TT}}|_{\text{modified}}^N = C_{\ell}^{\text{TT}}|_{\Omega_b, \Omega_{\text{CDM}}, H_0, \tau, A_S, n_S, \ell} \times T_i(C_0, C_1, C_2, \dots, C_N, \ell).$$

Confronting the concordance model of cosmology with Planck data

Hazra and Shafieloo, JCAP 2014

Consistent only at 2~3 sigma CL



Dates

Issue 01 (January 2014)

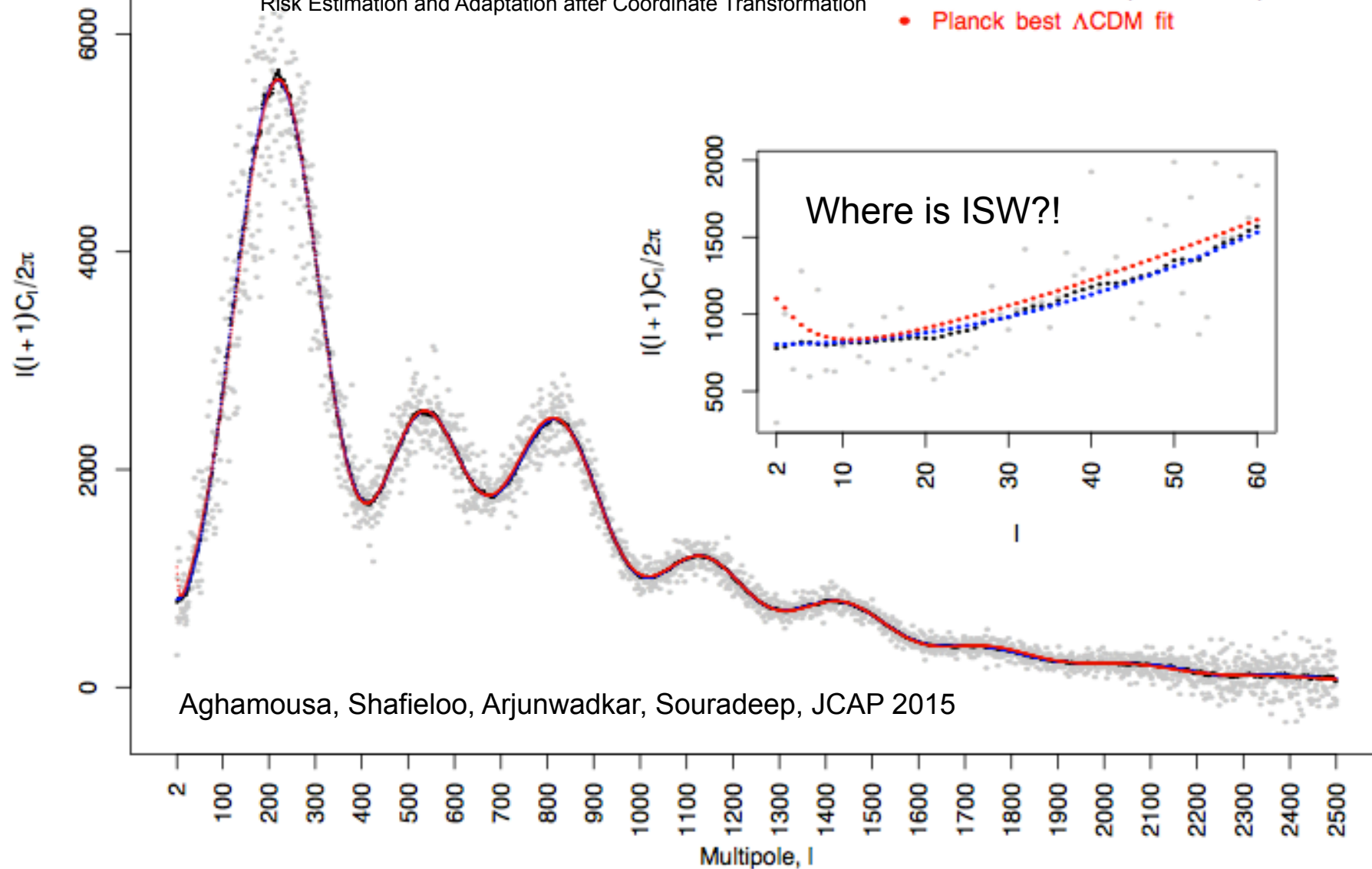
Received 13 January 2014, accepted for publication 14 January 2014

Published 28 January 2014

REACT Non-parametric fit

Risk Estimation and Adaptation after Coordinate Transformation

- Data
- Restricted - freedom fit (EDoF = 25)
- Full - freedom fit (EDoF = 130)
- Planck best Λ CDM fit



Summary:

- The nature of dark energy is unknown. We just know it exist (?!), long way to understand what it is.
- To study the behavior of dark energy we need to understand the expansion history of the universe and growth of fluctuations.
- Parametric and Non-Parametric approaches are both useful and each has some advantages and some disadvantages over the other one. Best is to combine them.
- First target can be testing the standard 'Vanilla' model. If it is not '*Lambda*' then we can look further. Falsifying DE models and in particular **Cosmological Constant** is more realistic and affordable than reconstructing dark energy and it can have a huge theoretical implications. This explains the importance of null tests like Ω_m , Ω_{mh^2} and Ω_{m3} and falsification methods.

Conclusion (Large Scales)

- Still something like 96% of the universe is missing. Something might be fundamentally wrong.
- We can (will) describe the constituents and pattern of the universe (soon). But still we do not understand it. Next challenge is to move from inventory to understanding, by the help of new generation of experiments.
- We should be happy that there are lots of problems unsolved. Problems that we might be able to solve some of them (to some extend) with our limited intelligence.