Lambda or Not Lambda

2nd APCTP-TUS Workshop on Dark energy

Tokyo University of Science, August 2-5 2015
Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological model.

Initial Conditions:

- **Form of the Primordial Spectrum** is Power-law
- **Dark Energy** is Cosmological Constant
- **Dark Matter** is Cold and weakly Interacting:
  - Baryon density
  - Neutrino mass and radiation density: fixed by assumptions and CMB temperature
- Universe is **Flat**: $\Omega_A = 1 - \Omega_b - \Omega_{dm}$

**Power-law**

- Epoch of reionization $\tau$
- Hubble Parameter and the Rate of Expansion $H_0$
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- Form of the Primordial Spectrum is Power-law
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  - Assumptions and CMB temperature
- Universe is Flat

Combination of Assumptions

Dark Energy is *Cosmological Constant*:

\[ \Omega_A = 1 - \Omega_b - \Omega_{dm} \]

Epoch of reionization

Hubble Parameter and the Rate of Expansion

\[ H_0, \tau \]
Why such assumptions?
Hints from Cosmological Observations

1991-94
COBE

2001-2010
WMAP

2009-2011
PLANCK

2020+
CMBPol/COrE
Statistics of CMB

CMB Anisotropy Sky map => Spherical Harmonic decomposition

\[ \Delta T(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi) \]

\[ \langle a_{lm} a_{l'm'}^* \rangle = C_l \delta_{ll'} \delta_{mm'} \]

Gaussian Random field => Completely specified by

**angular power spectrum** \( l(l+1)C_l \):

Power in fluctuations on angular scales of \( \sim \pi/l \)
Fig. 1.1. The Baryonic Acoustic Peak (BAP) in the correlation function – the BAP is visible in the clustering of the SDSS LRG galaxy sample, and is sensitive to the matter density (shown are models with $\Omega_m h^2 = 0.12$ (top), 0.13 (second) and 0.14 (third), all with $\Omega_b h^2 = 0.024$). The bottom line without a BAP is the correlation function in the pure CDM model, with $\Omega_b = 0$. From Eisenstein et al., 2005 (52).

$$P(k) = \int_{-\infty}^{\infty} \xi(r) \exp(-ikr)r^2dr.$$
Fig. 1.5. Rings of power superposed. Schematic galaxy distribution formed by placing the galaxies on rings of the same characteristic radius $L$. The preferred radial scale is clearly visible in the left hand panel with many galaxies per ring. The right hand panel shows a more realistic scenario - with many rings and relatively few galaxies per ring, implying that the preferred scale can only be recovered statistically.

$$P(k) = \int_{-\infty}^{\infty} \xi(r) \exp(-ikr) r^2 dr.$$
Measuring Distances in Astronomy

SNe Ia: Standardized Candles

distance-redshift measurements

must stretch by a factor of 1.83 to match; so SN 1997ap is at a redshift of 0.83

Very low redshift SNe Ia

Day $-2 \pm 2$

SN 1981B

SN 1997ap at $z = 0.83$

intensity

5000 10000 15000

wavelength (Angstroms, $10^{-10}$ meters)
Union 2.1 supernovae Ia Compilation

Constreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)

Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)

combination of *reasonable* assumptions, but.....
Beyond the Standard Model of Cosmology

• The universe might be more complicated than its current standard model (Vanilla Model).
• There might be some extensions to the standard model in defining the cosmological quantities.
• This needs proper investigation, using advanced statistical methods, high performance computational facilities and high quality observational data.
Standard Model of Cosmology
Mid 90's: Indirect evidences were seen in the distribution of the galaxies where SCDM could not explain the excess of power at large scales.

1998: Direct evidence came by Supernovae Type Ia Observations. Going to higher redshifts, supernovae are fainter than expected. One can explain this only (?!=Nobel Prize) by considering an accelerating universe.
Or better to say, ruling out zero-$\Lambda$ Universe

Hazra, Shafieloo, Souradeep, PRD 2013
Free PPS, No H0 Prior
FLAT LCDM
Non FLAT LCDM
Power-Law PPS

Union 2.1 SN Ia Compilation

WiggleZ BAO

D. Sherwin et.al, PRL 2011
Something seems to be there, but, what is it?
But which one is really responsible for the acceleration of the expanding universe?!
Universe is Accelerating

There are two models here!

Universe is not Accelerating

- Blue region: The expansion of the universe slowed down for a long time and then, with dark energy, sped up.
- Gold region: The expansion of the universe always slowed down after the first fraction of a second.
To find cosmological quantities and parameters, there are two general approaches:

1. Parametric methods
   - Easy to confront with cosmological observations to put constraints on the parameters, but the results are highly biased by the assumed models and parametric forms.

2. Non Parametric methods
   - Difficult to apply properly on the raw data, but the results will be less biased and more reliable and independent of theoretical models or parametric forms.

Reconstructing Dark Energy
Problems of Dark Energy Parameterizations

Chevallier-Polarski-Linder ansatz (CPL).

Brane Model
Quintessence DE?! Kink Model
Phantom DE?!

\[ w(z) = w_0 - w_a \frac{z}{1 + z}. \]
Model independent reconstruction of the expansion history

Crossing Statistic + Smoothing

Gaussian Processes

Shafieloo, JCAP (b) 2012
Shafieloo, Kim & Linder, PRD 2012
Dealing with observational uncertainties in matter density (and curvature)

- Small uncertainties in the value of matter density affect the reconstruction exercise dramatically.
- Uncertainties in matter density is in particular bound to affect the reconstructed $w(z)$.

\[
H(z) = \left[ \frac{d}{dz} \left( \frac{d_L(z)}{1+z} \right) \right]^{-1}
\]

\[
\omega_{DE} = \frac{\left( \frac{2(1+z)}{3} \frac{H'}{H} \right) - 1}{1 - \left( \frac{H_0}{H} \right)^2 \Omega_{0M} (1+z)^3}
\]
$\Omega_{0m}^{\text{erroneous}} = 0.22$

$\Omega_{0m}^{\text{true}} = 0.27$

$\Omega_{0m}^{\text{erroneous}} = 0.32$

Cosmographic Degeneracy

\[
d_L(z) = \frac{1 + z}{\sqrt{1 - \frac{\Omega_m}{\Omega_{\text{de}}}}} \sinh \left( \sqrt{1 - \frac{\Omega_m}{\Omega_{\text{de}}}} \int_0^z \frac{dz'}{h(z')} \right)
\]

\[
h(z)^2 \equiv \left( \frac{H(z)}{H_0} \right)^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \Omega_m (1 + z)^3 + (1 - \Omega_m - \Omega_{\text{de}})(1 + z)^2 + \Omega_{\text{de}} \exp \left[ 3 \int_0^z \frac{dz'}{1 + z'} \left[ 1 + w(z') \right] \right],
\]
Cosmographic Degeneracy

• Cosmographic Degeneracies would make it so hard to pin down the actual model of dark energy even in the near future.

Indistinguishable from each other!

Shafieloo & Linder, PRD 2011

\[
\omega_{DE} = \frac{\frac{2(1+z)}{3} \frac{H'}{H} - 1}{1 - \left(\frac{H_0}{H}\right)^2 \Omega_{0M} (1+z)^3}
\]
Considering (low) quality of the data and cosmographic degeneracies we should consider a new strategy sidewise to reconstruction: Falsification.

Yes-No to a hypothesis is easier than characterizing a phenomena. We should look for special characteristics of the standard model and relate them to observables.

But, How?
Instead of looking for \( w(z) \) and exact properties of dark energy at the current status of data, we can concentrate on a more reasonable problem: \( \Lambda \) or NOT \( \Lambda \).

**Falsification of Cosmological Constant**

Yes-No to a hypothesis is easier than characterizing a phenomena.
\[ w(z) = -0.7 \]

\[ w(z) = -1.3 \]

\[ H^2(z) = H_0^2 \left[ \Omega_{0m} (1+z)^3 + \Omega_{DE} \right] \]

\[ \Omega_{DE} = (1-\Omega_{0m}) \exp \left\{ 3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right\} \]
Falsification: Null Test of Lambda

\[ Om(z) = \frac{h^2(z) - 1}{(1 + z)^3} - 1 \]

We Only Need \( h(z) \)

\[ h(z) = \frac{H(z)}{H_0} \]

\( Om(z) \) is constant only for FLAT LCDM model

- \( w = -1 \rightarrow Om(z) = \Omega_{om} \)
- \( w < -1 \rightarrow Om(z) < \Omega_{om} \)
- \( w > -1 \rightarrow Om(z) > \Omega_{om} \)

Phantom
- \( w = -1.1 \)

Quintessence
- \( w = -0.9 \)
Om diagnostic is very well established.
A null diagnostic customized for reconstructing the properties of dark energy directly from BAO data

\[ \frac{O \text{m} 3(z_i,z_2,z_3)}{O \text{m}(z_3,z_1)} = \frac{h^2(z_2) - h^2(z_1)}{(1+z_2)^3 - (1+z_1)^3} = \frac{h^2(z_3) - h^2(z_1)}{(1+z_3)^3 - (1+z_1)^3} = \frac{H^2(z_2)}{H_0^2} - 1 = \frac{H^2(z_3)}{H_0^2} - 1 = \frac{H^2(z_2)}{H_0^2} - 1 = \frac{H^2(z_3)}{H_0^2} - 1 \]

\[ d(z) = \frac{r_s(z \text{CMB})}{D_V(z)} \]

Observables

\[ H(z_i; z_j) := \frac{H(z_i)}{H(z_j)} = \frac{z_i}{z_j} \left[ \frac{D(z_i)}{D(z_j)} \right]^2 \left[ \frac{D_V(z_j)}{D_V(z_i)} \right]^3 = \frac{z_i}{z_j} \left[ \frac{D(z_i)}{D(z_j)} \right]^2 \left[ \frac{d(z_i)}{d(z_j)} \right]^3 \]

Shafieloo, Sahni, Starobinsky, PRD 2013
Om is constant only for Flat LCDM model
Om3 is equal to one for Flat LCDM model

$$\text{Om3}(z_1; z_2; z_3) = \frac{H(z_2; z_1)^2 - 1}{x^3_2 - x^3_1} \div \frac{H(z_3; z_1)^2 - 1}{x^3_3 - x^3_1}, \text{ where } x = 1 + z,$$

$$H(z_i; z_j) = \left(\frac{z_j}{z_i}\right)^2 \left[\frac{D(z_i)}{D(z_j)}\right]^2 \left[\frac{A(z_j)}{A(z_i)}\right]^3 = \frac{z_i}{z_j} \left[\frac{D(z_i)}{D(z_j)}\right]^2 \left[\frac{d(z_i)}{d(z_j)}\right]^3,$$

Om3 is independent of H0 and the distance to the last scattering surface and can be derived directly using BAO observables.

Shafieloo, Sahni, Starobinsky, PRD 2013
Om is constant only for Flat LCDM model
Om3 is equal to one for Flat LCDM model

\[ Om3(z_1; z_2; z_3) = \frac{H(z_2; z_1)^2 - 1}{x_2^3 - x_1^3} \bigg/ \frac{H(z_3; z_1)^2 - 1}{x_3^3 - x_1^3}, \quad \text{where} \quad x = 1 + z, \]
Model Independent Evidence for Dark Energy Evolution from Baryon Acoustic Oscillation

\[ Omh^2(z_1, z_2) = \frac{H^2(z_2) - H^2(z_1)}{(1 + z_2)^3 - (1 + z_1)^3} = \Omega_{0m}H_0^2 \]

Only for LCDM


A very recent result.

Important discovery if no systematic in the SDSS Quasar BAO data

\[ Omh^2 = 0.1426 \pm 0.0025 \]

\[ Omh^2(z_1; z_2) = 0.124 \pm 0.045 \]

\[ Omh^2(z_1; z_3) = 0.122 \pm 0.010 \]

\[ Omh^2(z_2; z_3) = 0.122 \pm 0.012 \]
Gaussian Processes:

Modeling of the data around a mean function searching for likely features by looking at the likelihood space of the hyperparameters.

Bayesian Interpretation of Crossing Statistic:

Comparing a model with its own possible variations.

REACT:

Risk Estimation and Adaptation after Coordinate Transformation
Efficient in statistical modeling of stochastic variables
Derivatives of Gaussian Processes are Gaussian Processes
Provides us with all covariance matrices

\[
\begin{bmatrix}
y \\
f \\
f' \\
f''
\end{bmatrix} \sim \mathcal{N}
\begin{bmatrix}
m(Z) \\
m'(Z_1) \\
m''(Z_1)
\end{bmatrix}, \begin{bmatrix}
\Sigma_{00}(Z, Z) & \Sigma_{00}(Z, Z_1) & \Sigma_{01}(Z, Z_1) & \Sigma_{02}(Z, Z_1) \\
\Sigma_{00}(Z_1, Z) & \Sigma_{00}(Z_1, Z_1) & \Sigma_{01}(Z_1, Z_1) & \Sigma_{02}(Z_1, Z_1) \\
\Sigma_{10}(Z, Z) & \Sigma_{10}(Z_1, Z) & \Sigma_{11}(Z_1, Z_1) & \Sigma_{12}(Z_1, Z_1) \\
\Sigma_{20}(Z_1, Z) & \Sigma_{20}(Z_1, Z_1) & \Sigma_{21}(Z_1, Z_1) & \Sigma_{22}(Z_1, Z_1)
\end{bmatrix}
\]

\[
\begin{bmatrix}
f' \\
f''
\end{bmatrix} = \begin{bmatrix}
m(Z_1) \\
m'(Z_1) \\
m''(Z_1)
\end{bmatrix} + \begin{bmatrix}
\Sigma_{00}(Z_1, Z) \\
\Sigma_{10}(Z_1, Z) \\
\Sigma_{20}(Z_1, Z)
\end{bmatrix} \Sigma_{00}^{-1}(Z, Z) y
\]

\[
\text{Cov} \left( \begin{bmatrix}
f \\
f' \\
f''
\end{bmatrix} \right) = \begin{bmatrix}
\Sigma_{00}(Z_1, Z_1) & \Sigma_{01}(Z_1, Z_1) & \Sigma_{02}(Z_1, Z_1) \\
\Sigma_{10}(Z_1, Z_1) & \Sigma_{11}(Z_1, Z_1) & \Sigma_{12}(Z_1, Z_1) \\
\Sigma_{20}(Z_1, Z_1) & \Sigma_{21}(Z_1, Z_1) & \Sigma_{22}(Z_1, Z_1)
\end{bmatrix} - \begin{bmatrix}
\Sigma_{00}(Z_1, Z) \\
\Sigma_{10}(Z_1, Z) \\
\Sigma_{20}(Z_1, Z)
\end{bmatrix} \Sigma_{00}^{-1}(Z, Z) \left[ \Sigma_{00}(Z, Z_1), \Sigma_{01}(Z, Z_1), \Sigma_{02}(Z, Z_1) \right] \Sigma_{00}^{-1}(Z, Z)
\]

\[
2 \ln p(y|f) = -y^T \Sigma_{00}(Z, Z)^{-1} y - \ln \det \Sigma_{00}(Z, Z) - n \ln(2\pi)
\]
Detection of the features in the residuals

Simulations

Signal Detectable

Signal Undetectable

GP to test GR
Shafieloo, Kim, Linder, PRD 2013
\[
\mu_M^T(z) = \mu_M(p_i, z) \times T_N(C_1, ..., C_N, z)
\]

Comparing a model with its own variations

Chebyshev Polynomials as Crossing Functions

\[
T_I(C_1, z) = 1 + C_1 \left( \frac{z}{z_{\text{max}}} \right)
\]

\[
T_{II}(C_1, C_2, z) = 1 + C_1 \left( \frac{z}{z_{\text{max}}} \right) + C_2 \left[ 2 \left( \frac{z}{z_{\text{max}}} \right)^2 - 1 \right]
\]

Shafieloo. JCAP 2012 (a)
Shafieloo, JCAP 2012 (b)
Crossing Statistic

\[ C_{\ell}^{TT}_{\text{modified}} = C_{\ell}^{TT}_{\Omega_b,\Omega_{CDM},H_0,\tau,A_s,n_S,\ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell). \]

Consistent only at 2~3 sigma CL
REACT Non-parametric fit

Risk Estimation and Adaptation after Coordinate Transformation

Where is ISW?!

Aghamousa, Shafieloo, Arjunwadkar, Souradeep, JCAP 2015
• The nature of dark energy is unknown. We just know it exist (?!), long way to understand what it is.

• To study the behavior of dark energy we need to understand the expansion history of the universe and growth of fluctuations.

• Parametric and Non-Parametric approaches are both useful and each has some advantages and some disadvantages over the other one. Best is to combine them.

• First target can be testing the standard ‘Vanilla’ model. If it is not ‘\textit{Lambda}’ then we can look further. Falsifying DE models and in particular \textit{Cosmological Constant} is more realistic and affordable than reconstructing dark energy and it can have a huge theoretical implications. This explains the importance of null tests like Om, Omh2 and Om3 and falsification methods.
• We can (will) describe the constituents and pattern of the universe (soon). But still we do not understand it. Next challenge is to move from inventory to understanding, by the help of new generation of experiments.