On bigravity model: Graviton Oscillation in a viable bigravity model

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Based on

PTEP2014 043E01 De Felice, Nakamura and TT

JCAP 1406 (2014) 004 Yamashita and TT

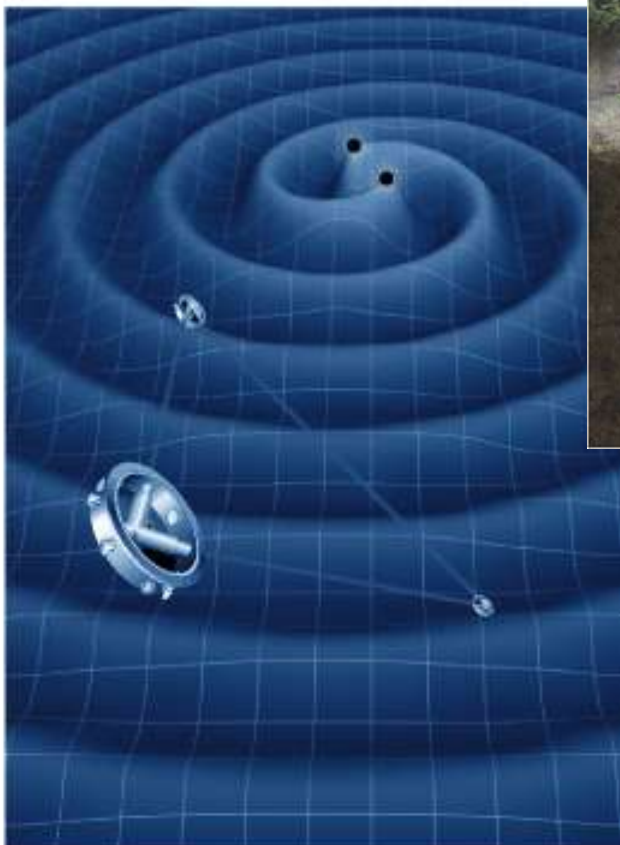
JCAP1406 (2014) 037 De Felice, Gumrukcuoglu, Mukohyama, Tanahashi and TT

Int. J. Mod. Phys. D23 (2014) 1443003 Yamashita, De Felice and TT

Phys. Rev. D91 (2015) 062007 Narikawa, Tagoshi, TT, Kanda and Nakamura

arXiv1508.XXXXX: Yamashita and TT

Gravitation wave detectors

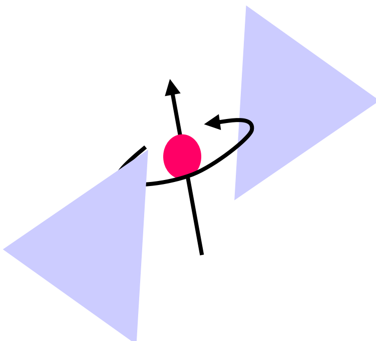


eLISA(NGO)
⇒DECIGO/BBO

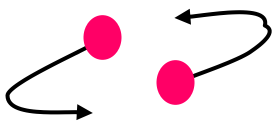
LIGO⇒adv LIGO

Test of GW generation

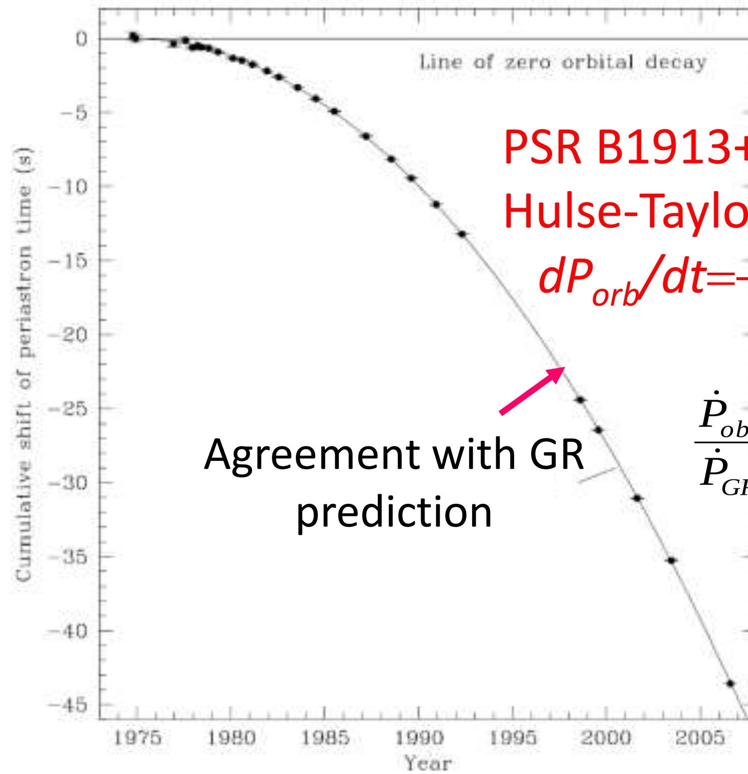
Pulsar : ideal clock



Test of GR by pulsar binaries



Periastron advance due to GW emission



PSR B1913+16

Hulse-Taylor binary

$dP_{orb}/dt = -2.423 \times 10^{-12}$

Agreement with GR prediction

$$\frac{\dot{P}_{obs}}{\dot{P}_{GR}} = 0.997 \pm 0.002$$

(J.M. Weisberg, Nice and J.H. Taylor, arXiv:1011.0718)

We know that GWs are emitted from binaries.

What is the possible big surprise when we directly detect GWs?

Is there possibility that graviton disappear during its propagation over cosmological distance?

Bi-gravity

$$L = \frac{\sqrt{-g}R}{16\pi G_N} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi G_N \kappa} + L_{matter}(g, \phi) + \cdots$$

Both massive and massless gravitons exist.

$\rightarrow \nu$ oscillation-like phenomena?

First question is whether or not we can construct a viable cosmological model.

Ghost free bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{2} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa} + \frac{\sqrt{-g}}{2}\sum_{n=0}^{4} c_n V_n + \frac{L_{matter}}{M_G^2}$$

$$V_0 = 1, \quad V_1 = \tau_1, \quad V_2 = \tau_1^2 - \tau_2, \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \qquad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

$\tilde{g}$ is promoted to a dynamical field.

Even in this case, it was shown that the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

$$ds^2 = a^2(t)\left(-dt^2 + dx^2\right)$$

$$d\tilde{s}^2 = b^2(t)\left(-c^2(t)dt^2 + dx^2\right)$$

$$T_{\mu\nu}^{(mass)} = 2\frac{\delta S^{(mass)}}{\delta g^{\mu\nu}}$$

$$\xi \equiv b/a$$

$$\nabla^\mu T_{\mu\nu}^{(mass)} = 0 \Rightarrow \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 :

Pathological: At the linear perturbation, expected scalar and vector perturbations are absent. Strong coupling? Unstable for the homogeneous anisotropic mode.

branch 2 :

Healthy: All perturbation modes are equipped.

Branch 2 background

$$\underbrace{\left(6c_3\xi^2+4c_2\xi+c_1\right)}_{\text{branch 1}}\underbrace{\left(cba'-ab'\right)}_{\text{branch 2}}=0 \qquad \xi\equiv b/a$$

branch 1 branch 2

branch 2 :

$$\rho-\frac{c_1}{\kappa\xi}+\left(c_0-\frac{6c_2}{\kappa}\right)+\left(3c_1-\frac{18c_3}{\kappa}\right)\xi+\left(6c_2-\frac{24c_4}{\kappa}\right)\xi^2+6c_3\xi^3=0$$

ξ becomes a function of ρ. $\xi\to\xi_c$ for $\rho\to 0$.

$$H^2=\frac{\rho+\rho_{mass}}{3M_G^2} \qquad \rho_{mass}:=c_0+3c_1\xi+6c_2\xi^2+6c_3\xi^3$$

effective energy density due to mass term

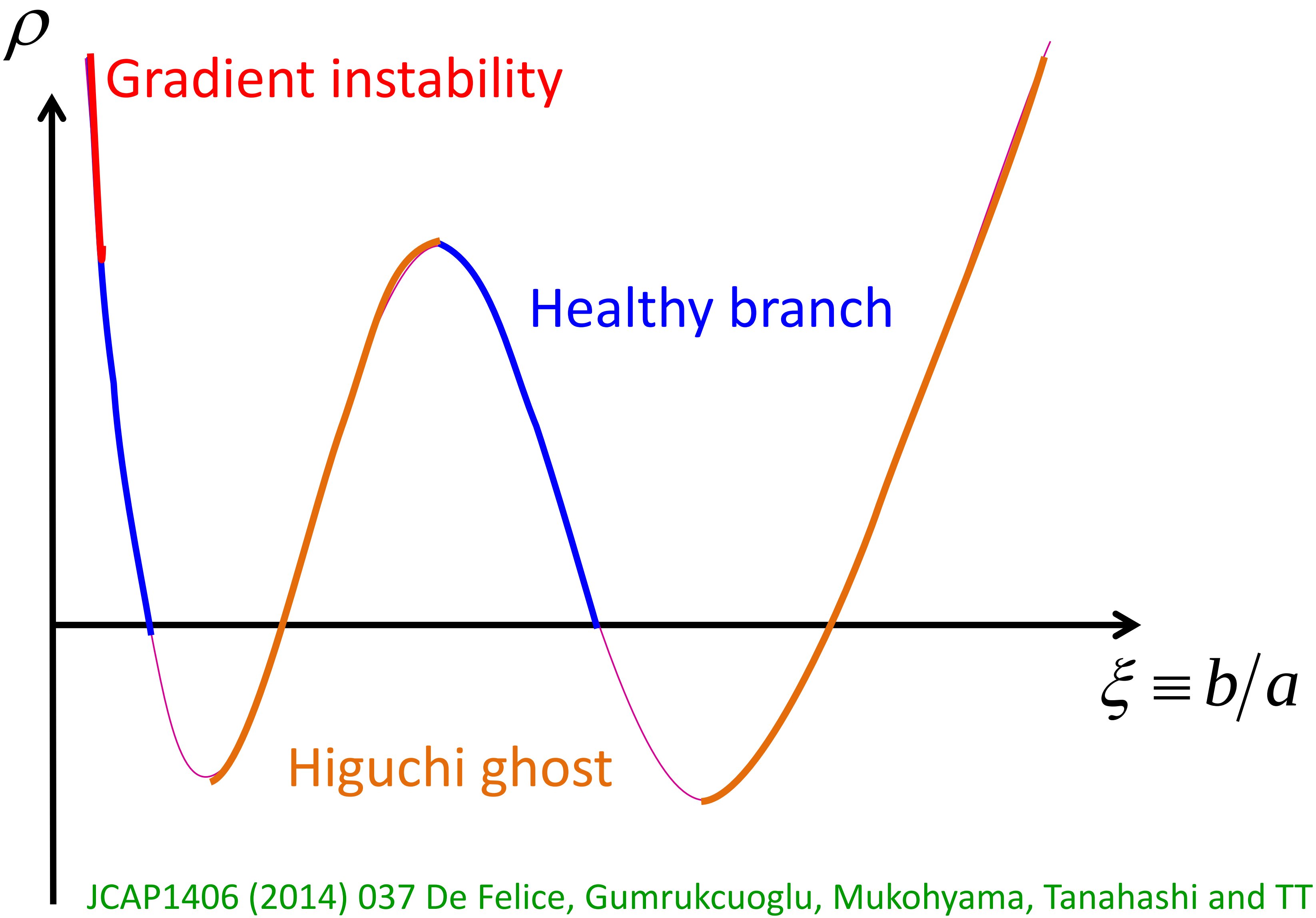
$$\frac{1}{c-1}\frac{\xi'}{\xi}=\frac{a'}{a} \quad\Rightarrow\quad c-1=\frac{3(\rho+P)\kappa\xi_c}{\Gamma_c\left(1+\kappa\xi_c^2\right)M_G^2} \qquad \Gamma=\frac{d\rho_{mass}}{d\xi}$$

Natural Tuning to c=1 for $\rho\to 0$.

$$H^2=\frac{\rho}{3\left(1+\kappa\xi_c^2\right)M_G^2}$$

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

Stability of linear perturbation



JCAP1406 (2014) 037 De Felice, Gumrukcuoglu, Mukohyama, Tanahashi and TT

Gravitational potential around a star

(PTEP2014 043E01 De Felice, Nakamura and TT)

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R, and truncating at the second order

$$\Rightarrow \quad \left(\Delta - \mu^2\right)u - \frac{A}{\mu^2}\left((\Delta u)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$$A \propto \frac{d(\log \Gamma)}{d(\log \xi)}$$ can be tuned to be extremely large.

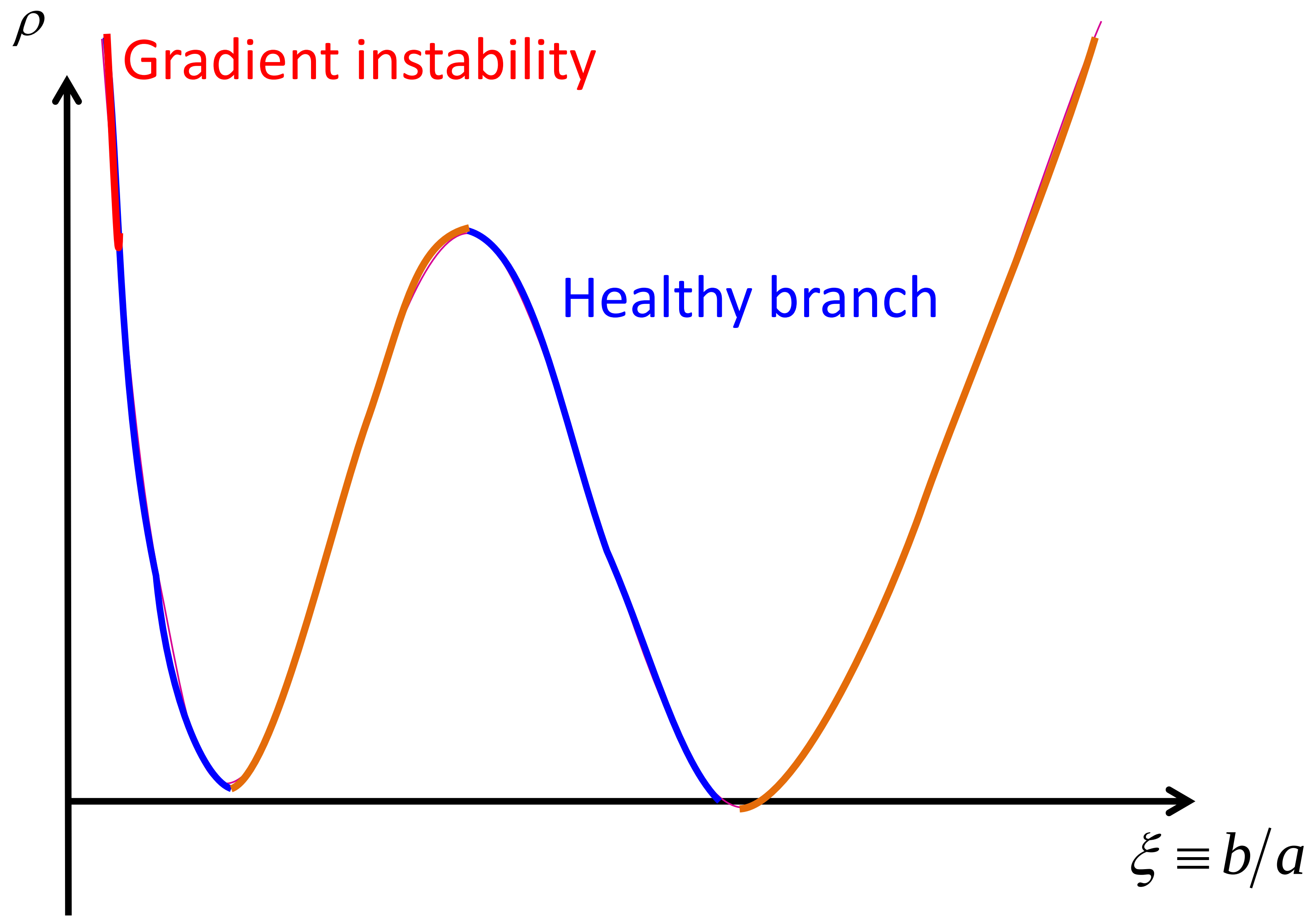
Then, the Vainshtein radius $r_V \approx \left(\frac{A r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} << 300\text{Mpc}$.

Solar system constraint: $\sqrt{A}\mu^{-1} \geq 300\text{Mpc}$

$\Delta v \approx \Delta \tilde{v} \approx \frac{\rho_m}{\tilde{M}_G^2}$ Both v and $\tilde{v}$ are excited as in GR. $H^2 = \frac{\rho}{3\tilde{M}_G^2}$

Meaning of A >>1



Gravitational wave propagation

Short wavelength approximation :

$$k >> m_g >> H$$

$$h'' - \Delta h + m_g^2 \left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' - c^2 \Delta \tilde{h} - \frac{c m_g^2}{\kappa \xi_c^2}\left(h - \tilde{h}\right) = 0$$

$$m_g^2 = \xi\left(c_1 + 2c_2(c+1)\xi + 6cc_3\xi^3\right)$$

(Comelli, Crisostomi, Pilo (2012))

$$\mu^2 := m_g^2 \frac{1+\kappa\xi^2}{\kappa\xi^2}$$

$$k_c := \frac{\mu}{\sqrt{2(c-1)}} \approx 100\text{Hz}\left(\frac{1+\kappa\xi_c^2}{100}\right)^{-1/2}\left(\frac{\mu}{(0.08\,pc)^{-1}}\right)^2$$

k

mass term is important.

Eigenmodes are

$$h + \kappa\xi_c^2 \tilde{h}, \quad h - \tilde{h}$$

modified dispersion relation due to the effect of mass

$c \neq 1$ is important.

Eigenmodes are

$$h, \quad \tilde{h}$$

modified dispersion relation due to different light cone

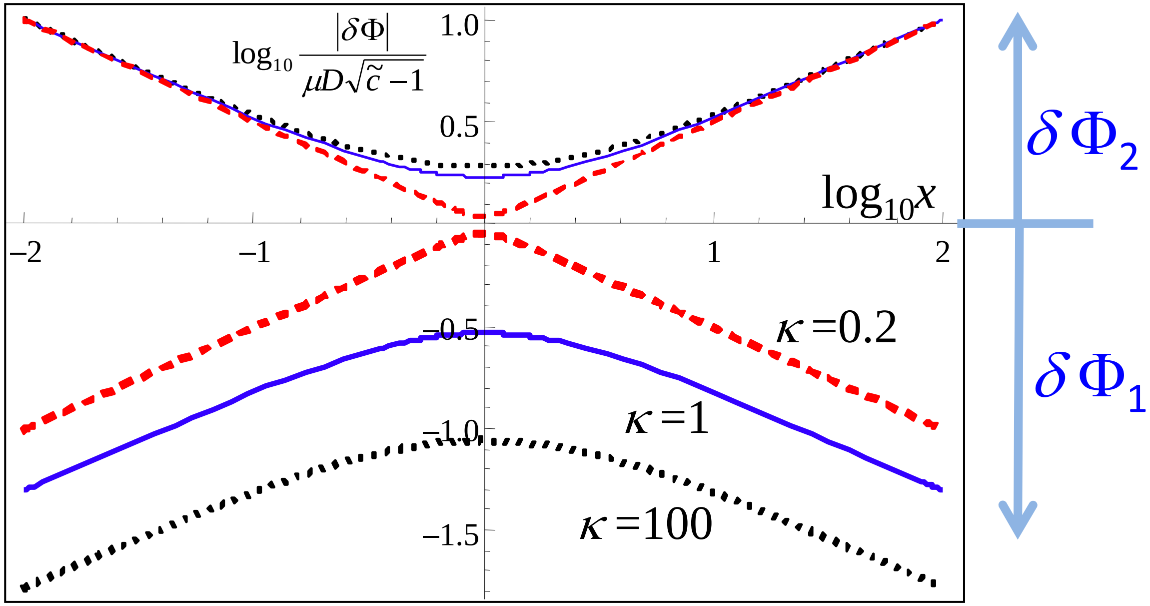
Gravitational wave propagation over a long distance D

Phase shift due to the modified dispersion relation:

$$\delta\Phi_{1,2} \equiv -\frac{\Delta k^2}{2\omega} D = -\frac{\mu D\sqrt{c-1}}{2\sqrt{2x}}\left(1+x\mp\sqrt{1+2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2}+x^2}\right)$$

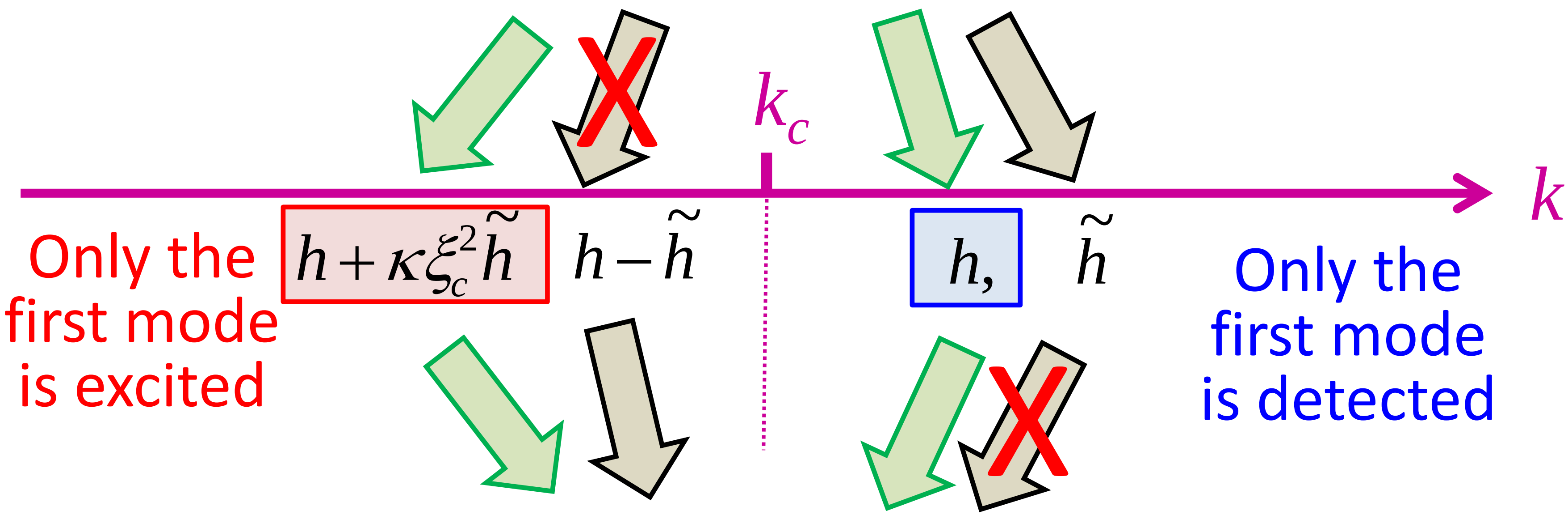
$$\mu D\sqrt{c-1} \approx HD\sqrt{3\left(1+\kappa\xi_c^2\right)\Omega_0}$$

becomes $O(1)$ after propagation over the horizon distance



$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

At the GW generation, both h and $\tilde{h}$ are equally excited.



We can detect only h.

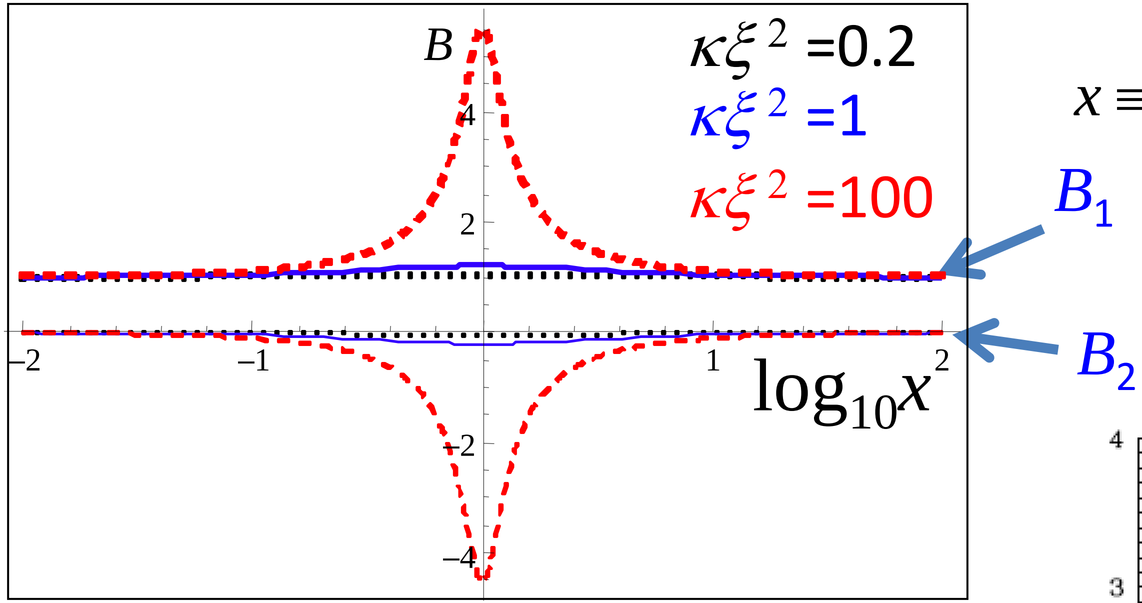
Only modes with $k \sim k_c$ pick up the non-trivial dispersion relation of the second mode.

Interference between two modes ⇒ Graviton oscillations

If the effect appears ubiquitously, the model would be already ruled out.

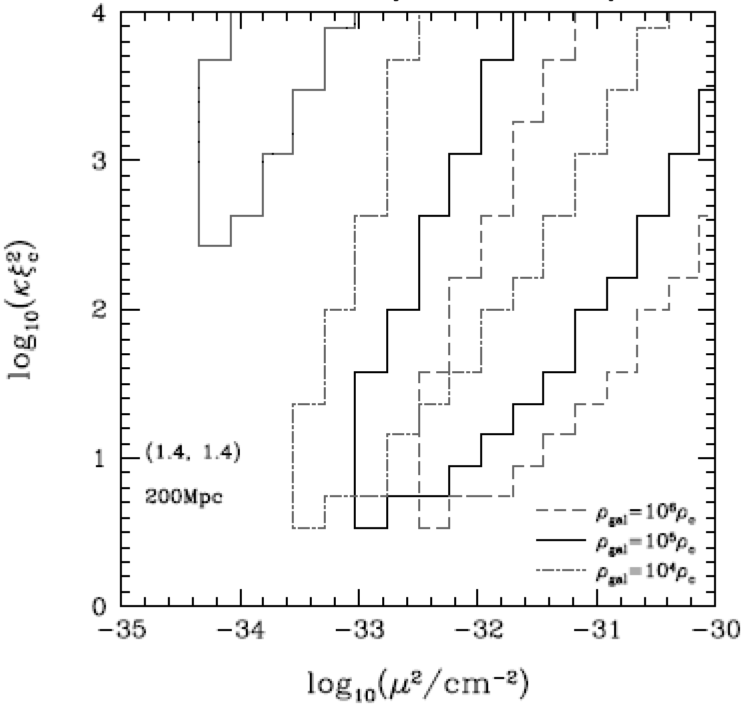
Gravitational wave oscillations

$$h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$



$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

Detectable range of parameters by KAGRA, assuming NS-NS binary at 200Mpc.



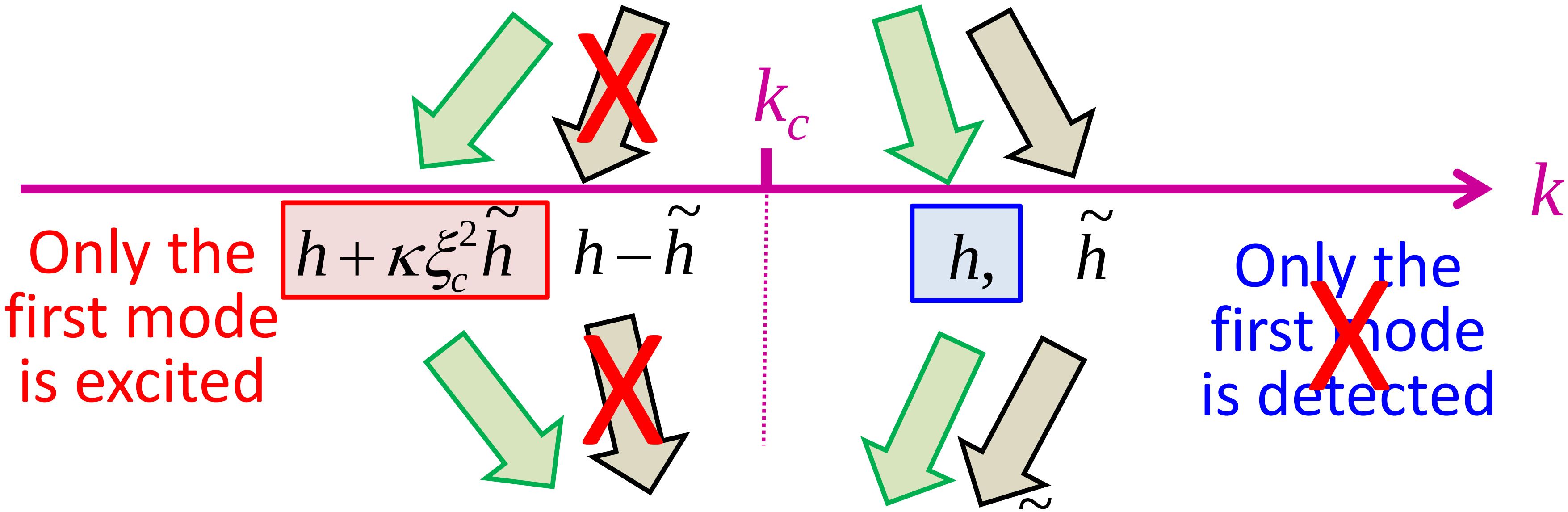
At low frequencies only the first mode is excited.

At high frequencies only the first mode is observed.

(Phys. Rev. D91 (2015) 062007 Narikawa, Tagoshi, TT, Kanda and Nakamura)

If Vainshtein mechanism works for GW detectors…

At the GW generation, both h and $\tilde{h}$ are equally excited.



We detect both h and $\tilde{h}$.

All modes with $k > k_c$ pick up the non-trivial dispersion relation of the second mode.

Interference between two modes ⇒ Graviton oscillations

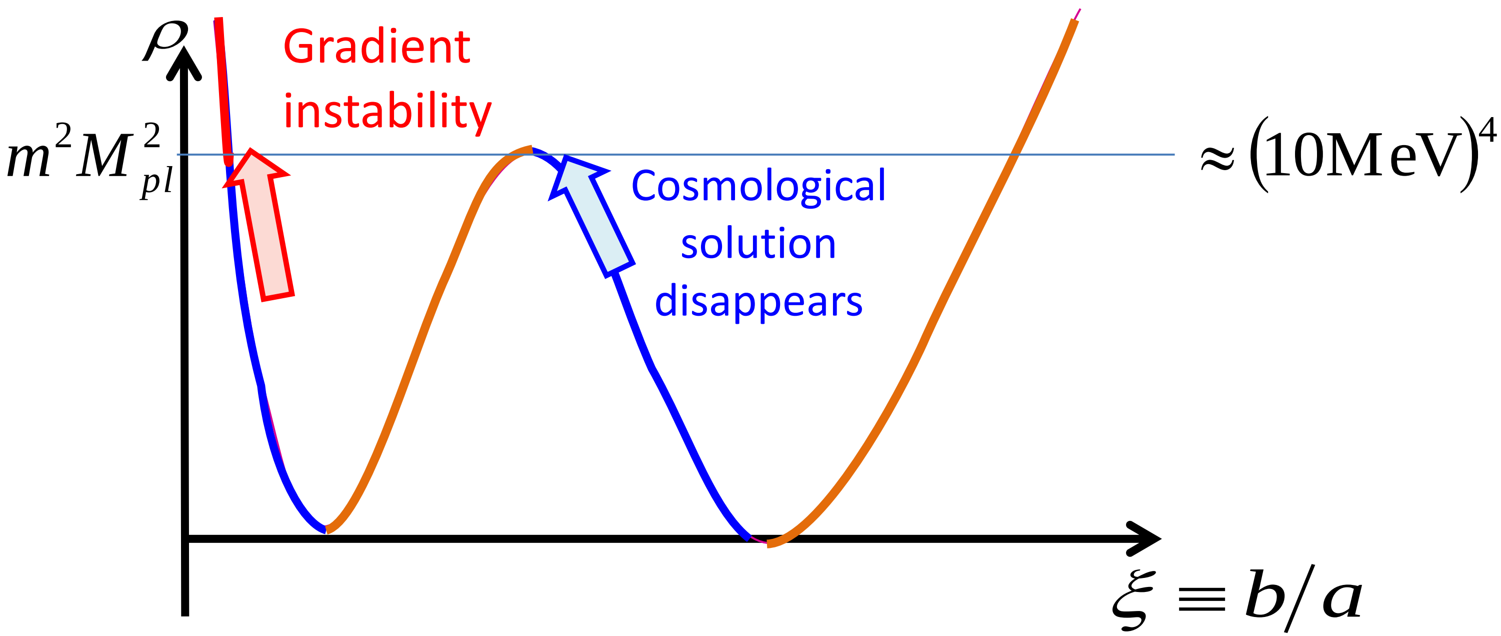
Solar system constraint: $\sqrt{C}\mu^{-1} \geq 300\mathrm{Mpc}$

For detection by aLIGO, aVirgo and KAGRA: $\mu^{-1} \approx 0.1\mathrm{pc}$

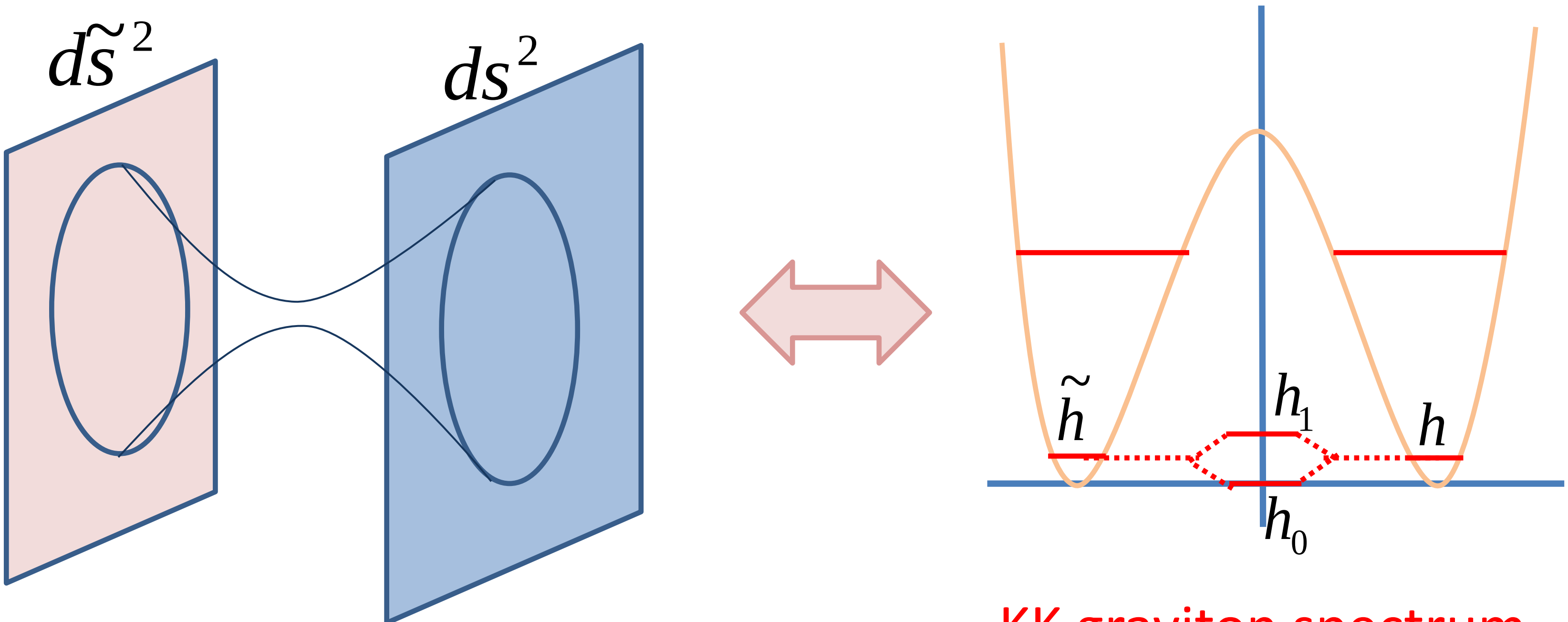
No Vainshtein effect in the inter-galactic space: $\sqrt{C}\mu^{-1} < 3000\mathrm{Mpc}$

⇨ Window is narrow but open.

Bare mass: $m \approx \sqrt{C}\mu \approx (10\mathrm{MeV})^2 / M_G$



Can Bigravity with large A be naturally realized as a low energy effective theory?



Higher dimensional model?!

Matter on right brane couples to h.

KK graviton spectrum
Only first two modes remain at low energy

If the internal space is stabilized ⇒ $d\tilde{s}^2 = \xi_c^2 ds^2$ ⇒ $c = 1$

However, pinched throat configuration looks quite unstable…

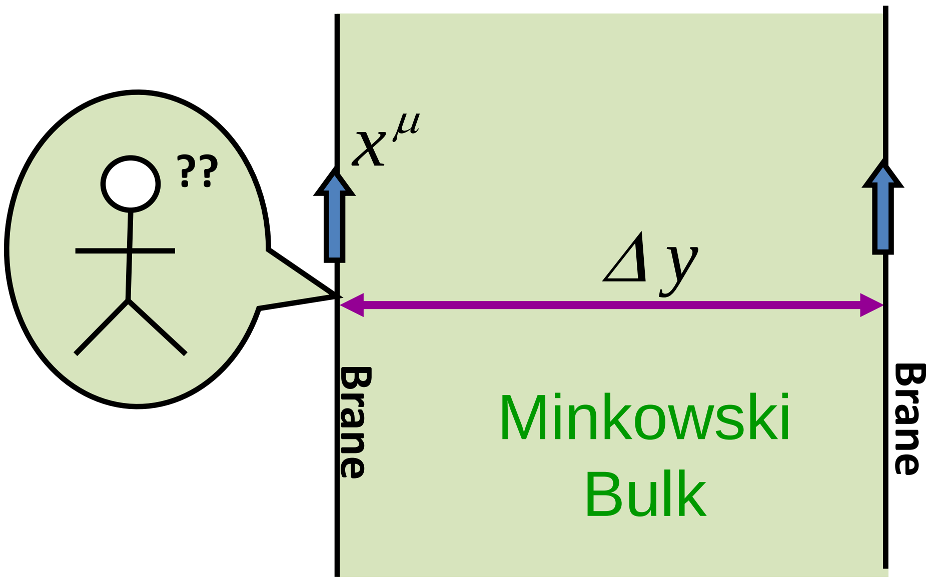
Induced gravity on the branes

Dvali-Gabadadze-Porrati model (2000)

$$S = M_5^3 \int d^5x \sqrt{g} R + \int d^4x \sqrt{-g_{(+)}} \left(M_4^2 R_{(+)} + L_{matt(+)} \right) + \chi \int d^4x \sqrt{-g_{(-)}} \left(M_4^2 R_{(-)} + L_{matt(-)} \right)$$

$$M_5^3 = M_4^2 / 2r_c$$

Critical length scale



- Induced gravity terms play the role of potential well.
- Lowest KK graviton mass $\mu \approx 1/\sqrt{r_c \Delta y}$
- KK graviton mass

$$m \approx 1/\Delta y$$

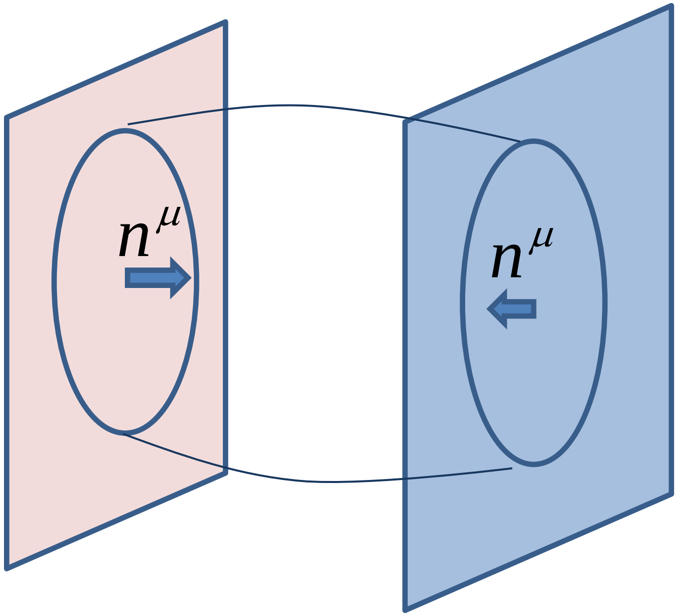
⇨ $\Delta y << r_c$ is required to reproduce bigravity.

(JCAP 1406 (2014) 004 Yamashita and TT)

- To construct a viable model,
  the radion (=brane separation) must be stabilized.
- Radions can be made as heavy as KK gravitons.
- However, the energy density cannot be made large.

Why?

We know that self-accelerating branch has a ghost.



Normal branch condition:

$$r_c n^\mu \partial_\mu (\log a) \equiv r_c K < 2$$

$$K \approx H_{(4D)}$$

$H_{(4D)}$ cannot be made larger than $1/r_c \; (<< \mu)$ in this model.

Deriving bigravity without fine tuning

(Yamashita and TT, in preparation)

- Despite the fore-mentioned limitation, it would be interesting to see how bigravity derives from brane setup without fine tuning of coupling constants.
- We neglect radion stabilization, for simplicity.
- Thus, we consider a system of bigravity with radion

Gradient expansion

We solve the bulk equations of motion for given boundary metrics

$$\frac{1}{N}\partial_y K_{\mu\nu} = -2K^{\rho}_{\mu}K_{\rho\nu} + KK_{\mu\nu} + \frac{4}{l^2}g_{\mu\nu} - R_{\mu\nu} + \frac{1}{N}\nabla_\mu\nabla_\nu N$$

$$K^2 - K^{\nu}_{\mu}K^{\mu}_{\nu} = -\frac{12}{l^2} + R$$

with the scaling assumptions:

$K\Delta y << 1$ $\quad K$:bulk extrinsic curvature

$Kr_c \lesssim 1$ $\quad \Delta y$:Brane separation

$$\mu^2 \sim H^2 \sim \partial^2 \sim l^{-2} \sim 1/r_c\Delta y$$

Substituting back the obtained bulk solution into the action, we obtain at the quadratic order in perturbation

$$S=\frac{M_{pl}^2}{2}\Big[\int d^4x\sqrt{-g_{(+)}}\left(R_{(+)}-6H_{(+)}^2\right)+\chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)}-6H_{(-)}^2\right)$$
$$+\mu_*^2\int d^4x\sqrt{-\gamma}\left\{\Delta g^{\mu\nu}\Delta g_{\mu\nu}-(\Delta g)^2+\frac{3}{4}X\left(1-\frac{\alpha^2H^2}{\mu_*^4}\left(\Box+4H^2\right)\right)X\right\}$$

with

$$X=\Delta g+\frac{2}{3}\frac{\alpha}{\mu_*^2}\left(1-\frac{\alpha^2H^2}{\mu_*^4}\left(\Box+4H^2\right)\right)^{-1}\left(\nabla_\mu\nabla_\nu-\left(\Box+3H^2\right)\gamma_{\mu\nu}\right)\left(h_{(+)}^{\mu\nu}+h_{(-)}^{\mu\nu}-\frac{\alpha H^2}{2}\gamma^{\mu\nu}\Delta g\right)$$

$$\alpha=Kr_c/16 \qquad H_{(\pm)}=H/a_{(\pm)}$$

This looks very complicated but can be recast into the form of bigravity + radion, which is coupled to the averaged metric:

$$S=\frac{M_{pl}^2}{2}\Big[\int d^4x\sqrt{-g_{(+)}}\left(R_{(+)}-6H_{(+)}^2\right)+\chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)}-6H_{(-)}^2\right)$$
$$\mu_*^2\int d^4x\sqrt{-\gamma}\left\{\Delta g^{\mu\nu}\Delta g_{\mu\nu}-(\Delta g)^2-\frac{3}{4}\Phi\left(\Box+4H^2\right)\Phi\right.$$
$$\left.\left.-\frac{3}{4}\Phi\left(\Phi-2\Delta g+\frac{2\alpha}{3\mu_*^2}\underline{\left(\nabla_\mu\nabla_\nu-\left(\Box+3H^2\right)\gamma_{\mu\nu}\right)\left(h_{(+)}^{\mu\nu}+h_{(-)}^{\mu\nu}\right)}\right)\right\}\right]$$
$$=\left(\delta R_{(+)}+\delta R_{(-)}\right)$$

Summary

Gravitational wave observations give us a new probe to modified gravity.

Even graviton oscillations are not immediately denied, and hence we may find something similar to the case of solar neutrino experiment in near future.

Although natural realization of such a scenario is not easy to obtain, there is a possibility that the artificial tuning of model parameters can be explained from the higher dimensional view.