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Possibility of realizing weak gravity in red-shift space distortions

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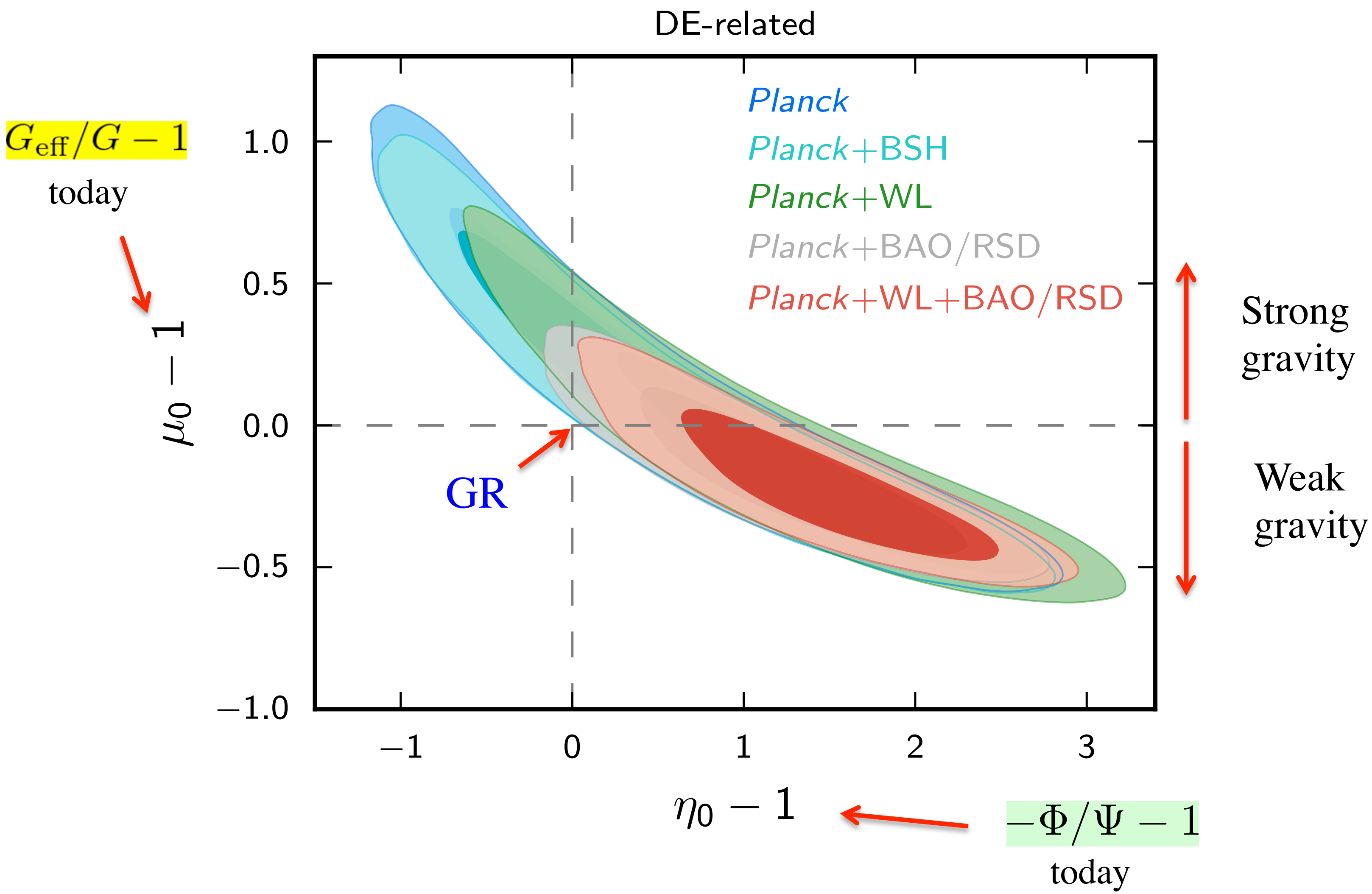
The problem of dark energy and dark matter is an interesting intersection between astronomy and physics!

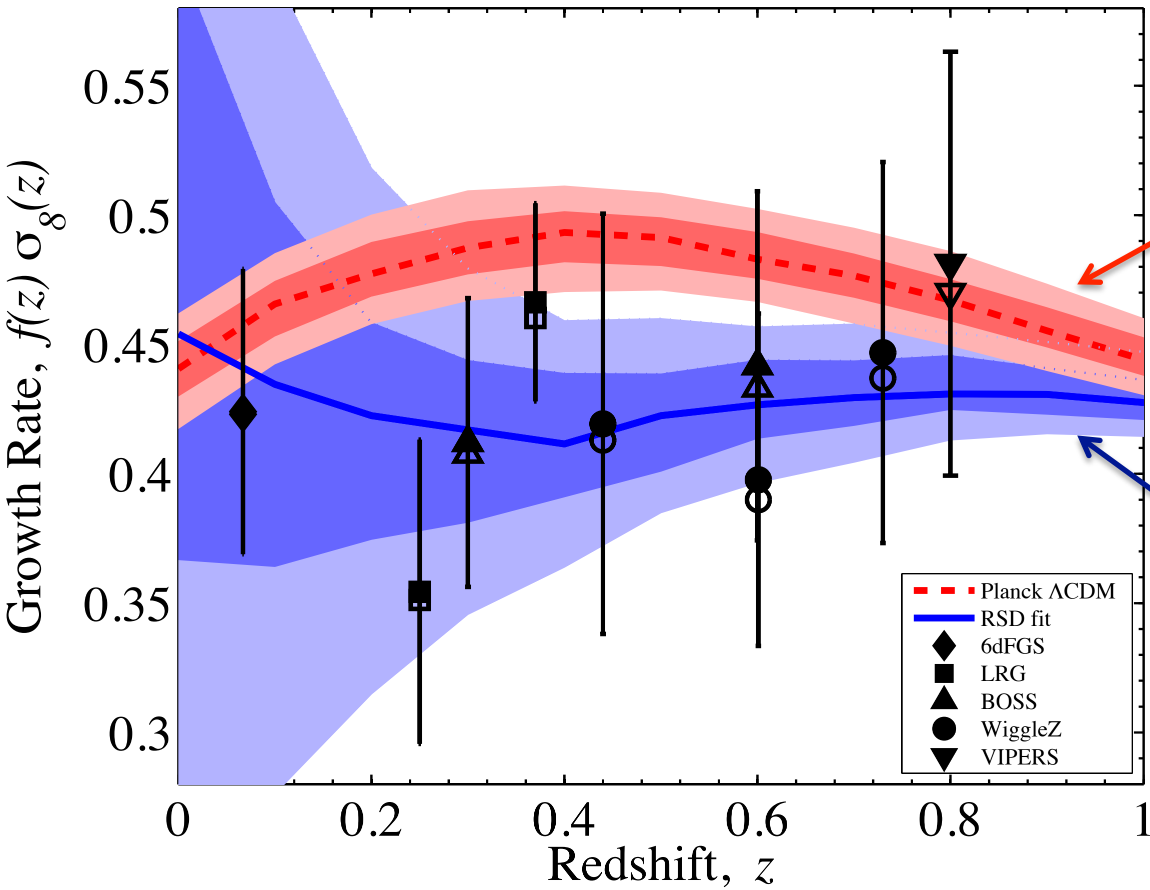
Planck constraints on the effective gravitational coupling and the gravitational slip parameter (Ade et al, 2015)



Weak gravity

The recent observations of redshift-space distortions (RSD) measured the lower growth rate of matter perturbations lower than that predicted by the LCDM model.

Macaulay et al, PRL (2014)



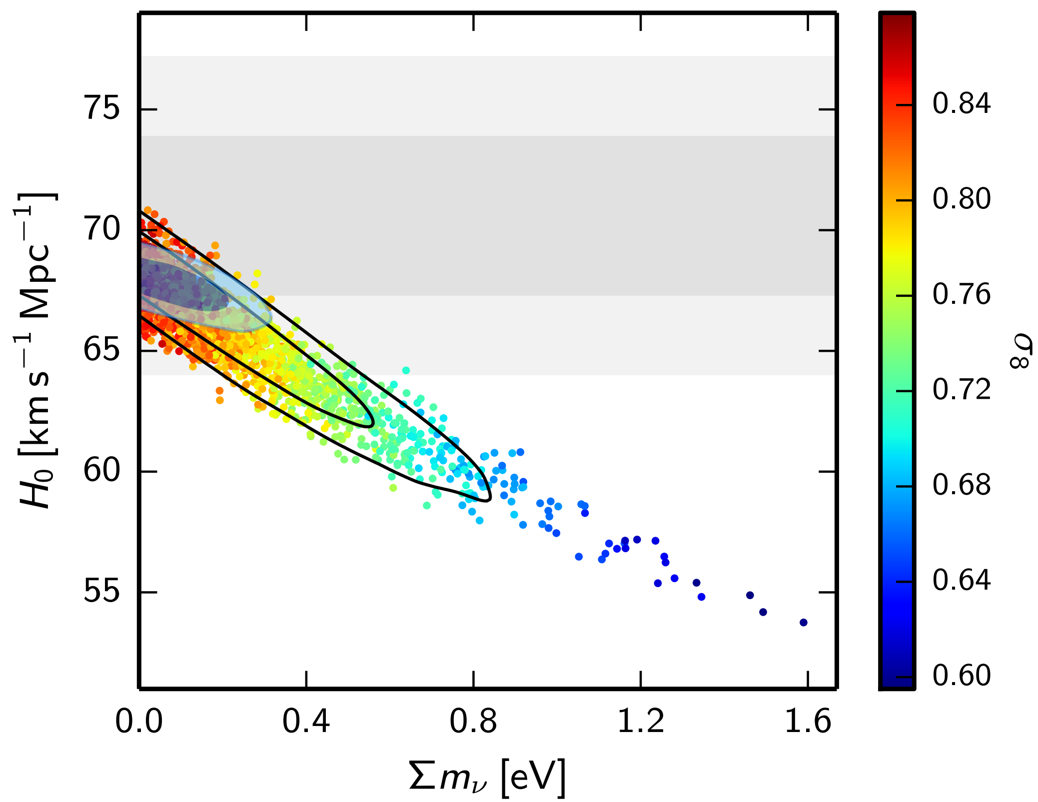
Planck LCDM fit

Tension between Planck and RSD data

RSD fit

One possibility for reconciling the discrepancy: Massive neutrinos

Battye and Moss (2013)



Increasing the neutrino mass m_ν leads to the lower values of σ_8, but it also decreases H_0.

⇨ Tension with the direct measurement of H_0

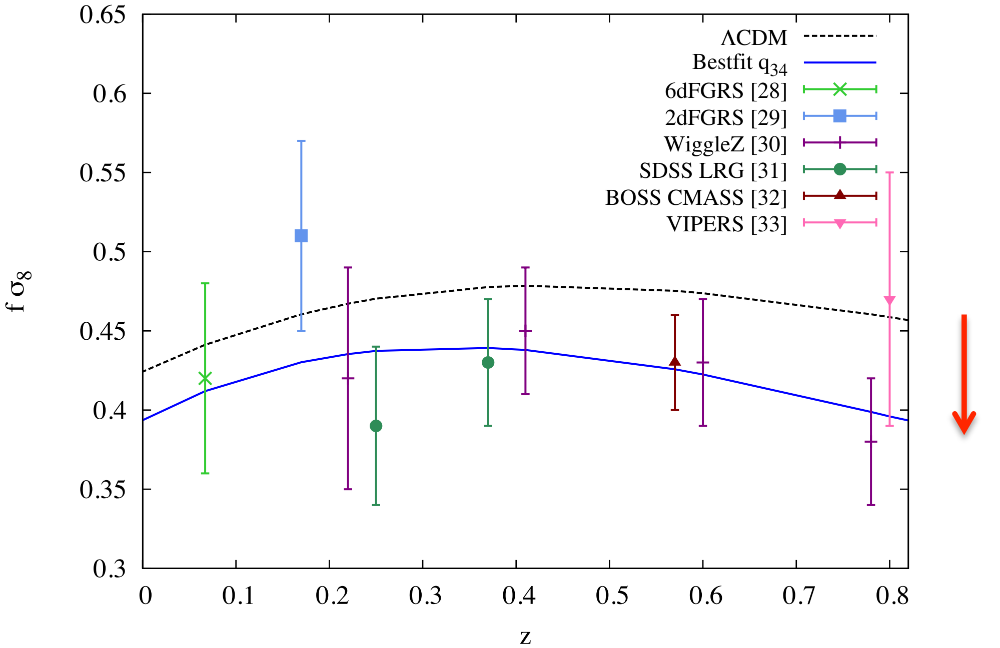
Another possibility: Interacting dark matter/vacuum energy

There is an energy transfer between CDM and vacuum:

Salvatelli et al (2014)

$$\dot{\rho}_c + 3H\rho_c = -Q \qquad \dot{V} = Q$$

The coupling Q is usually taken in an ad-hoc way (like $Q = -qHV$).



Lower growth rate than in LCDM for $Q > 0$.

However there is no concrete Lagrangian explaining the origin of such a coupling. It is likely that the low growth rate is associated with the appearance of ghosts.

Another possibility: Modified gravity with a concrete Lagrangian

In this case we can explicitly derive conditions for the absence of ghosts and instabilities.

The question is

Is it possible to realize the cosmic growth rate lower than that in LCDM in modified gravity models, while satisfying conditions for the absence of ghosts and instabilities?

In doing so, we begin with most general second-order scalar-tensor theories with single scalar degree of freedom (Horndeski theories).

Horndeski theories

$$S = \int d^4x \sqrt{-g}\, L$$

Most general scalar-tensor theories with second-order equations

Horndeski (1973)
Deffayet et al (2011)
Charmousis et al (2011)
Kobayashi et al (2011)

$$\begin{aligned} L &= G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right] \\ &\quad + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \end{aligned}$$

Single scalar field ϕ with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

R and $G_{\mu\nu}$ are the 4-dimensional Ricci scalar and the Einstein tensors, respectively.

The Lagrangian of Horndeski theories is constructed to keep the equations of motion up to second order, such that the theories are free from the Ostrogradski instability.

- General Relativity corresponds to $G_4 = M_{\rm pl}^2/2$.
- Horndeski theories accommodate a wide variety of gravitational theories like Brans-Dicke theory, $f(R)$ gravity, and covariant Galileons.

Cosmological perturbations in Horndeski theories

The scalar degree of freedom in Horndeski theories can give rise to

- the late-time cosmic acceleration at the background level
- interactions with the matter sector (CDM, baryons)

We take into account non-relativistic matter with the energy density

$$\rho_m = \underbrace{\rho_m(t)}_{\text{Background}} + \underbrace{\delta\rho_m(t, \mathbf{x})}_{\text{Perturbations}}$$

The perturbed line element in the longitudinal gauge is

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$$

The four velocity of non-relativistic matter is

$$u^\mu = (1-\Psi, \nabla^i v)$$

v is the rotational-free velocity potential.

Matter perturbations in the Horndeski theories

$\delta \equiv \delta\rho_m/\rho_m$ and $\theta \equiv \nabla^2 v$ obey

$$\dot{\delta} = -\theta/a - 3\dot{\Phi}$$

→ $\dot{\delta}$ is related with v.

$$\dot{\theta} = -H\theta + (k^2/a)\Psi$$

In RSD observations, the growth rate of matter perturbations is constrained from peculiar velocities of galaxies.

The gauge-invariant density contrast $\delta_m \equiv \delta + \frac{3aH}{k^2}\theta$ obeys

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi = 3\left(\ddot{I} + 2H\dot{I}\right)$$

where $I \equiv (aH/k^2)\theta - \Phi$

Ψ is related with δ_m through the modified Poisson equation:

$$\frac{k^2}{a^2}\Psi \simeq -4\pi G_{\rm eff}\rho_m\delta_m$$

→ $G_{\rm eff}$ is the effective gravitational coupling with matter.

Effective gravitational coupling in Horndeski theories

For the modes deep inside the Hubble radius ($k \gg aH$) we can employ the quasi-static approximation under which the dominant terms are those including k^2/a^2, δ_m, and $M^2 \equiv -K_{,\phi\phi}$. It then follows that

De Felice, Kobayashi, S.T. (2011).

$$G_{\rm eff} = \frac{2M_{\rm pl}^2[(B_6 D_9 - B_7^2)\,(k/a)^2 - B_6 M^2]}{(A_6^2 B_6 + B_8^2 D_9 - 2A_6 B_7 B_8)\,(k/a)^2 - B_8^2 M^2} G$$

Schematically

$$G_{\rm eff} = \frac{a_0 (k/a)^2 + a_1}{b_0 (k/a)^2 + b_1}$$

M corresponds to the mass of a scalar degree of freedom and

$$A_6 = -2XG_{3,X} - 4H\,(G_{4,X} + 2XG_{4,XX})\,\dot{\phi} + 2G_{4,\phi} + 4XG_{4,\phi X}$$
$$+4H\,(G_{5,\phi} + XG_{5,\phi X})\,\dot{\phi} - 2H^2 X\,(3G_{5,X} + 2XG_{5,XX})$$

$$D_9 = -K_{,X} + \text{derivative terms of } G_3, G_4, G_5$$

In GR, $G_4 = M_{\rm pl}^2/2,\ B_6 = B_8 = 2M_{\rm pl}^2,\ A_6 = B_7 = 0,\ D_9 = -K_{,X}$ ⇒ $G_{\rm eff} = G$

In the massive limit ($M^2 \to \infty$) with $B_6 \simeq B_8 \simeq 2M_{\rm pl}^2$ we also have $G_{\rm eff} \simeq G$

In the massless limit $M^2 \to 0$ we have

$$G_{\rm eff} = \frac{2M_{\rm pl}^2 (B_6 D_9 - B_7^2)}{A_6^2 B_6 + B_8^2 D_9 - 2A_6 B_7 B_8} G$$

⇒ The effect of modified gravity manifests itself.

Conditions for the absence of ghosts and instabilities

The second-order action for tensor perturbations γ_{ij} is

$$S_2^{(h)} = \int d^4x\, a^3 q_{\rm t} \delta^{ik}\delta^{jl}\left(\dot{\gamma}_{ij}\dot{\gamma}_{kl} - \frac{c_{\rm t}^2}{a^2}\partial\gamma_{ij}\partial\gamma_{kl}\right)$$

where

$$q_{\rm t} = \frac{1}{4}\left(G_4 - 2XG_{4,X} - H\dot{\phi}XG_{5,X} - \frac{1}{2}XG_{5,\phi}\right),$$

$$c_{\rm t}^2 = \frac{2G_4 + XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}}{2G_4 - 4XG_{4,X} - 2H\dot{\phi}XG_{5,X} - XG_{5,\phi}}.$$

We require $q_{\rm t} > 0$ and $c_{\rm t}^2 > 0$ to avoid ghosts and Laplacian instabilities.

In GR we have $q_{\rm t} = M_{\rm pl}^2/8$ and $c_{\rm t}^2 = 1$.

For scalar perturbations we also have corresponding quantities $q_{\rm s}$ and $c_{\rm s}^2$ which must be positive.

ST (2015)

Simple form of the effective gravitational coupling

In the massless limit, the effective gravitational coupling in Horndeski theories reads

$$\frac{G_{\rm eff}}{G} = \frac{M_{\rm pl}^2 c_{\rm t}^2}{8q_{\rm t}} \left(1 + \frac{8Q^2}{\alpha_{\mathcal{W}}^2 c_{\rm s}^2 c_{\rm t}^2} \frac{q_{\rm t}}{q_{\rm s}}\right)$$

Q and $\alpha_{\mathcal{W}}$ are functions of G_i and their derivatives.

Tensor contribution

Scalar contribution

This correspond to the intrinsic modification of the gravitational part.

Always positive under the no-ghost and no-instability conditions:
$q_{\rm s} > 0, \quad q_{\rm t} > 0, \quad c_{\rm s}^2 > 0, \quad c_{\rm t}^2 > 0$

The necessary condition to realize weaker gravity than that in GR is

$$\frac{M_{\rm pl}^2 c_{\rm t}^2}{8q_{\rm t}} < 1$$

⇒ This is not a sufficient condition for realizing $G_{\rm eff} < G$.

The scalar-matter interaction always enhances the effective gravitational coupling.

Examples

(i) f(R) gravity: $f(R) = R - \mu R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n}+1}$

Hu and Sawicki, Starobinsky, ST.

$$G_{\rm eff} = \frac{G}{f_{,R}}\left(1+\frac{1}{3}\right)$$

Typically, $f_{,R}$ varies from 1 (matter era) to the value like 0.9 (today), so the scalar-matter interaction leads to $G_{\rm eff} > G$.

(ii) Covariant Galileons

Deffayet et al.

$$G_2 = c_2 X\,, \quad G_3 = c_3 X\,, \quad G_4 = M_{\rm pl}^2/2 + c_4 X^2\,, \quad G_5 = c_5 X^2$$

For late-time tracking solutions, it is possible to realize $M_{\rm pl}^2 c_{\rm t}^2/(8q_{\rm t}) < 1$ due to the decrease of $c_{\rm t}^2$ (< 1), but the scalar-matter interaction overwhelms this decrease.

⇒ $G_{\rm eff} > G$ typically.

De Felice, Kase, ST (2011)

Two crucial quantities for the realization of weak gravity

$$q_{\rm t} = \frac{1}{4}\left(G_4 - 2XG_{4,X} - H\dot{\phi}XG_{5,X} - \frac{1}{2}XG_{5,\phi}\right),$$

$$c_{\rm t}^2 = \frac{2G_4 + XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}}{2G_4 - 4XG_{4,X} - 2H\dot{\phi}XG_{5,X} - XG_{5,\phi}}.$$

To recover the GR behavior in regions of the high density, the dominant contribution to the Horndeski Lagrangian is the $M_{\rm pl}^2/2$ term in G_4.

➡ $q_{\rm t} \simeq M_{\rm pl}^2/8$ and $c_{\rm t}^2 \simeq 1$ during most of the matter era.

➡ The large variations of $q_{\rm t}^2$ and $c_{\rm t}^2$ from the end of the matter era to today are required to satisfy the condition $\dfrac{M_{\rm pl}^2 c_{\rm t}^2}{8q_{\rm t}} < 1$.

If we go beyond the Horndeski domain, it is possible to realize $c_{\rm t}^2 < 1$ even in the deep matter era.

Horndeski Lagrangian in the ADM Language

In the ADM formalism, we can construct a number of geometrical scalars:

$$K \equiv K^{\mu}{}_{\mu}\,, \quad \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu}\,, \quad \mathcal{R} \equiv \mathcal{R}^{\mu}{}_{\mu}\,, \quad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,.$$

where $K_{\mu\nu}$ and $\mathcal{R}_{\mu\nu}$ are extrinsic and intrinsic curvatures, respectively.

In the unitary gauge ($\delta\phi = 0$), the Horndeski Lagrangian on the FLRW background is equivalent to

Gleyzes et al (2013)

$$L = A_2 + A_3 K + A_4(K^2 - \mathcal{S}) + B_4\mathcal{R} + A_5\mathcal{K}_3 - B_5(K\mathcal{R}/2 - \mathcal{U})$$

$$(\mathcal{K}_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S}) + O(3))$$

with two particular relations

$$A_4 = 2XB_{4,X} - B_4, \quad A_5 = -XB_{5,X}/3$$

(Horndeski conditions)

What happens if we do not impose these two conditions ?

⇒ Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theories (PRL, 2014)

A simple dark energy model in GLPV theories

$$L = A_2 + A_4(K^2 - \mathcal{S}) + B_4\mathcal{R}$$

De Felice, Koyama, and ST(2015)

where

$$A_2 = -\frac{1}{2}X - V(\phi)\,, \qquad A_4 = -\frac{1}{2}M_{\rm pl}^2\,, \qquad B_4 = \frac{1}{2}M_{\rm pl}^2 F(\phi)$$

A canonical scalar field in GR corresponds to the case $F(\phi) = 1$.

The tensor propagation speed squared is

$$c_{\rm t}^2 = -\frac{B_4}{A_4} = F(\phi)$$

The deviation parameter $\alpha_{\rm H}$ from Horndeksi theories is given by

$$\alpha_{\rm H} = c_{\rm t}^2 - 1$$

The difference of $c_{\rm t}^2$ from 1 quantifies the difference from Horndeski theories.

How about observational signatures in this model ?

Model of constant tensor propagation speed

For constant $F(\phi)$, $c_{\rm t}^2 = $ constant.

If $c_{\rm t}^2$ deviates from 1, this leads to the growth of $c_{\rm s}^2$ as we go back to the past. The $c_{\rm s}^2$ can remain constant for the scaling dark energy model:

$$V(\phi) = V_1 e^{-\lambda_1 \phi/M_{\rm pl}} + V_2 e^{-\lambda_2 \phi/M_{\rm pl}} \qquad (\lambda_1 \gg 1,\ \lambda_2 \lesssim 1)$$

Provided that the oscillating mode of scalar perturbations is initially suppressed, the effective gravitational coupling $G_{\rm eff}$ and the anisotropy parameter $\eta = -\Phi/\Psi$ are given by

$$\frac{G_{\rm eff}}{G} = 1 + \frac{1-c_{\rm t}^2}{c_{\rm s}^2} \qquad \eta \simeq 1 + \frac{5(1-c_{\rm t}^2)(c_{\rm s}^2 - c_{\rm t}^2)}{3c_{\rm t}^2(1+c_{\rm s}^2-c_{\rm t}^2)}$$

during the scaling matter era

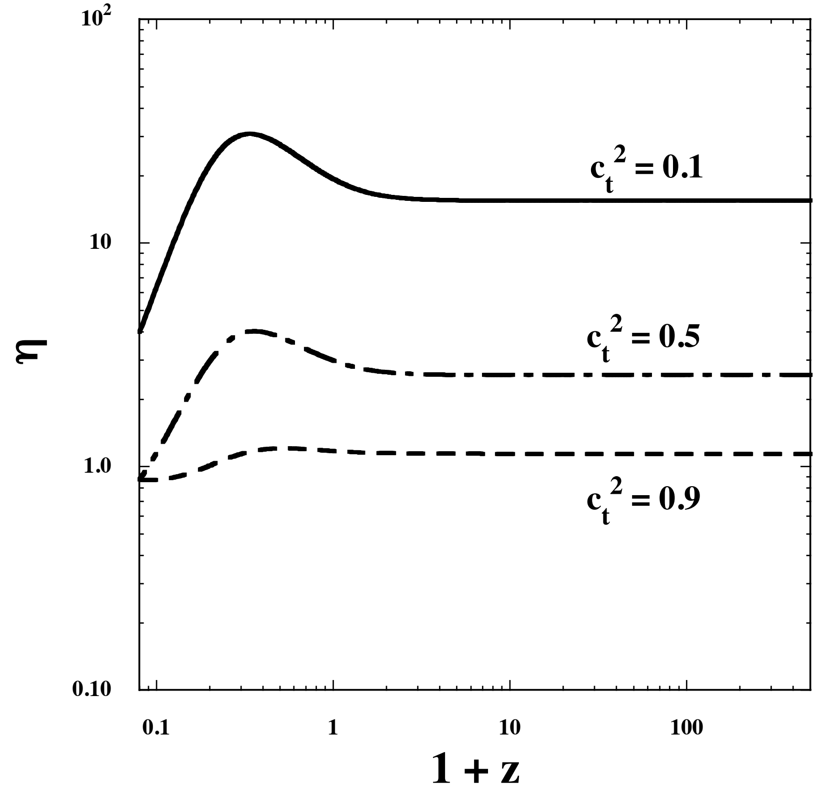
In the sub-luminal regime ($c_{\rm t}^2 < 1$), the Laplacian instability associated with negative $c_{\rm s}^2$ can be avoided.

→ $G_{\rm eff} > G$ (strong gravity), but the deviation from G is not large.

$\eta > 1$ for $c_{\rm t}^2$ away from 1

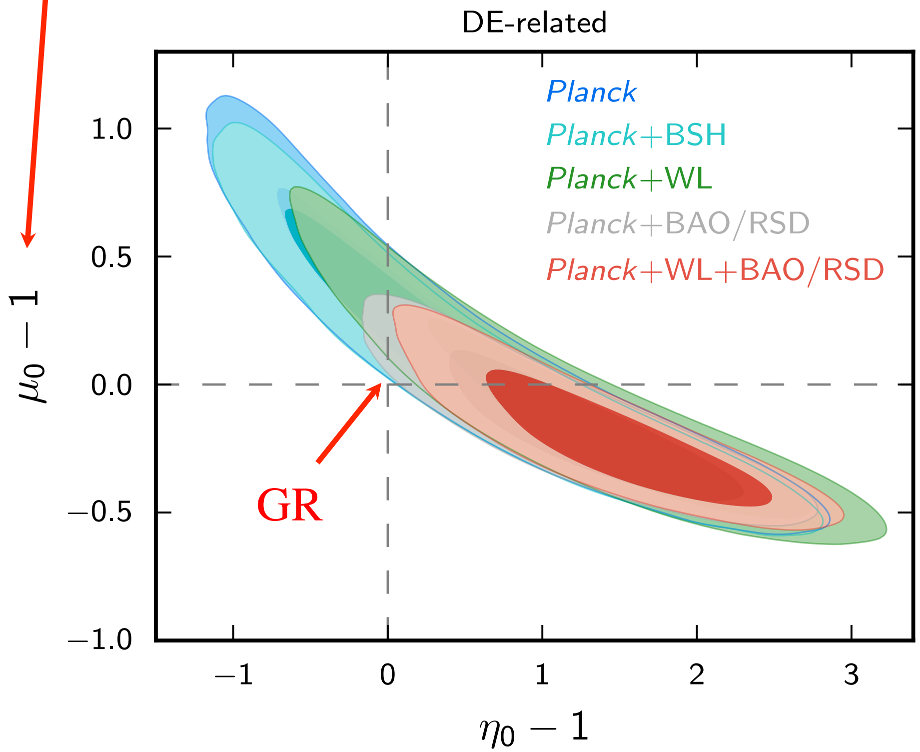
The anisotropy parameter

De Felice, Koyama and ST (2015)



$G_{\rm eff}/G - 1$

Planck constraints (2015)



It is possible to realize $\eta > 1$ in this model.

A model realizing weak gravity (class of GLPV theories)

$$L = A_2 + A_4(K^2 - \mathcal{S}) + B_4 \mathcal{R}$$

ST (2015), PRD to appear

$$A_2 = -\frac{1}{2}X - V(\phi)\,, \qquad A_4 = -\frac{1}{2}M_{\rm pl}^2\,, \qquad B_4 = \frac{1}{2}M_{\rm pl}^2 F(\phi)$$ where $F(\phi) = c_{ti}^2 e^{-2\beta\phi/M_{\rm pl}}$

The decrease of $c_{\rm t}^2$ leads to $G_{\rm eff}$ smaller than G.

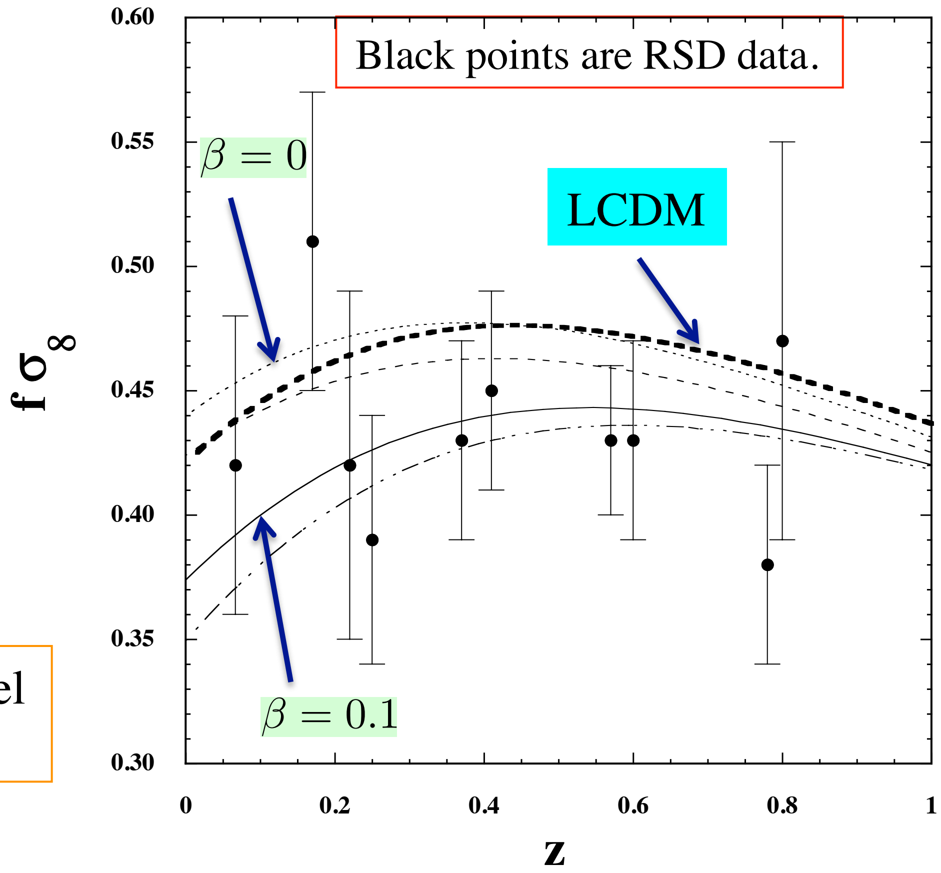
In the scaling matter era,

$$\frac{G_{\rm eff}}{G} = 1 + \frac{1-c_{\rm t}^2}{c_{\rm s}^2} + \frac{5}{3c_{\rm t}^2}\epsilon_{\alpha_{\rm H}}$$

Negative for

$$\epsilon_{\alpha_{\rm H}} \equiv \frac{2c_{\rm t}\dot{c}_{\rm t}}{H} < 0\,.$$

Compared to the LCDM, the model shows a better fit to the RSD data.



Summary and outlook

1. Motivated by the observational tension between CMB and RSD data, we have studied the possibility of realizing weak gravity for the growth of matter perturbations.

2. The necessary condition for realizing weak gravity in Horndeski theories is $\frac{M_{\rm pl}^2 c_{\rm t}^2}{8 q_{\rm t}} < 1$, but this is not sufficient due to the scalar-matter coupling (which is always positive under no-ghost and no-instability conditions).

3. In GLPV theories, $c_{\rm t}^2$ can deviate from 1 even during the matter era. We have constructed a concrete model of weak gravity in GLPV theories.

It remains to see whether the signature of weak gravity persists in future observations. Even if it does not persist, our theoretical study will be useful to distinguish a host of dark energy models.