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Redshift space distortion and halo velocity bias

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Zheng, Yong, Oh, et al., 2015, in preparation

Zhang, Zheng & Jing, 2014, arXiv: 1405.7125, PRD accepted

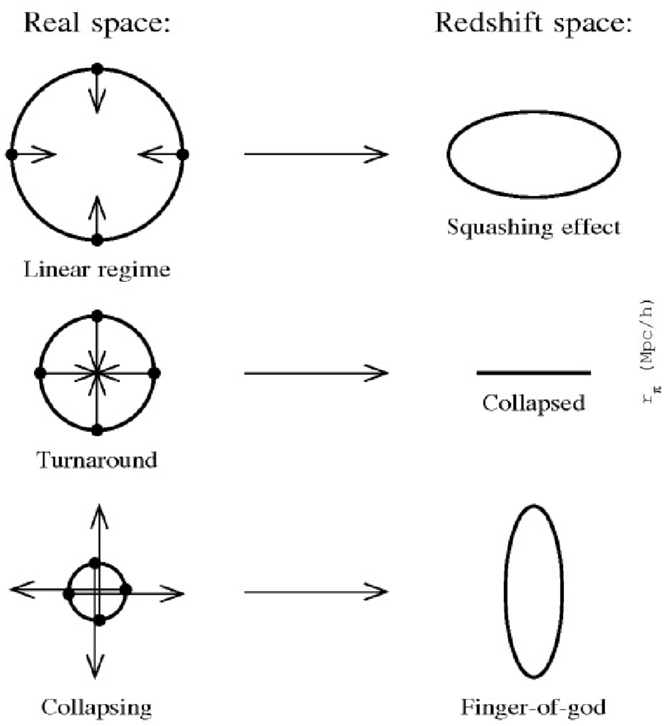
Zheng, Zhang & Jing, 2014a, arXiv: 1409.6809, PRD accepted

Zheng, Zhang & Jing, 2014b, arXiv: 1410.1256, PRD accepted

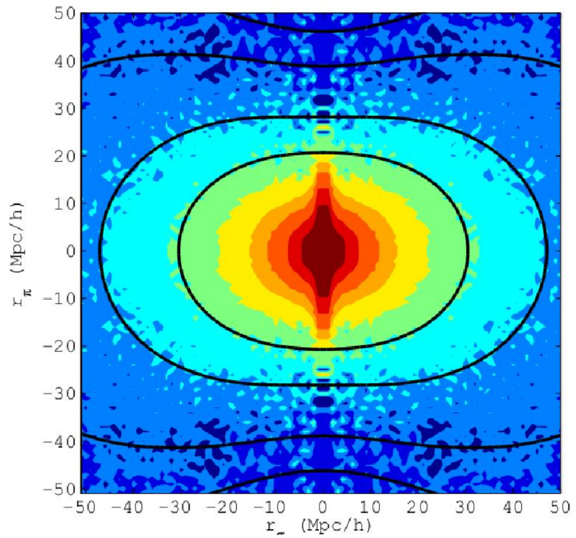
Outline

- Redshift space distortion
  - Key question: if the velocity dispersion σ_v^2 in the FoG term is scale independent or not?
  - Scoccimarro model
  - TNS model
- Halo velocity bias
  - Theoretical prediction from Gaussian statistics (k^2 scale dependence)
  - Measurement from simulation ($b_v = 1$ at $k \leq 0.1\ h/\mathrm{Mpc}$ in 2% accuracy)

Introduction



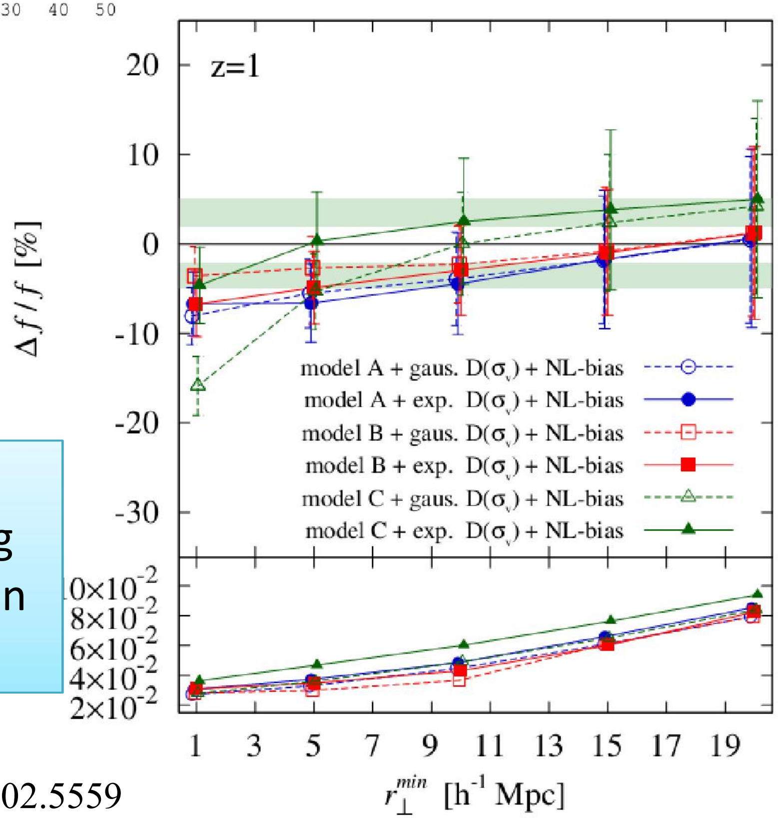
A. J. S. Hamilton, astro-ph/9708102



Reid et al. 2012

The redshift position **s**:

$$\boldsymbol{s} = \boldsymbol{x} + v_z/(aH)\,\hat{\boldsymbol{z}}$$



Torre & Guzzo, 1202.5559

5-10%

O(1%)

Observationally:

Euclid, BigBOSS
et all:

Theoretically:
1. Nonlinear mapping
2. Nonlinear evolution
3. Bias modelling

Phenomenological RSD model

$$P^{(S)}(k,\mu) = D_{\rm FoG}[k\mu f\sigma_{\rm v}]P_{\rm Kaiser}(k,\mu),$$

$$P_{\rm Kaiser}(k,\mu) = \begin{cases} (1+f\mu^2)^2 P_{\delta\delta}(k) & \text{linear,} \\ P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) & \text{nonlinear.} \end{cases}$$

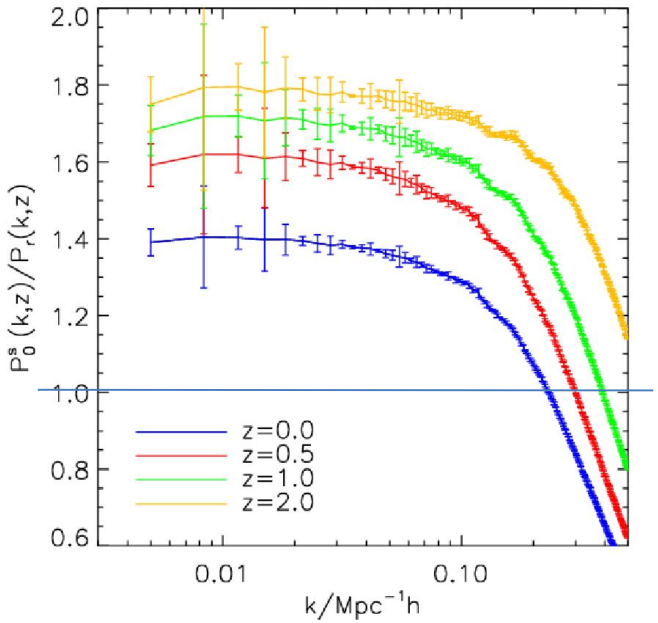
$$D_{\rm FoG}[x] = \begin{cases} \exp(-x^2) & \text{Gaussian,} \\ 1/(1+x^2) & \text{Lorentzian.} \end{cases}$$

interpreted to be global (pairwise) velocity dispersion, a constant fitting parameter (?)

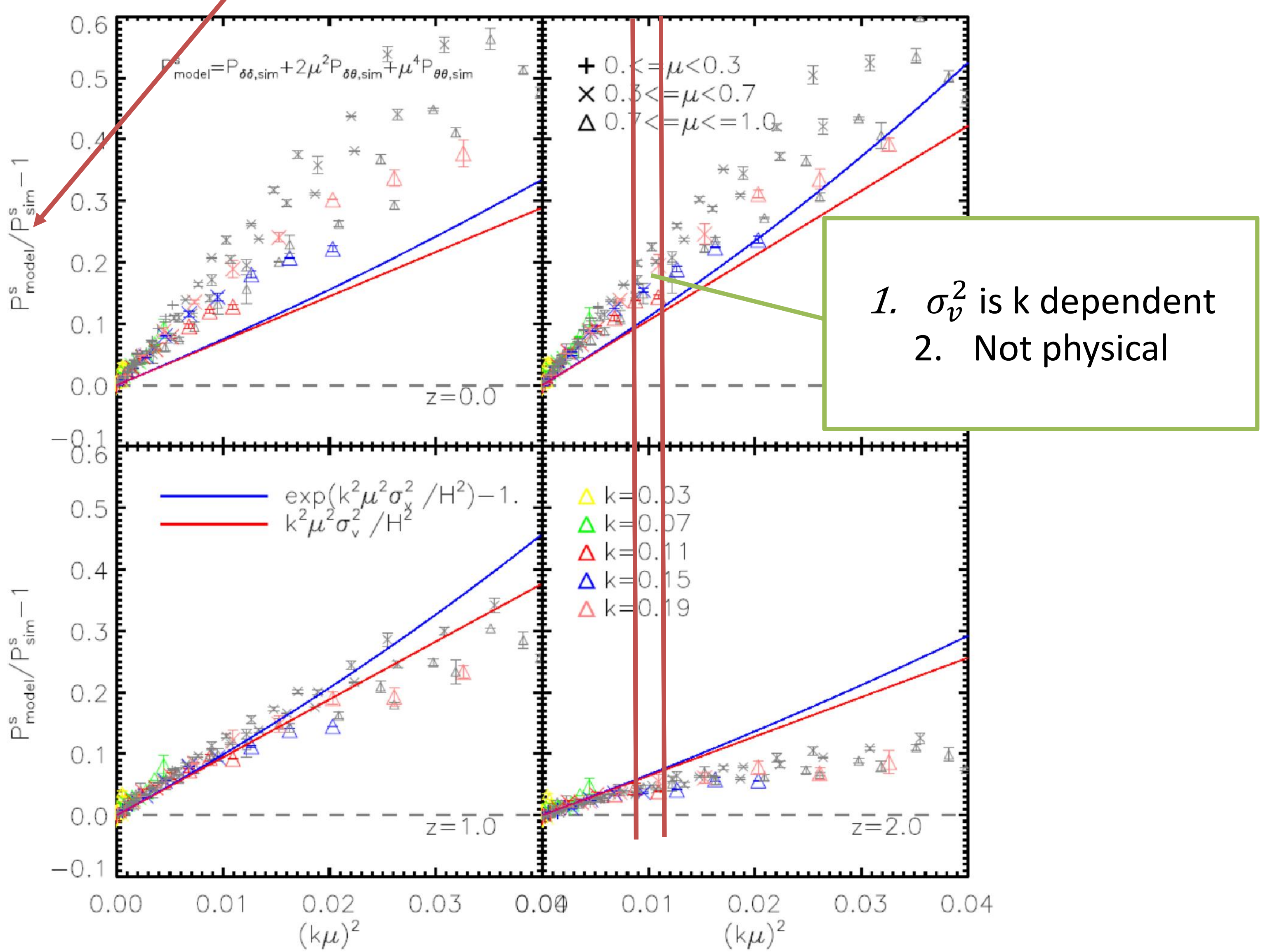
Taruya et al. arXiv:1006.0699

$$G(k\mu;\sigma_v) = \begin{cases} e^{-k^2\mu^2\sigma_v^2/2} & \text{Gaussian,} \\ (1+k^2\mu^2\sigma_v^2/2)^{-2} & \text{Lorentzian.} \end{cases}$$

arXiv:1506.05814v1



$$P^{(S)}(k, \mu) = D_{\rm FoG}[k\mu f \sigma_v] P_{\rm Kaiser}(k, \mu),$$



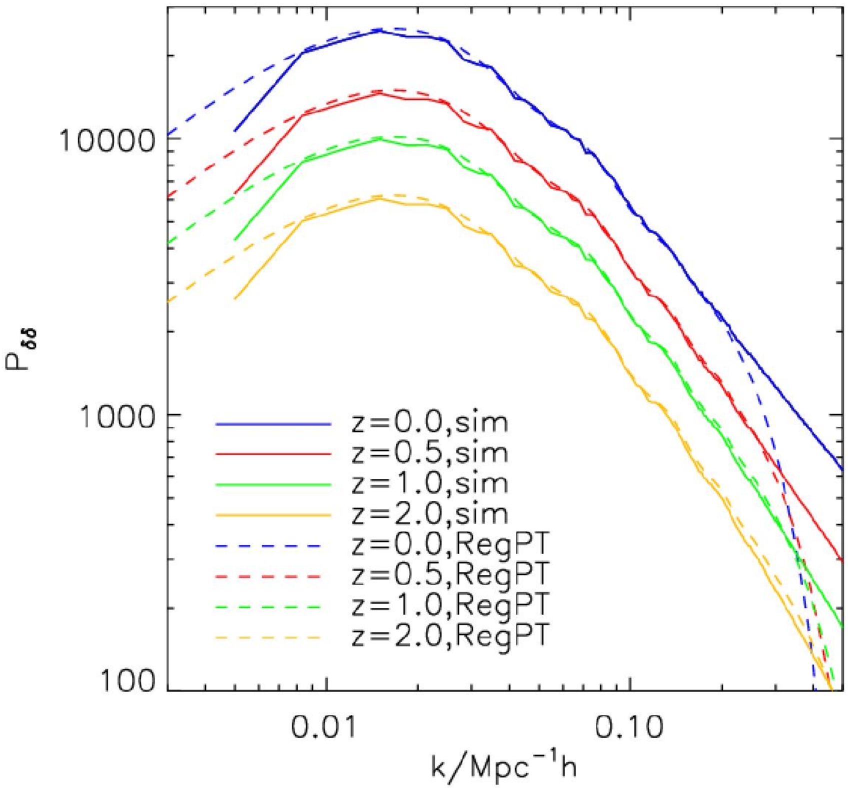
Mathematically show the point: scoccimarro's model

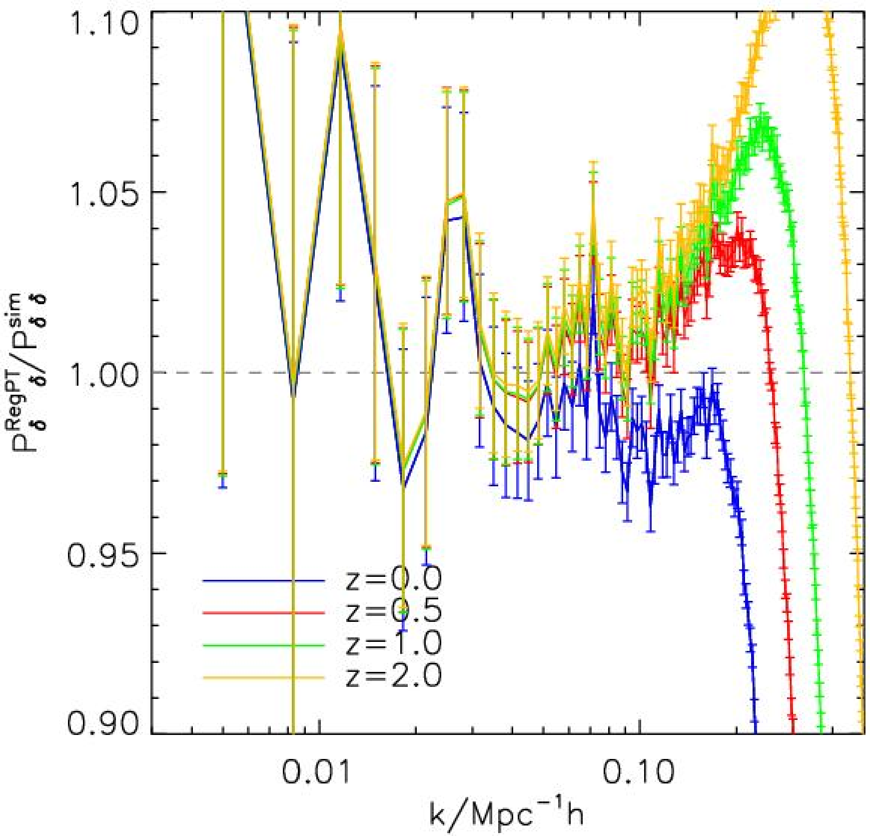
$$P_{\mathrm{Kaiser}}(k, \mu) = \begin{cases} (1 + f\mu^2)^2 P_{\delta\delta}(k) & \text{linear,} \\ P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) & \text{nonlinear.} \end{cases}$$

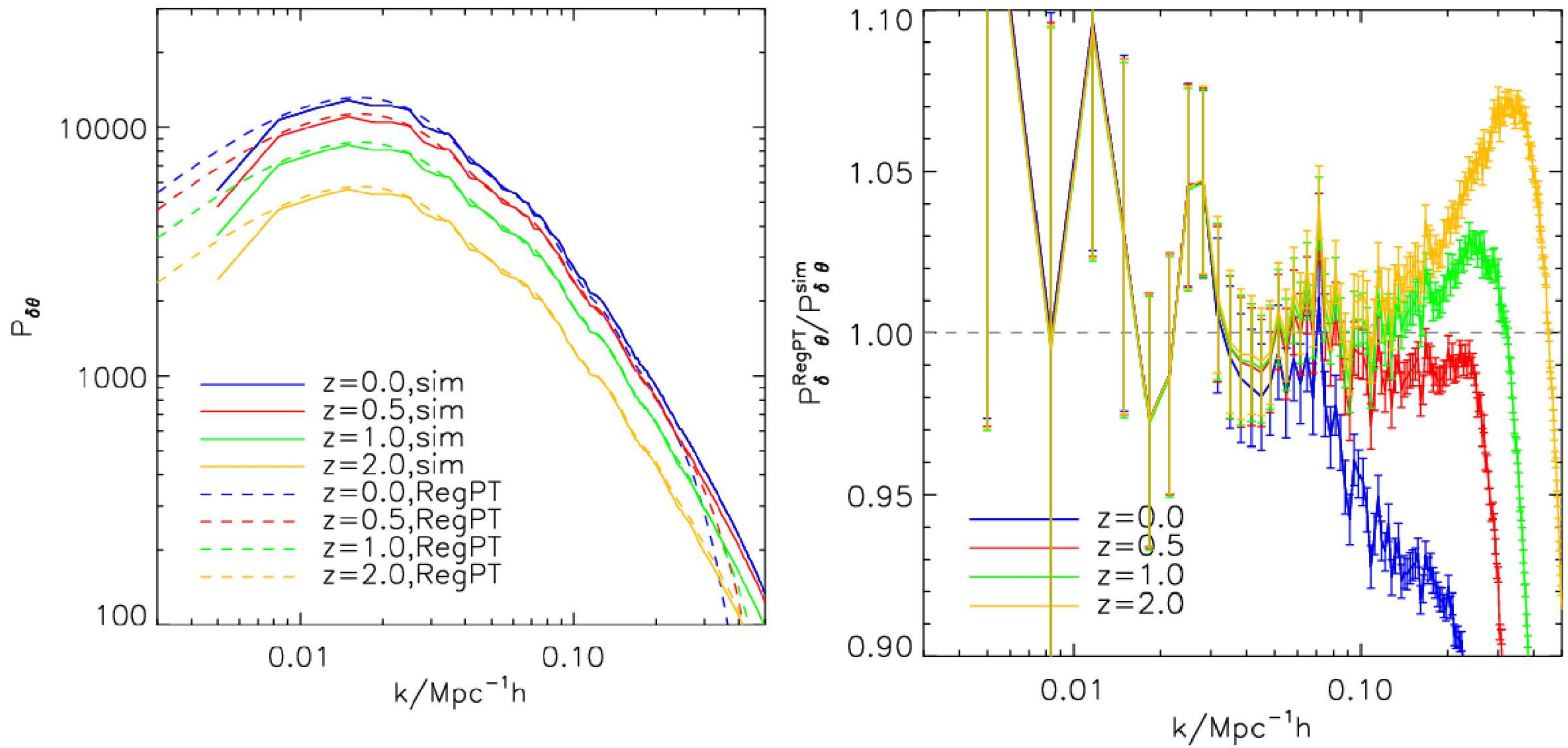
For each k bin, we restrict to $k\mu < 0.1$, and fit the σ_v^2.

RegPT: Taruya et al. PHYSICAL REVIEW D 86, 103528

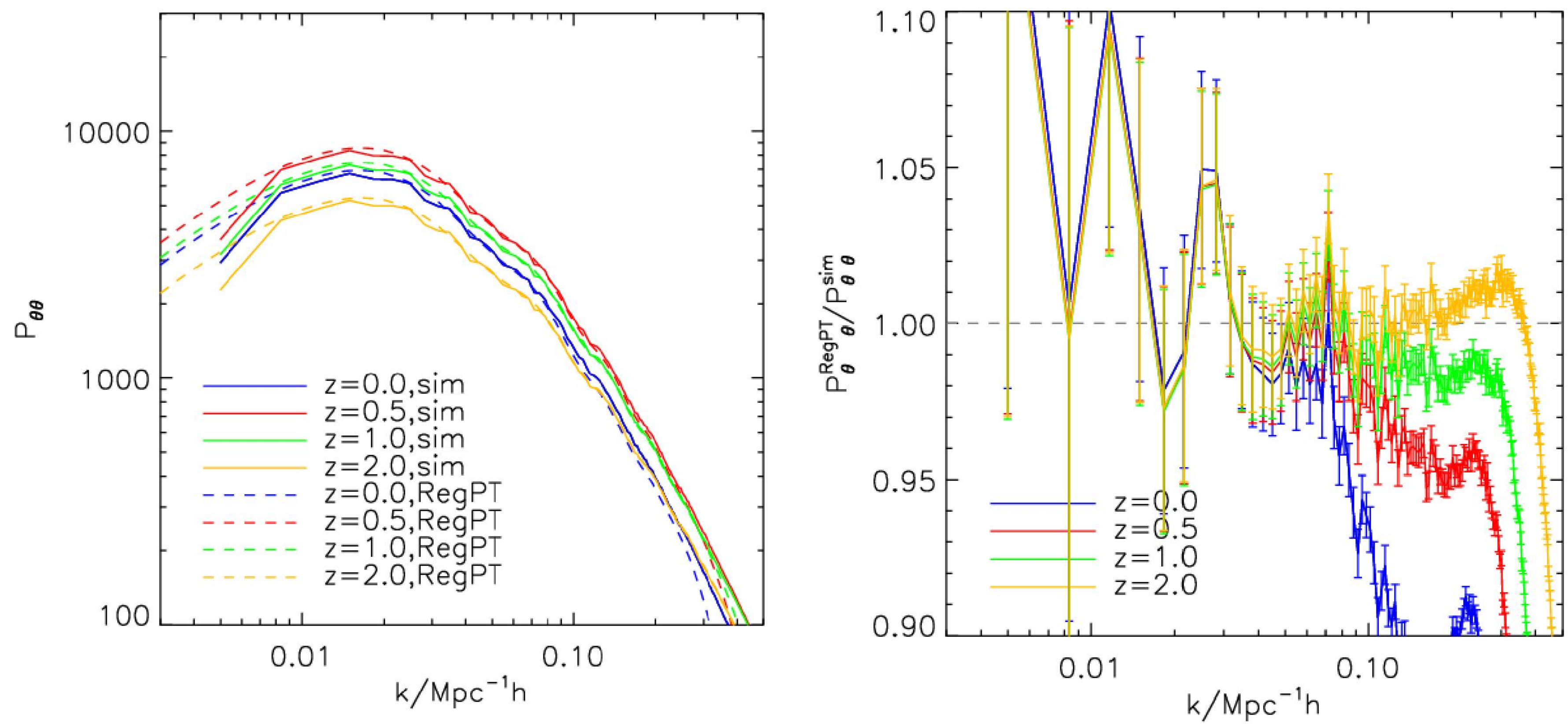
$$P_{ab}(k;\eta) = \Gamma^{(1)}_{a,\mathrm{reg}}(k;\eta)\Gamma^{(1)}_{b,\mathrm{reg}}(k;\eta)P_0(k) + 2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\Gamma^{(2)}_{a,\mathrm{reg}}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};\eta)\Gamma^{(2)}_{b,\mathrm{reg}}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};\eta)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$
$$+ 6\int \frac{d^6\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^6}\Gamma^{(3)}_{a,\mathrm{reg}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};\eta)\Gamma^{(3)}_{b,\mathrm{reg}}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};\eta)P_0(p)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)$$



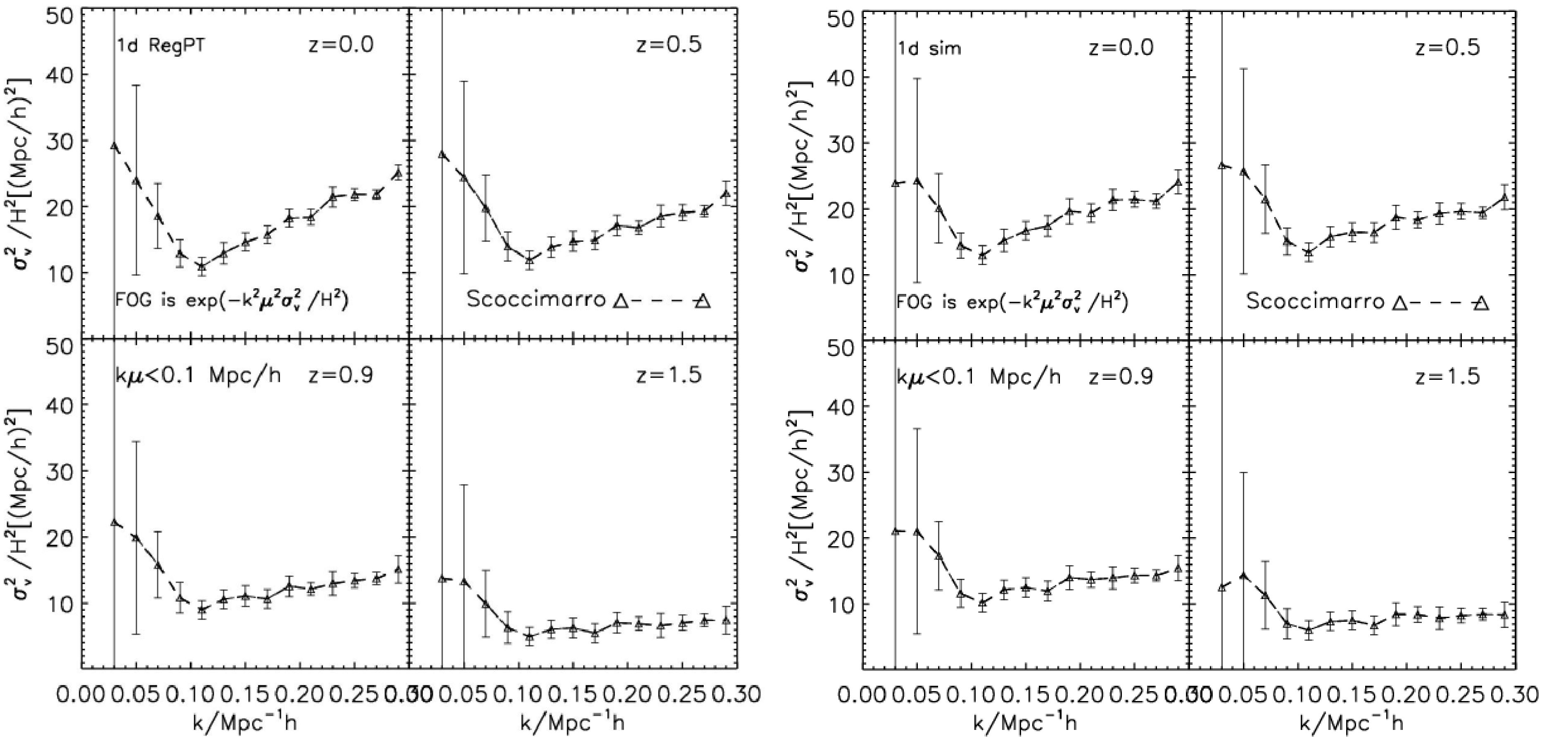




P_δθ
10000
1000
100
0.01
0.10
k/Mpc⁻¹h
z=0.0,sim
z=0.5,sim
z=1.0,sim
z=2.0,sim
z=0.0,RegPT
z=0.5,RegPT
z=1.0,RegPT
z=2.0,RegPT
P^RegPT_δθ/P^sim_δθ
1.10
1.05
1.00
0.95
0.90
0.01
0.10
k/Mpc⁻¹h
z=0.0
z=0.5
z=1.0
z=2.0



P_θθ
10000
1000
100
0.01
0.10
k/Mpc⁻¹h
z=0.0,sim
z=0.5,sim
z=1.0,sim
z=2.0,sim
z=0.0,RegPT
z=0.5,RegPT
z=1.0,RegPT
z=2.0,RegPT
P_θθ^RegPT/P_θθ^sim
1.10
1.05
1.00
0.95
0.90
0.01
0.10
k/Mpc⁻¹h
z=0.0
z=0.5
z=1.0
z=2.0



1d RegPT
z=0.0
z=0.5
FOG is exp(−k²μ²σ²_v /H²)
Scoccimarro
kμ<0.1 Mpc/h
z=0.9
z=1.5
σ²_v /H²[(Mpc/h)²]
k/Mpc⁻¹h
1d sim
z=0.0
z=0.5
FOG is exp(−k²μ²σ²_v /H²)
Scoccimarro
kμ<0.1 Mpc/h
z=0.9
z=1.5
σ²_v /H²[(Mpc/h)²]
k/Mpc⁻¹h

Taruya's model

$$P^{(\mathrm{S})}(k,\mu) = D_{\mathrm{FoG}}[k\mu f\sigma_{\mathrm{v}}]\{P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + A(k,\mu) + B(k,\mu)\}.$$

$$j_1 = -ik\mu f, \qquad A_1 = u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'),$$

$$A_2 = \delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r}), \qquad A_3 = \delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}').$$

$$A(k,\mu) = j_1 \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle A_1 A_2 A_3 \rangle_c,$$

$$B(k,\mu) = j_1^2 \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c.$$

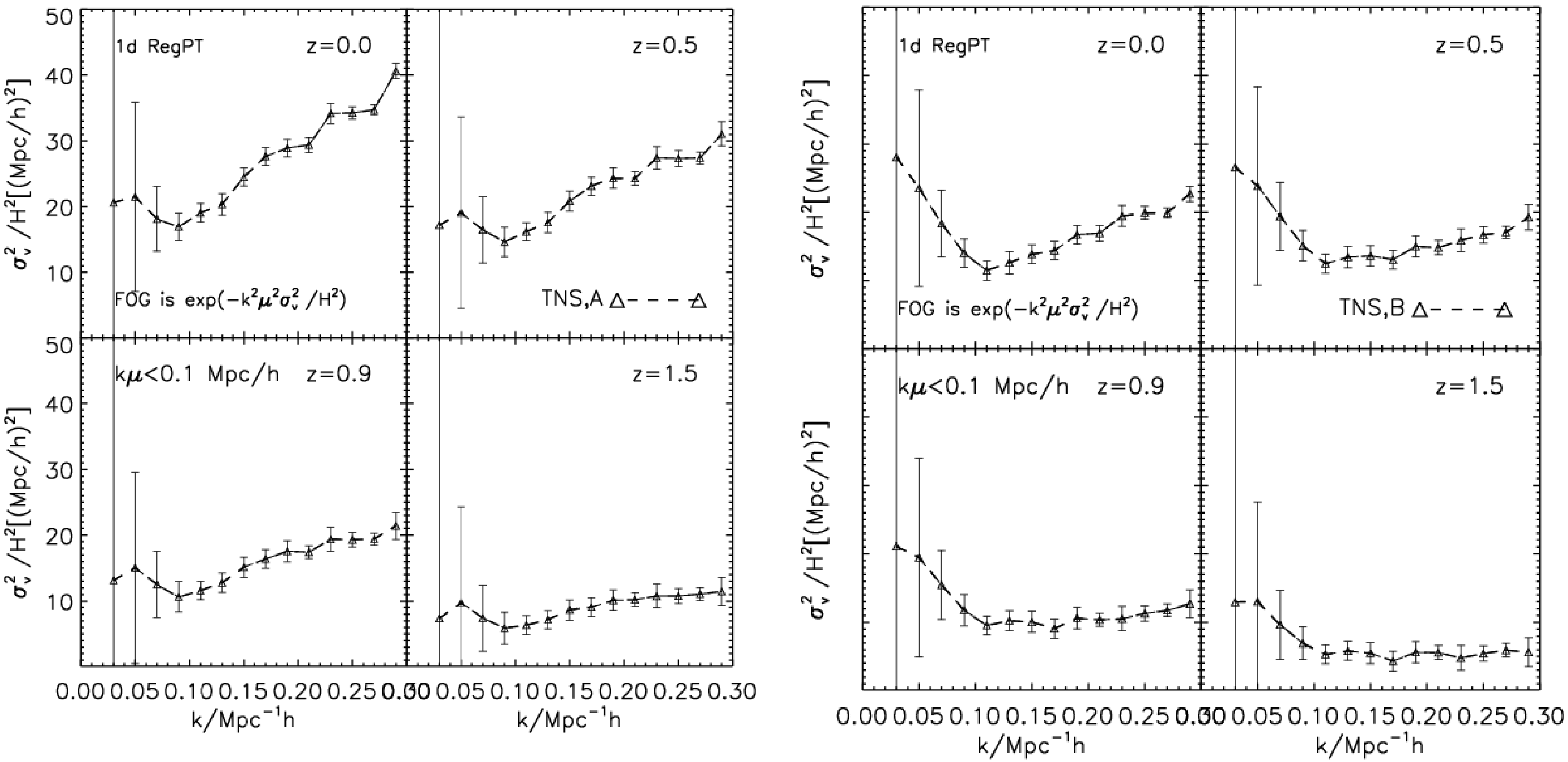
Perturbation:

$$A(k,\mu) = (k\mu f)\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\frac{p_z}{p^2}\{B_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) - B_\sigma(\boldsymbol{p},\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{p})\},$$

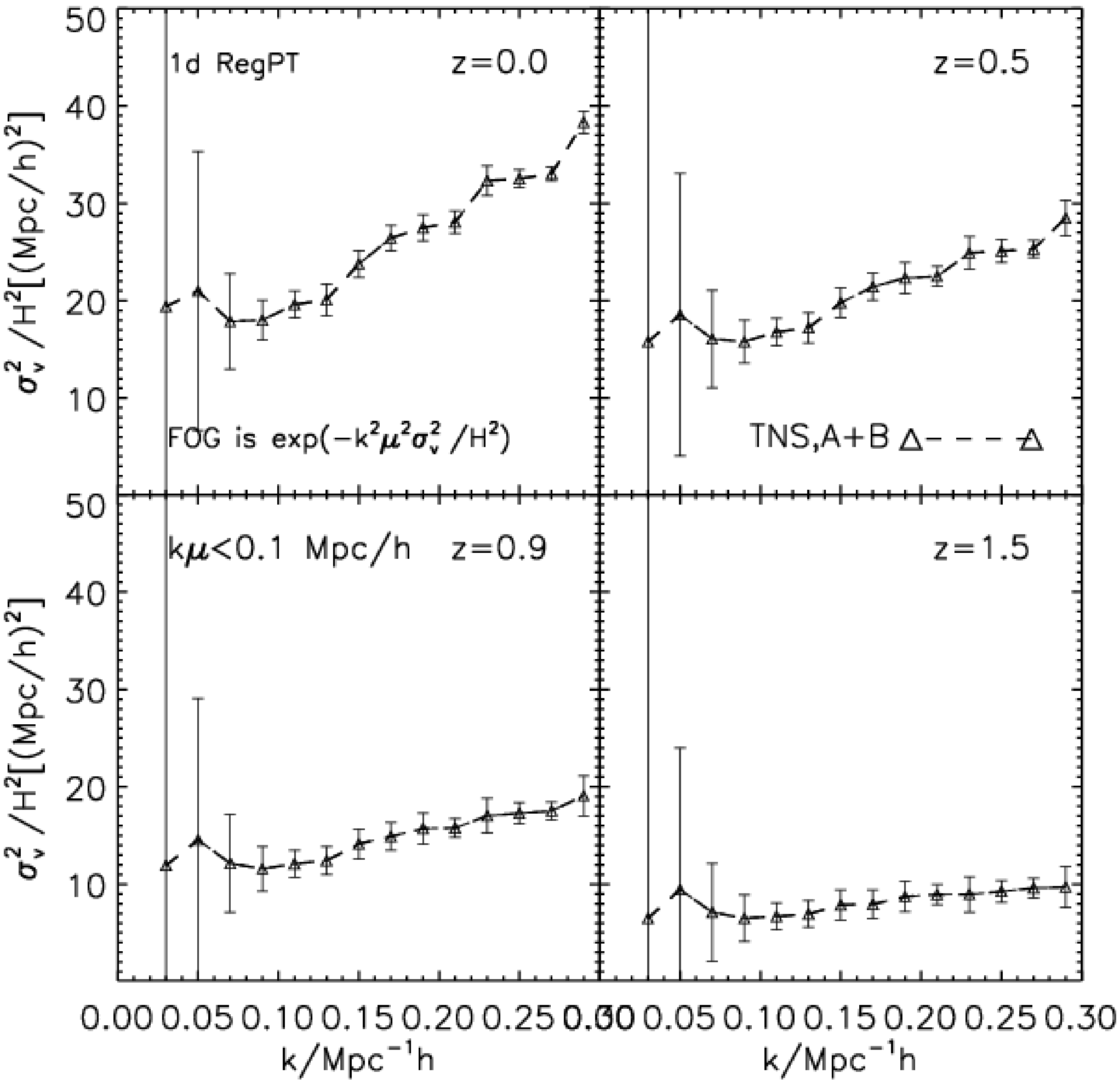
$$\left\langle \theta(\boldsymbol{k}_1)\left\{\delta(\boldsymbol{k}_2) + f\frac{k_{2z}^2}{k_2^2}\theta(\boldsymbol{k}_2)\right\}\left\{\delta(\boldsymbol{k}_3) + f\frac{k_{3z}^2}{k_3^2}\theta(\boldsymbol{k}_3)\right\}\right\rangle = (2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).$$

$$B(k,\mu) = (k\mu f)^2\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}F(\boldsymbol{p})F(\boldsymbol{k}-\boldsymbol{p});$$

$$F(\boldsymbol{p}) = \frac{p_z}{p^2}\left\{P_{\delta\theta}(p) + f\frac{p_z^2}{p^2}P_{\theta\theta}(p)\right\},$$

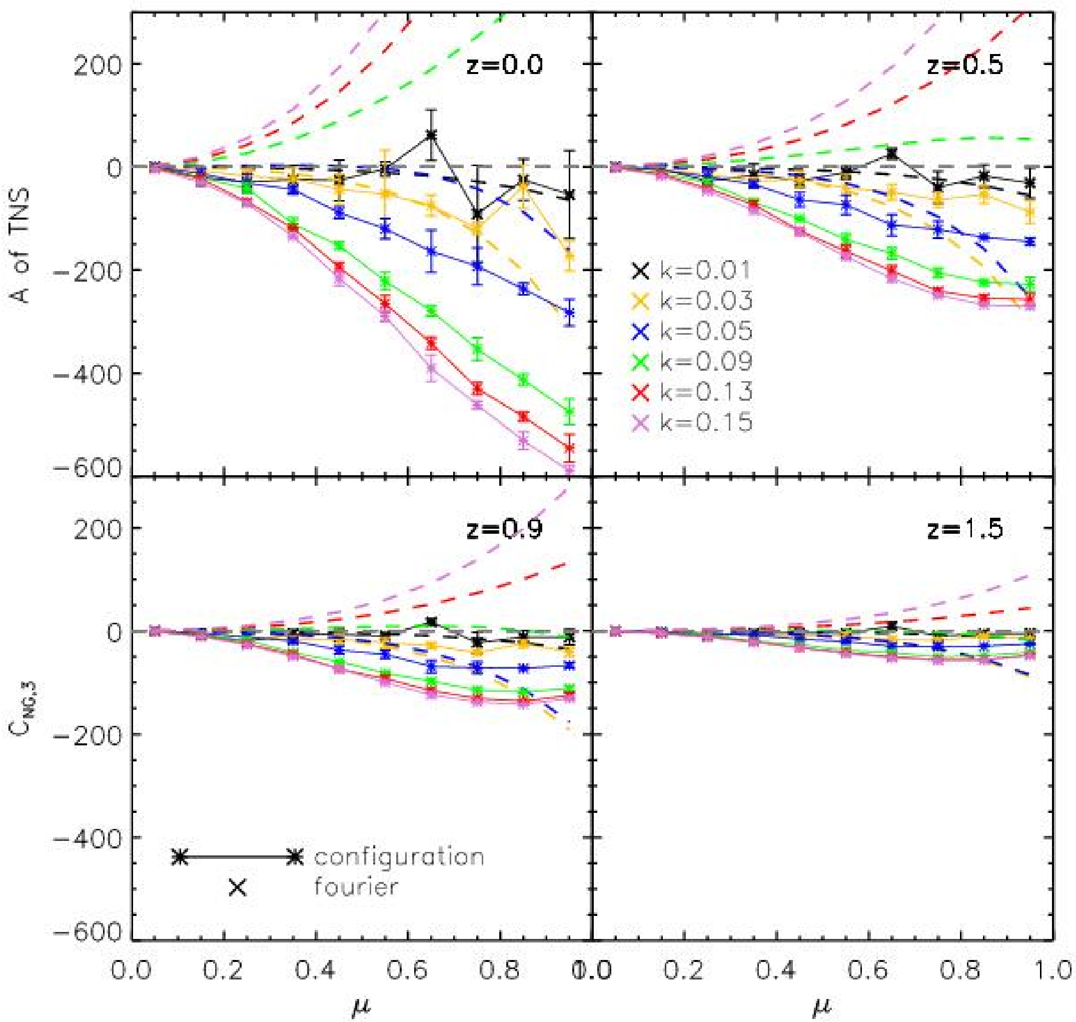


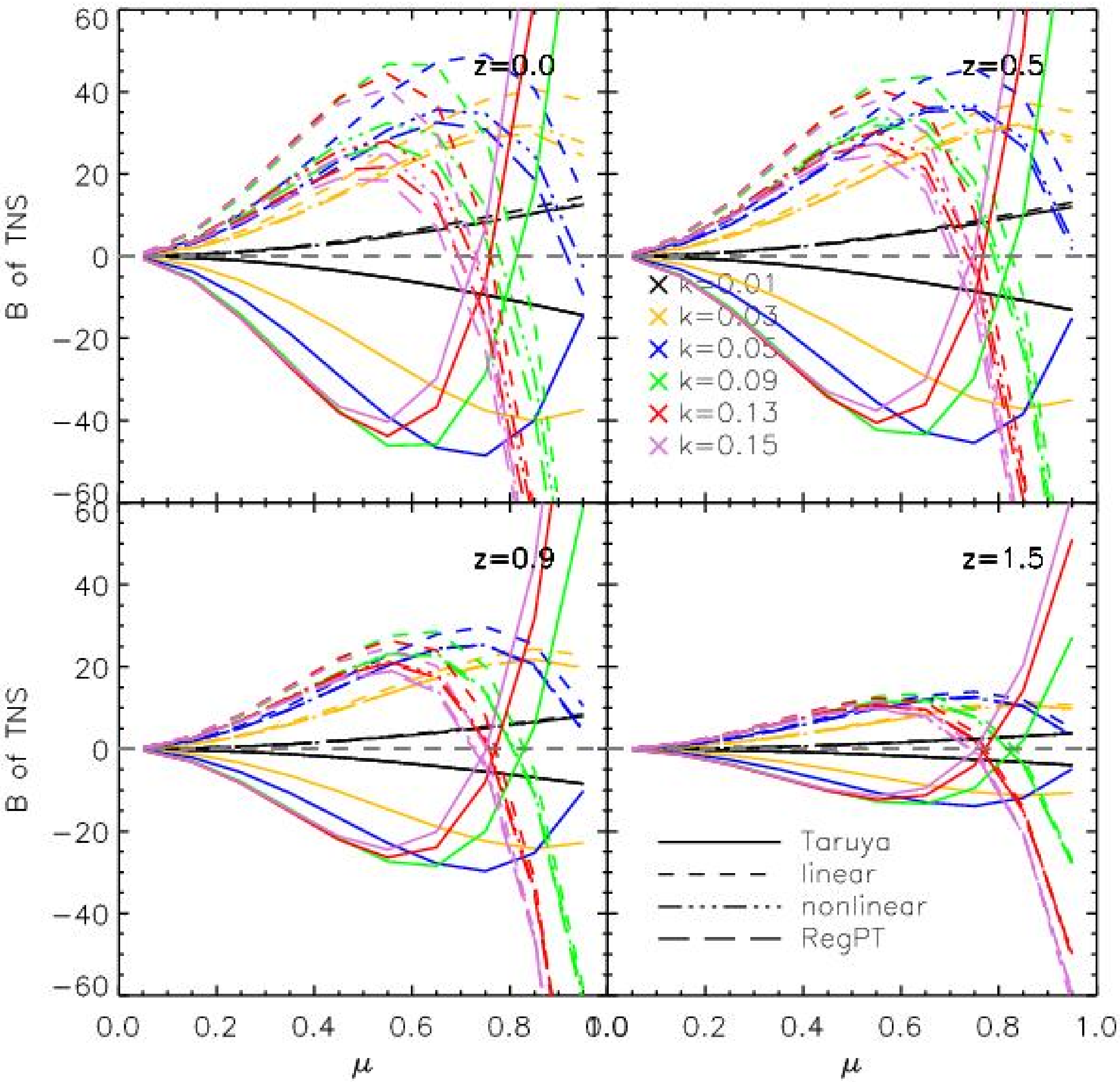
1d RegPT
z=0.0
z=0.5
FOG is exp(−k²μ²σ²ᵥ/H²)
TNS,A
kμ<0.1 Mpc/h
z=0.9
z=1.5
TNS,B
σ²ᵥ/H²[(Mpc/h)²]
k/Mpc⁻¹h



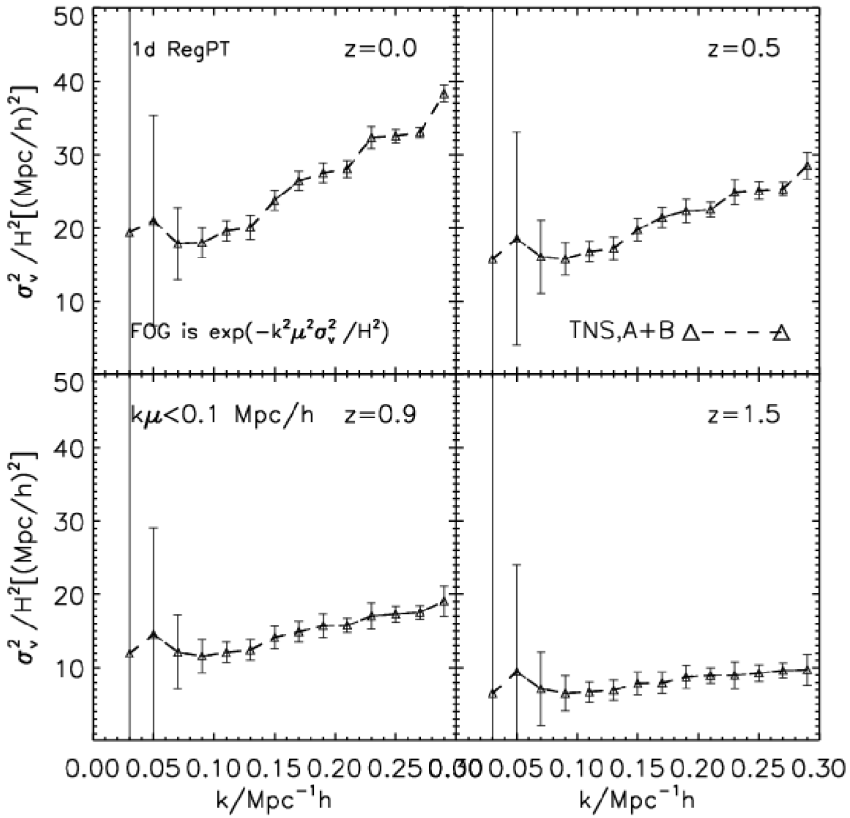
1d RegPT
z=0.0
z=0.5
FOG is exp(−k²μ²σ_v²/H²)
TNS,A+B
kμ<0.1 Mpc/h
z=0.9
z=1.5
σ_v²/H²[(Mpc/h)²]
k/Mpc⁻¹h

Test of A, B terms

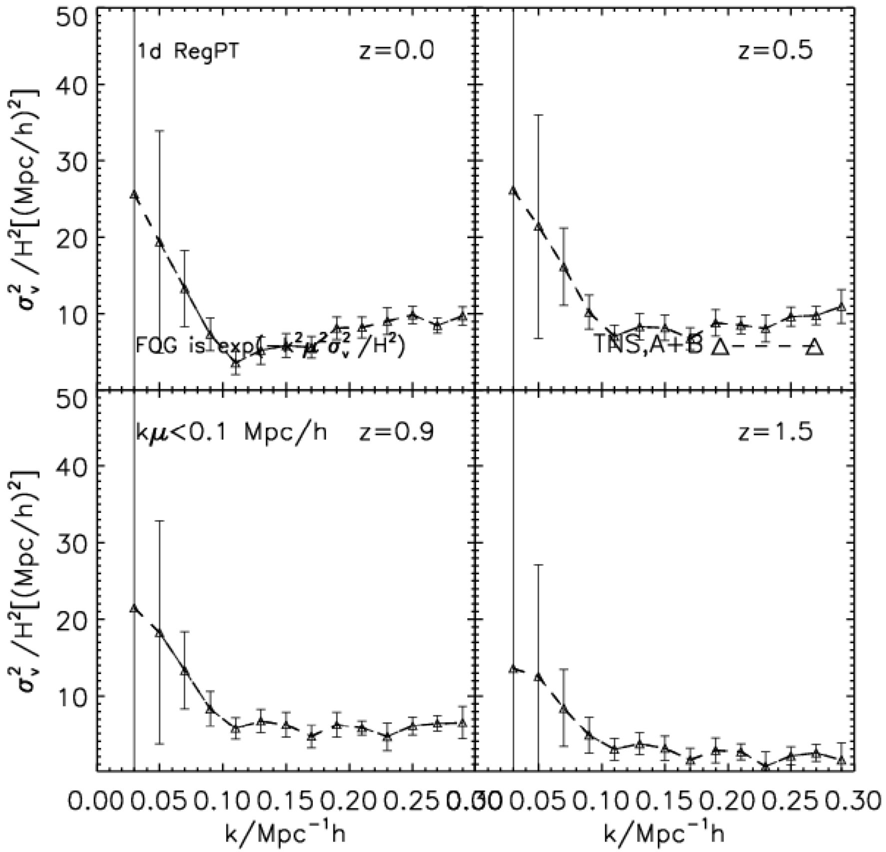




z=0.0
z=0.5
z=0.9
z=1.5
B of TNS
μ
k=0.01
k=0.03
k=0.05
k=0.09
k=0.13
k=0.15
Taruya
linear
nonlinear
RegPT


1d RegPT
z=0.0
z=0.5
z=0.9
z=1.5
kμ<0.1 Mpc/h
FOG is exp(−k²μ²σ²_v /H²)
TNS,A+B △- - - -△
σ²_v /H²[(Mpc/h)²]
k/Mpc⁻¹h

simulation


1d RegPT
z=0.0
z=0.5
z=0.9
z=1.5
kμ<0.1 Mpc/h
FOG is exp(−k²μ²σ²_v /H²)
TNS,A+B △- - - -△
σ²_v /H²[(Mpc/h)²]
k/Mpc⁻¹h

Further improvment

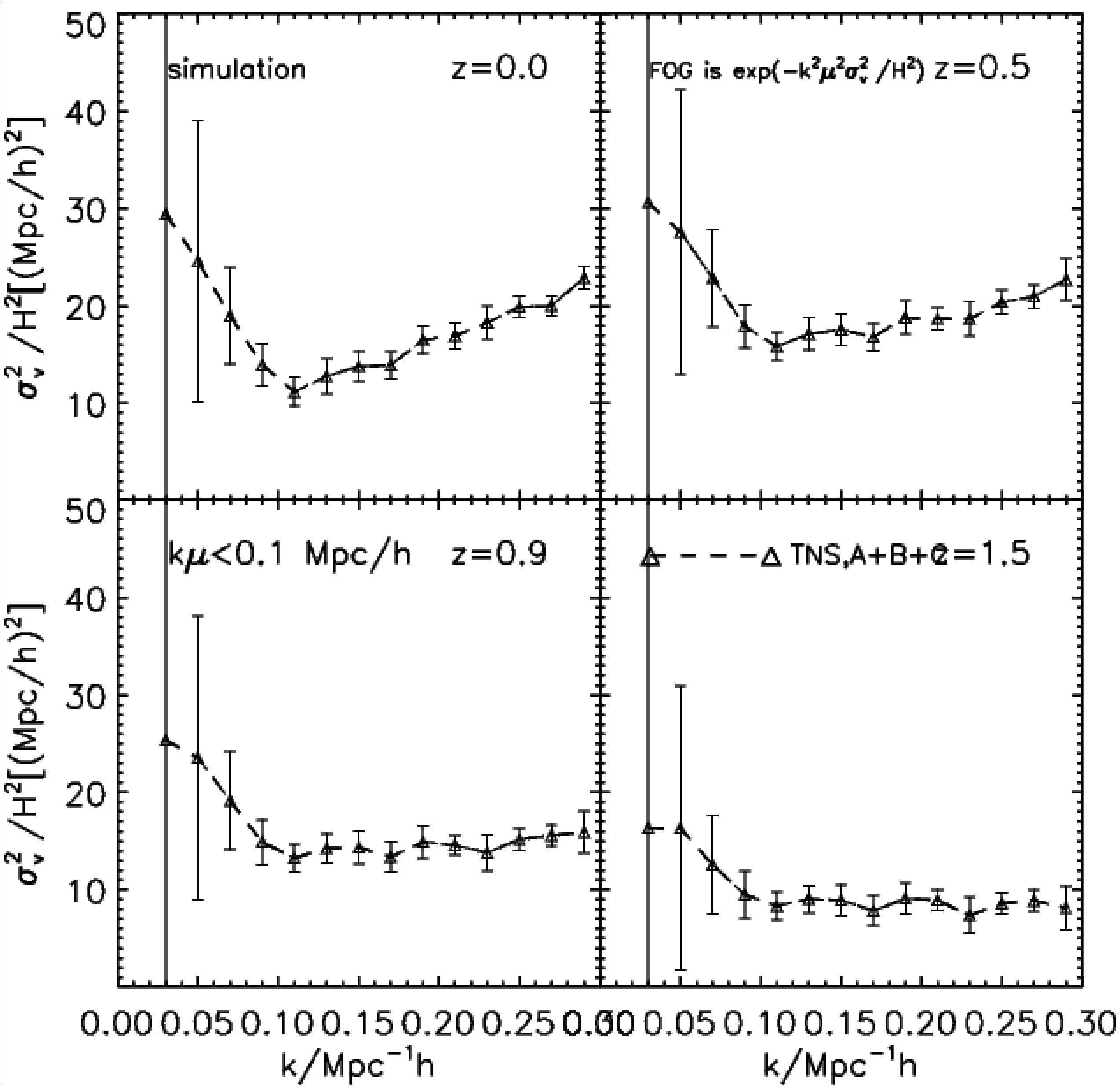
(ii) For cumulants $\langle A_1^n \rangle_c = \langle [u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}')]^n \rangle_c$ of any integer value n, the spatial correlations between different positions are ignored, and the nonvanishing cumulants are assumed to be expressed as $\langle A_1^n \rangle_c \simeq 2\langle u_z^n \rangle_c = 2c_n \sigma_{\rm v}^n$ for even n, with c_n being constants. Taruya et al. arXiv:1006.0699

- C term: velocity correlation

$$C(k,\mu) = (k\mu f)^2 \int \frac{d^3\boldsymbol{p}\, d^3\boldsymbol{q}}{(2\pi)^3} \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}) \frac{\mu_p^2}{p^2} P_{\theta\theta}(p) \times \{P_{\delta\delta}(q) + 2f\mu_q^2 P_{\delta\theta}(q) + f^2\mu_q^4 P_{\theta\theta}(q)\}$$

$$\simeq (k\mu f)^2 \int \frac{d^3\boldsymbol{p}\, d^3\boldsymbol{q}}{(2\pi)^3} \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}) \frac{\mu_p^2}{p^2} \times (1 + f\mu_q^2)^2 P_{\rm lin}(p) P_{\rm lin}(q) \quad (24)$$

- Zhang13 model. (P. Zhang, J. Pan, and Y. Zheng, 1207.2722.)



simulation
z=0.0
FOG is exp(−k²μ²σ²v /H²) z=0.5
kμ<0.1 Mpc/h
z=0.9
TNS,A+B+C
z=1.5
σ²v /H²[(Mpc/h)²]
k/Mpc⁻¹h

Velocity bias or not?

A fundamental problem in peculiar velocity cosmology

In linear theory:

Desjacques & Sheth 0909.4544

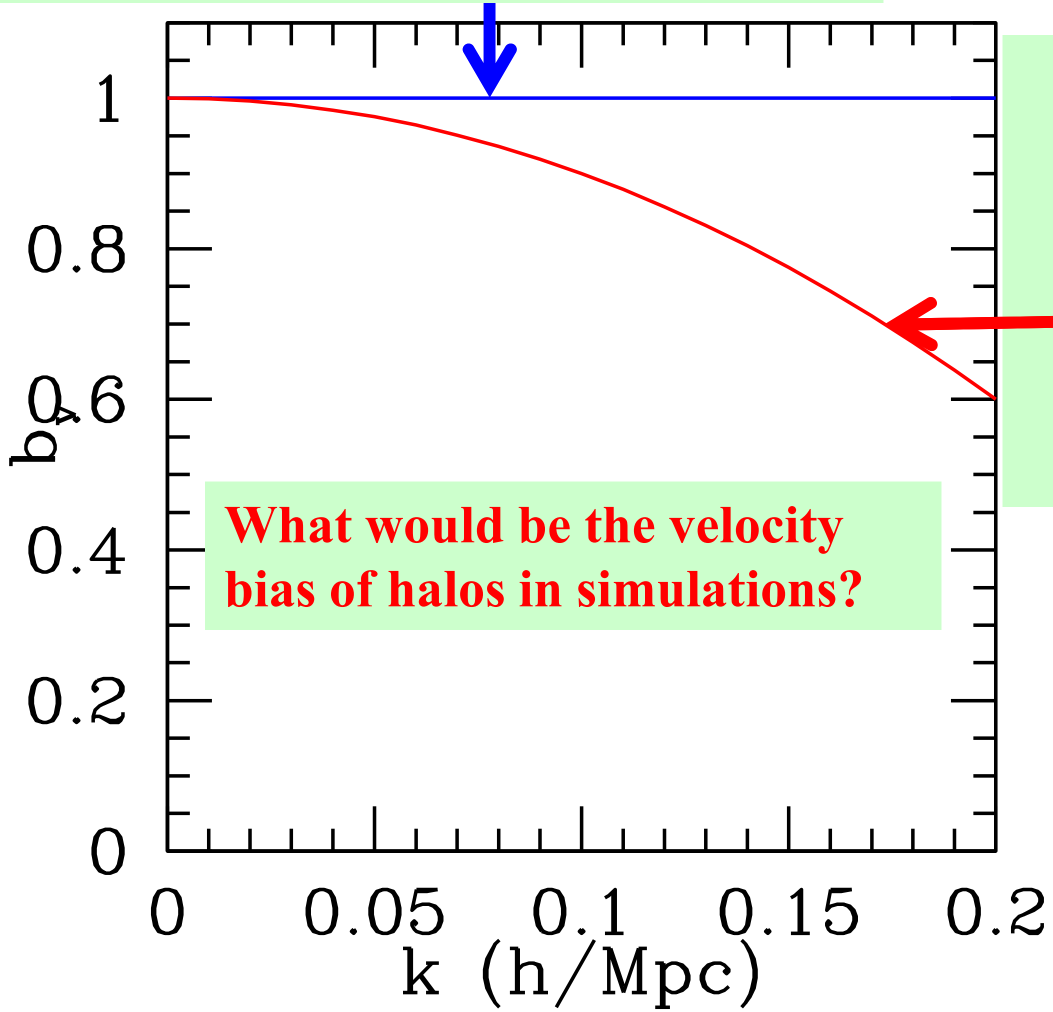
$$P^{s}_{\rm pk}(k,\mu) = \exp(-f^2\sigma^2_{\rm vel}\, k^2\mu^2)\ \left[b_{\rm pk}(k) + b_{\rm vel}(k) f\mu^2\right]^2\ P_\delta(k)$$

Cosmological observations: galaxy velocity field (power spectrum)

$$b_v(k) \equiv \sqrt{\frac{P^v_g(k)}{P^v_{\rm DM}(k)}} = 1?$$

Cosmological observations: dark matter velocity field (power spectrum)

Peculiar velocity cosmology by default assumes no velocity bias at large scale:



What would be the velocity bias of halos in simulations?

Environmental effect: halos/galaxies locate at special regions around density peaks. Proto-halos/linearly evolved density peaks (BBKS 1986; Desjacques & Sheth 2010) have velocity bias

$$b_v \simeq 1 - \frac{\sigma_0^2}{\sigma_1^2} k^2$$

$$\frac{\sigma_v^2(\mathrm{halos})}{\sigma_v^2(\mathrm{DM})} = 1 - \frac{\sigma_0^2}{\sigma_{-1}\sigma_{+1}}$$

$$\sigma_n^2 \propto \int_0^\infty dk k^{2(n+1)} P_m(k,z) W^2(k,M)$$

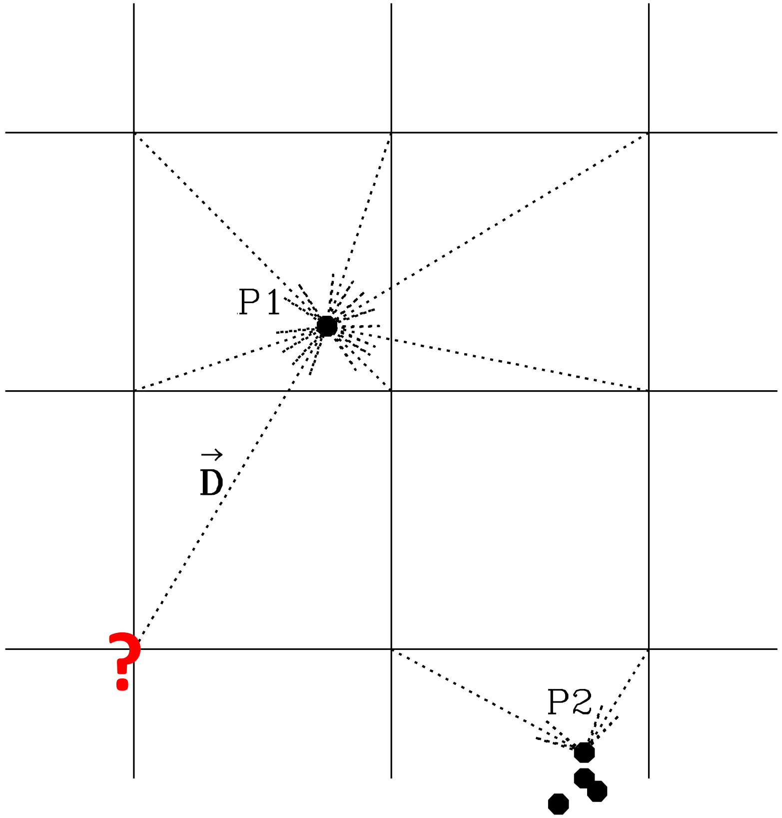
PJZ's PPT

Truth is:

the velocity bias disappear. First is the stochasticity in the proto-halo–halo relation. A fraction of halos today do not correspond to initial density peaks and a fraction of initial density peaks do not evolve into halos today (e.g., [15]). Second, halos move from their initial positions. They tend to move towards each other and, hence, modify their velocity correlation. Third, the density and velocity evolution has non-negligible nonlinearity (e.g., [15,22]) and, hence, non-Gaussianity. This alters the predicted velocity bias based on Gaussian statistics.

The sampling artifact

- Velocity field from most RSD models is volume weighted
- Where there is no particle, the velocity is usually non-zero. It can be large. The sampling on the **volume weighted** velocity field is biased/imperfect
- The sampling artifact: inevitable for inhomogeneously distributed objects. Severe for sparse populations. Persists for NP, Voronoi and Delaunay tessellation.
- The sampling and the signal are neither completely uncorrelated nor completely correlated. Hard to correct straightforwardly
- Can be fully described in the language of the **D** field (ZPJ, Zheng & Jing, 2014), similar to CMB lensing



PJZ's PPT

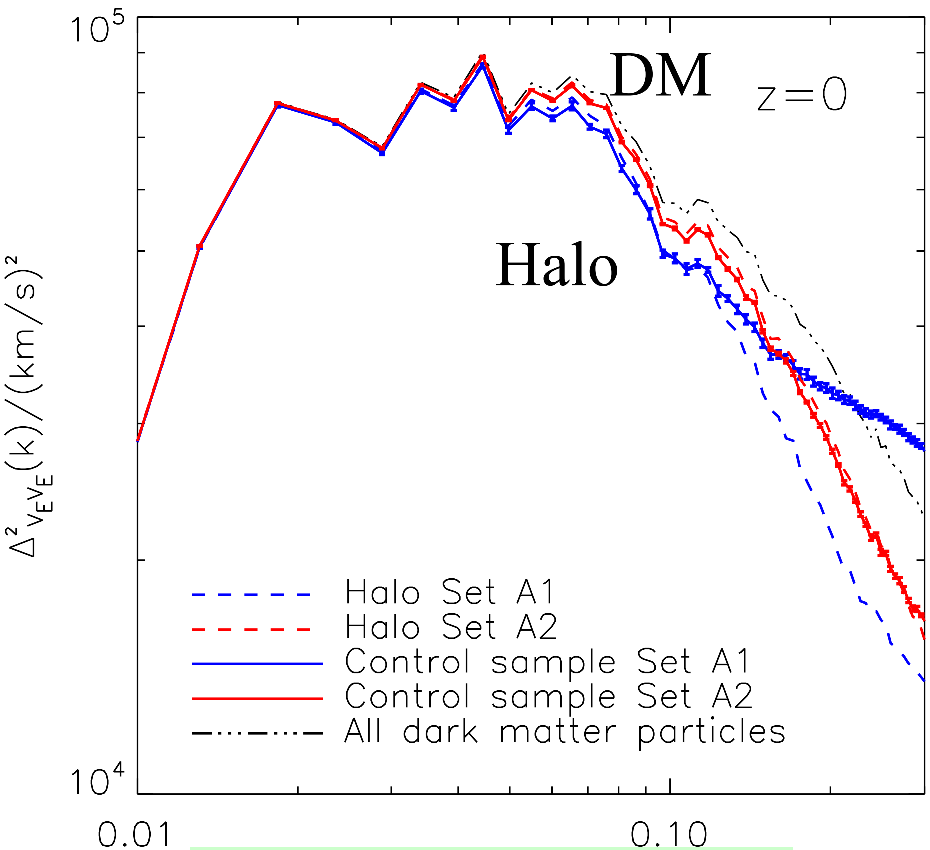
PJZ's PPT

Severe sampling artifact

can be misinterpreted as a "velocity bias"

$$b_v \equiv \sqrt{\frac{P_{h,vv}}{P_{DM,vv}}}$$

- Naive comparison between the raw measurements of halo velocity and DM velocity gives a apparent $b_v<1$
- **Illusion caused by the sampling artifact**
- Unphysical numerical effect. But can be misinterpreted as a "velocity bias"



Yipeng's P^3M simulation:
1200 Mpc/h, 1024^3 particles

Zheng, ZPJ, Jing, 2014b

Understanding the sampling artifact

$$\hat{\mathbf{v}}(\mathbf{x}) = \mathbf{v}(\mathbf{x}_P(\mathbf{x})) \ . \qquad \mathbf{D}(\mathbf{x}) \equiv \mathbf{x}_P(\mathbf{x}) - \mathbf{x}$$

Neglect v-D correlation.

$$\hat{P}_{ij}(\mathbf{k}) = \sum_{\mathbf{q}} P_{ij}(\mathbf{q}) W(\mathbf{q}, \mathbf{q}^{'}) \ . W(\mathbf{q}, \mathbf{q}^{'}) \equiv \frac{1}{N^2_{\rm grid}} \sum_{\mathbf{x} \neq \mathbf{x}^{'}} S(\mathbf{q}, \mathbf{r}) e^{i\mathbf{q}^{'} \cdot \mathbf{r}} \quad S(\mathbf{q}, \mathbf{r}) = \left\langle e^{i\mathbf{q}\cdot(\mathbf{D}^{'} - \mathbf{D})} \right\rangle$$

Neglect correlation in D

Including the correlation in D.Not exact.

$$\hat{P}^{(1)}_E(k) \simeq P_E(k) S(k) \ .$$

$$S(k) \equiv S(\mathbf{k}, \mathbf{r} \to \infty) = \left| \langle e^{i\mathbf{k}\cdot\mathbf{D}} \rangle \right|^2 = e^{-\frac{1}{3}k^2\sigma^2_D + \cdots}$$

ZPJ, Zheng & Jing, 2014

$$\hat{P}_E(k) \simeq P_E(k) S(k) e^{\blacksquare\frac{1}{3}k^2 \xi_D(r_{\rm eff} = \alpha/k)}$$

$$= P_E(k) \langle e^{i\mathbf{q}\cdot\mathbf{D}} \rangle^2 e^{\blacksquare\frac{1}{3}k^2 \xi_D(r_{\rm eff} = \alpha/k)}$$

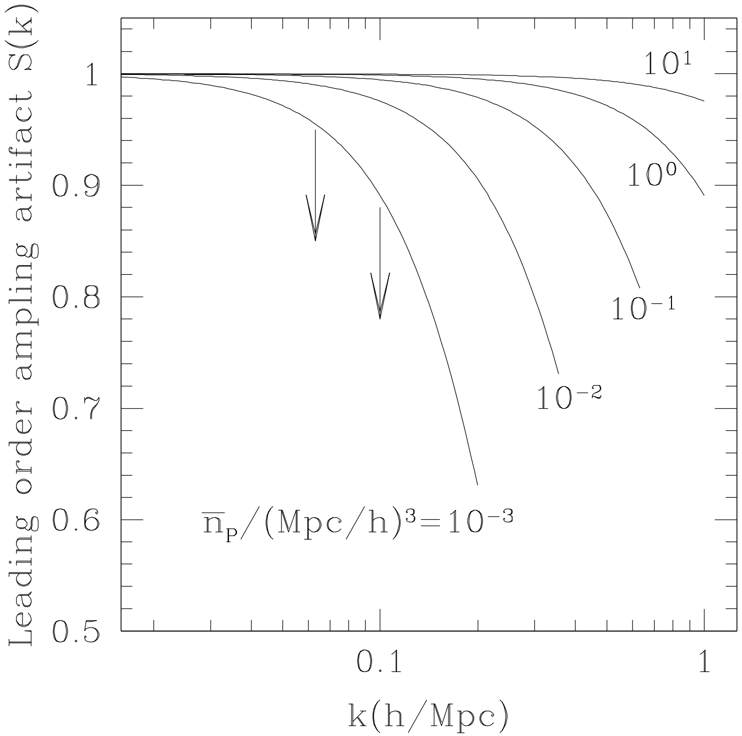
Zheng, ZPJ & Jing, 2014a

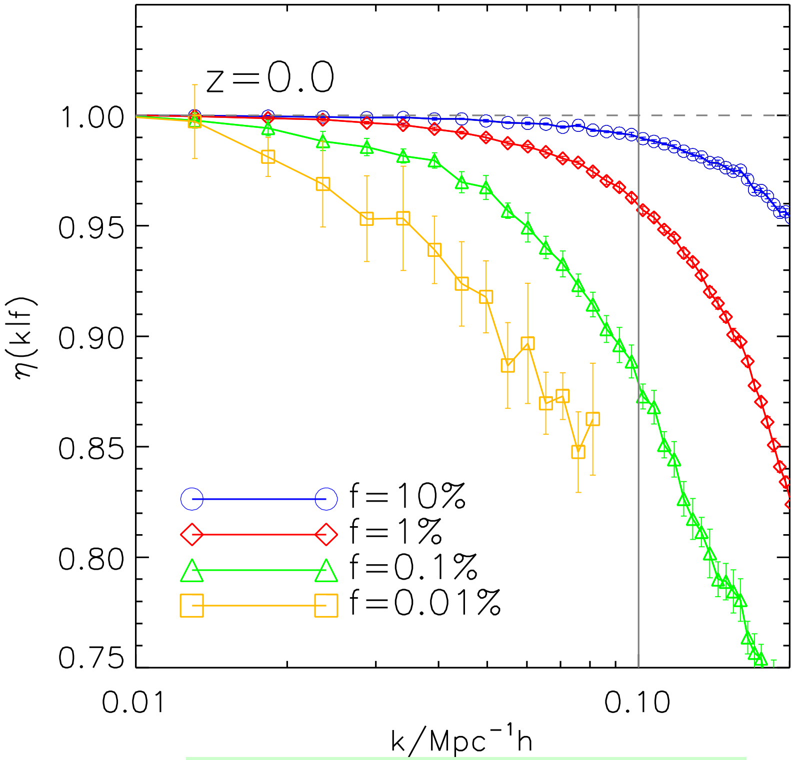
- **Our model works.**
- **But improvements are needed**
  - **Take correlation in D fully into account**
  - **Take v-D correlation into account**

Theory and simulation of the sampling artifact

The measured velocity power spectrum

The real velocity power spectrum

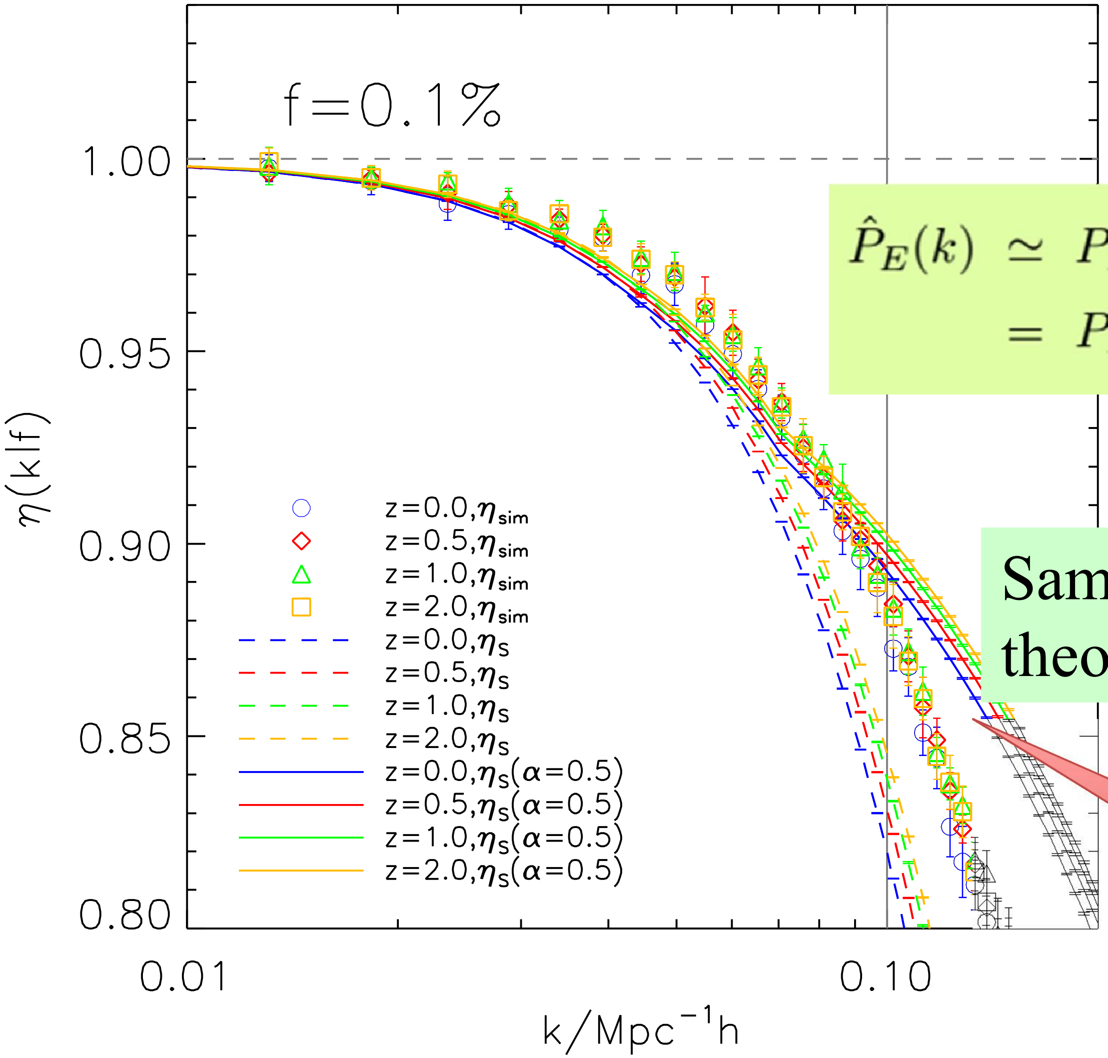




Theory prediction:
ZPJ, Zheng & Jing, 2014

Simulation verification
Zheng, ZPJ & Jing, 2014a

ZPJ's PPT



$$
\begin{aligned}
\hat{P}_E(k) &\simeq P_E(k)S(k)e^{\frac{1}{3}k^2\xi_D(r_{\rm eff}=\alpha/k)} \\
&= P_E(k)\langle e^{i\mathbf{q}\cdot\mathbf{D}}\rangle^2 e^{\frac{1}{3}k^2\xi_D(r_{\rm eff}=\alpha/k)}
\end{aligned}
$$

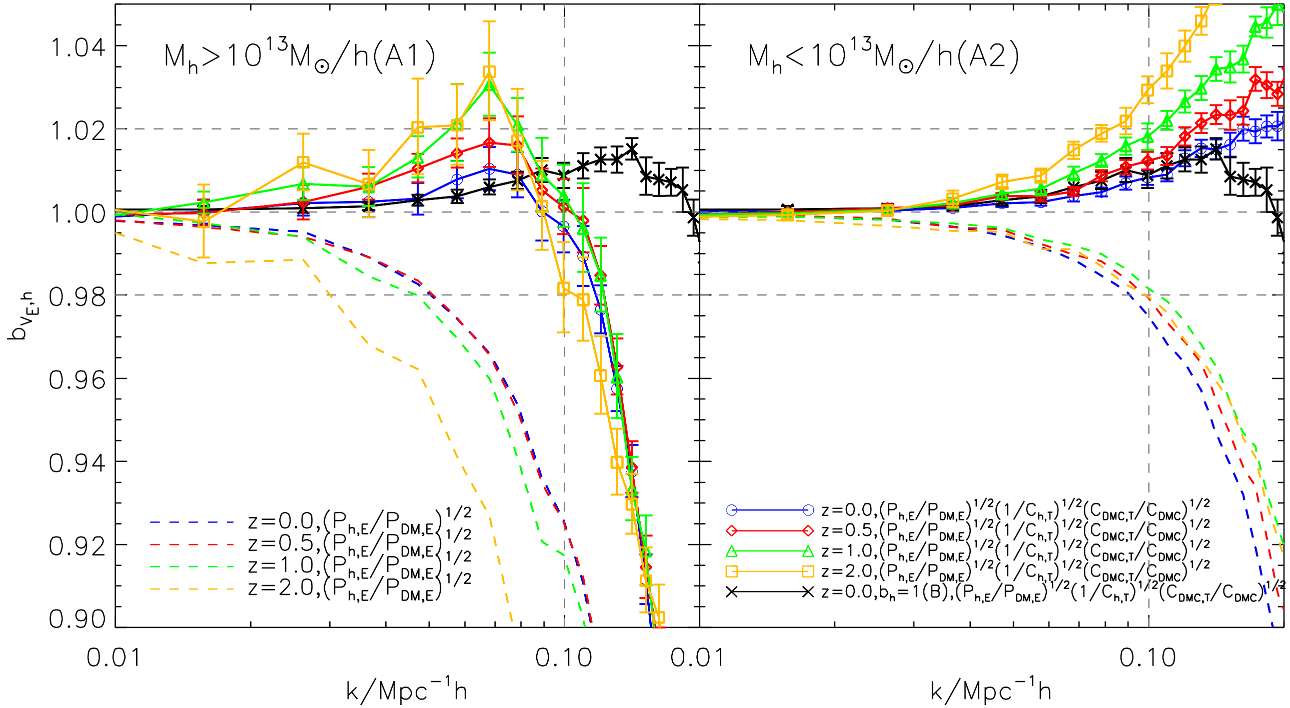
Sampling artifact: theory vs. simulation

1% accuracy <0.1h/Mpc

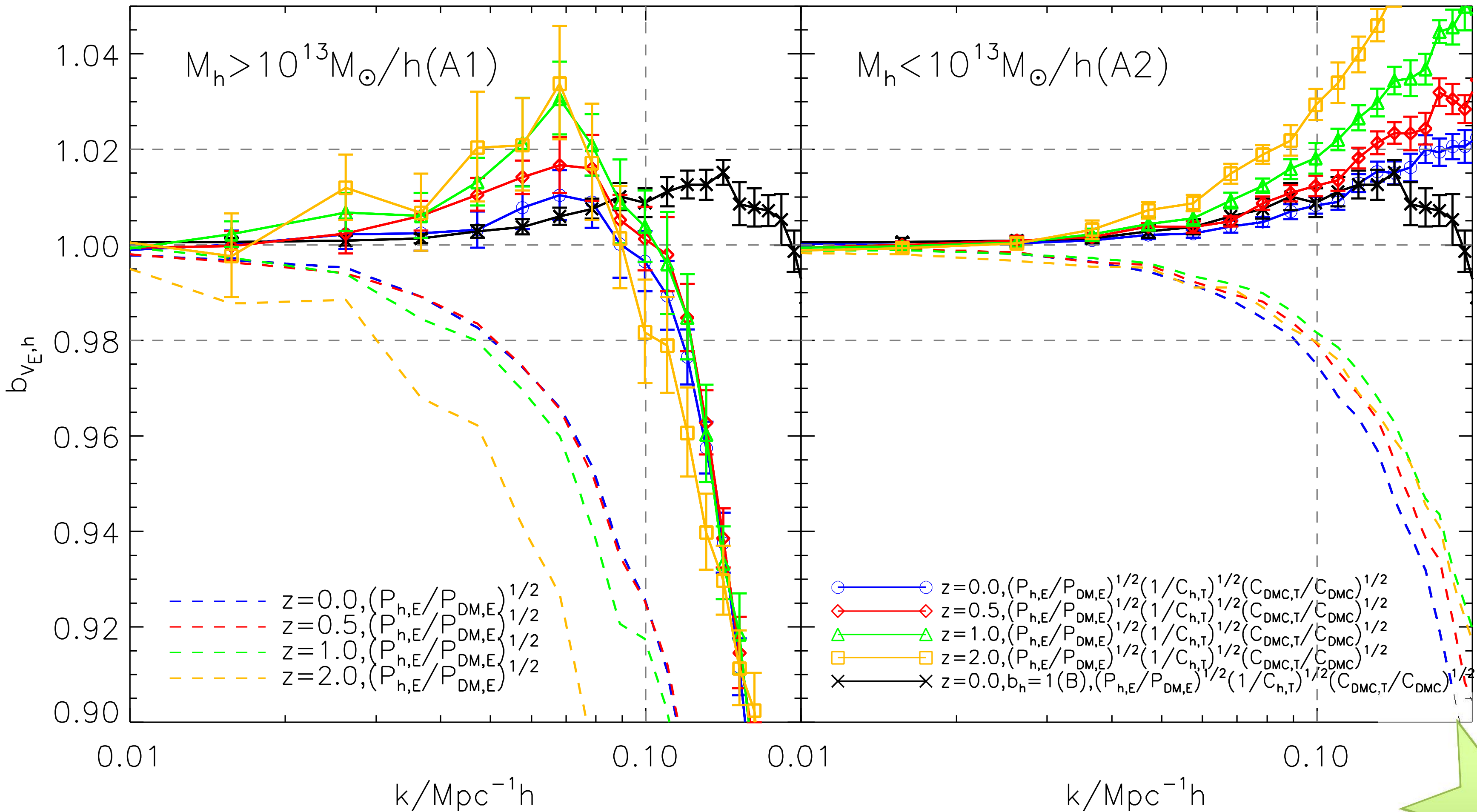
FIG. 4: Same as Fig. 3, but for $f = 0.1\%$ ($\bar{n}_P = 6.2 \times 10^{-4}(\mathrm{Mpc}/h)^{-3}$). Eq. 15 (solid curves) improves over Eq. 7 (dashed curves) from $\sim 6\%$ at $k = 0.1h/\mathrm{Mpc}$ to 1%.

Raw measurement	Step one Correction	Step two correction

$$b_v(k) \simeq \sqrt{\frac{\hat{P}^v_{h,E}(k)}{P^v_{\mathrm{DM},E}(k)}}\sqrt{\frac{1}{C_{h,T}(k)}}\sqrt{\frac{C_{\mathrm{DMC,T}}(k)}{C_{\mathrm{DMC}}(k)}}$$



ZPJ's PPT



1. Velocity bias vanishes at k<0.1 h/Mpc. Consolidates peculiar velocity cosmology
2. Velocity bias at k>0.1h/Mpc? Poses a challenge

ZPJ’s PPT

For discussions

- Needs theory explanation
- Needs more accurate quantification
  - Need improved understanding of the sampling artifact
- Needs to extend to galaxies (mock catalog)
- Perhaps needs new velocity assignment (e.g. Jun Zhang's idea)
- Cosmological applications
  - Could be smoking guns of MG
  - Promising tests of the equivalence principle (ongoing work with Zheng Yi, Yipeng, Baojiu Li & De-Chang Dai)

ZPJ's PPT