Simultaneous Optimization of Topology and Orientation of Anisotropic Material Using Isoparametric Projection Method

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Acknowledgments

Collaborators

- Dr. Ercan M. Dede (Toyota Research Institute of North America)
- Prof. Shintaro Yamasaki (Osaka University)
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- Dr. Tadayoshi Matsumori (Toyota Central R&D Labs., Inc.)
- Prof. Kazuhiro Izui (Kyoto University)
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Recent fabrication technologies and Orientation design

Tailored Fiber Placement

Super Robotics Sewing Machine

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Resin transfer molding

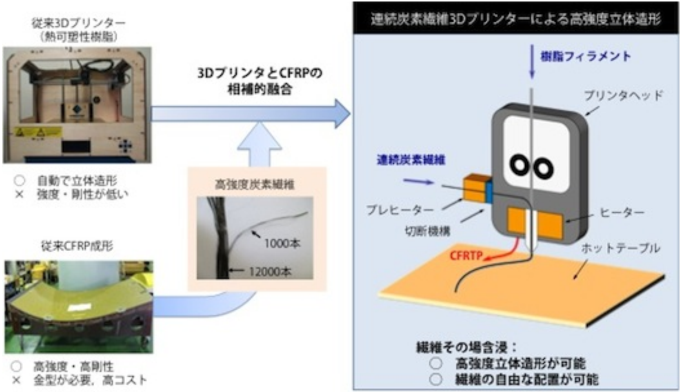
[a]

[a] https://www.behance.net/gallery/11063817/L1-lightweight-stool

[a] http://www.ipfdd.de/en/ departments/institute- of-polymer-materials/ composite-materials/ fields-of-research/ complex-structural- components-tailored-fibre- placement-tfp/

Recent fabrication technologies and Orientation design

Continuous fiber printing



Matsuzaki Lab / Tokyo University of Science[1]

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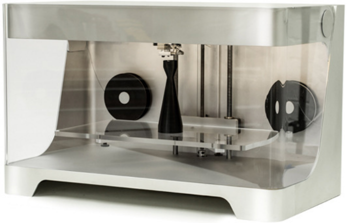
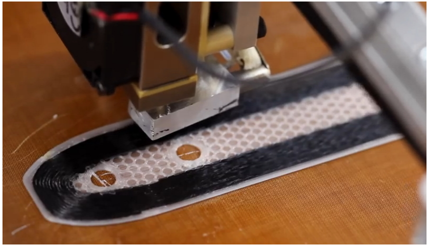
[1] http://www.rs.tus.ac.jp/rmatsuza/

Recent fabrication technologies and Orientation design

Continuous fiber printing





Markforged MarkOne [1]

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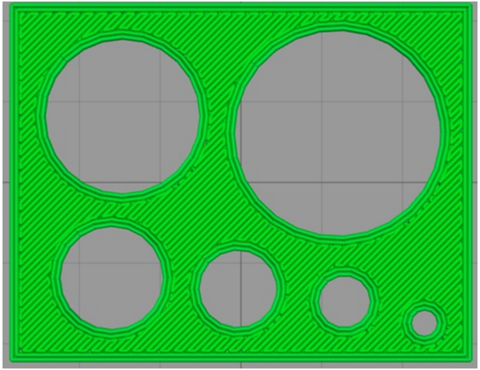
[1] http://www.markforged.com/

Recent fabrication technologies and Orientation design

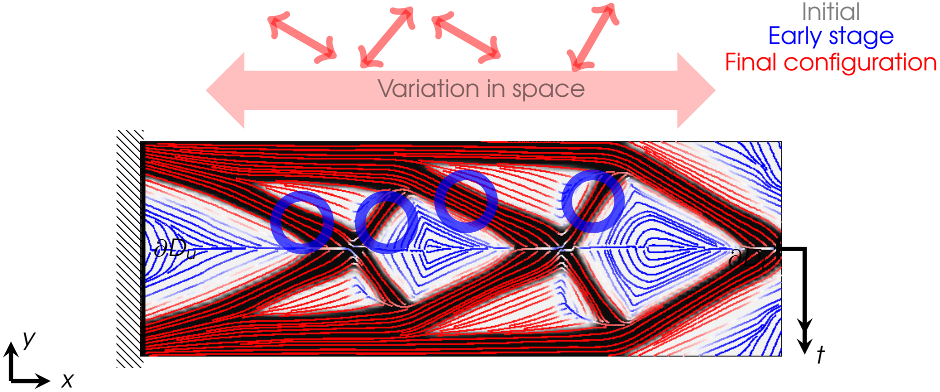
Toolpath depended anisotropy (Even without fiber)

 [a]

[a] http://www.re3d.org/

Topology and Orientation design of 2D / shell is becoming practical and more important

Challenges of orientation design problems



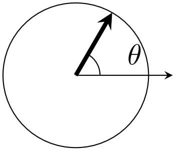
- Large local change in orientation
  - Has to be flexible for local change
- Complex structure due to topological change
  - Spatial regularization (**filtering**) is necessary

Parameterization schemes for Orientation design 1/2

Continuous Fiber Angle Optimization

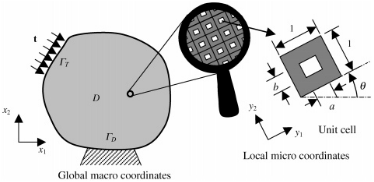
Pedersen P.1989

Directly use angle as design variable



Homogenization Design Method

Bendsøe M. P and Kikuchi N, 1988



Local minima issue

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \qquad \mathbf{C}^G_{ijkl} = \sum_{I,J,K,L=1}^{2} \mathbf{C}^H_{IJKL}\mathbf{R}_{Ii}(\theta)\mathbf{R}_{Jj}(\theta)\mathbf{R}_{Kk}(\theta)\mathbf{R}_{Ll}(\theta)$$

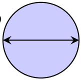
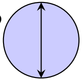
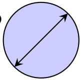
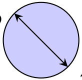
Parameterization schemes for Orientation design 1/2

More restrictive

Discrete Material Optimization (DMO)

Stegmann J and Lund E. 2005

- Based on multimaterial topology optimization

$\mathbf{C}_{ijkl} = \Big(\rho_1^p$  $+ \rho_2^p$  $+ \rho_3^p$  $+ \cdots + \rho_n^p$  $\Big)$ (1)

$\rho_1 + \rho_2 + \rho_3 + \cdots + \rho_n + \leqslant 1$

- Each orientation corresponds to a material

More permissive

Free Material Optimization

- Design all independent tensor element under loose constraints

Parametarization schemes for Orientation design 2/2

- Vector element parameterization

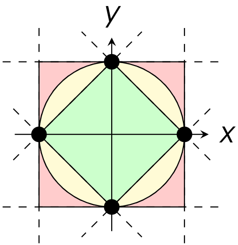
JS Choi, J Yoo, 2008

Orientation vector

$$\begin{cases} \mathbf{B}_r = B_r \begin{bmatrix} x \\ y \end{bmatrix} \\ -1 \leqslant \{x, y\} \leqslant 1, \\ \|x\| + \|y\| \leqslant 1 \end{cases} \quad (2)$$

Box constraint

Linear constraint



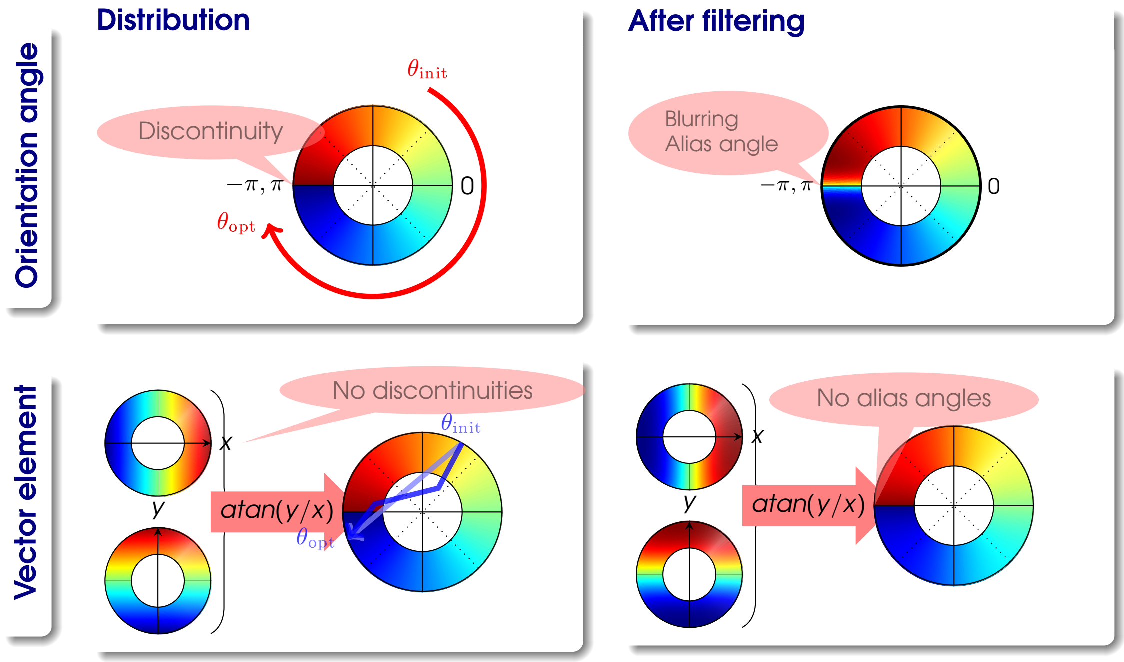
Unit circle Expensive (Pointwise nonlinear constraint)

Box constraint Low cost (free) but insufficient (infeasible properties)

Linear constraint Acceptable cost but too tight

Converges into discrete design but depends on global coordinate axis

Advantage of vector parameterization



PDE density filter / Vector case

Actual design variable

- Precursor variable, $\begin{bmatrix}\bar{\xi}\\ \bar{\eta}\end{bmatrix}$, Filtered variable, $\begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix}$, Projected variable, $\begin{bmatrix}\xi\\ \eta\end{bmatrix}$

- Helmholtz filtering

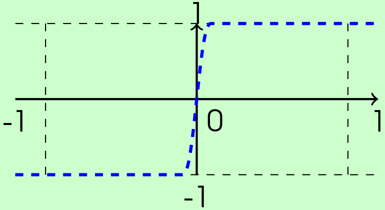
$$-\mathbf{R}_v \nabla^2 \begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix} + \begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix} = \begin{bmatrix}\bar{\xi}\\ \bar{\eta}\end{bmatrix}, \quad (5)$$

$\mathbf{R}_v$: $\left(\begin{matrix}\text{Filter}\\ \text{radius}\end{matrix}\right)^2 \mathbf{I}$

- Heaviside projection

$$\begin{bmatrix}\xi(\mathbf{x})\\ \eta(\mathbf{x})\end{bmatrix} = \begin{bmatrix}2\tilde{H}(\tilde{\xi}(\mathbf{x}))-1\\ 2\tilde{H}(\tilde{\eta}(\mathbf{x}))-1\end{bmatrix}. \quad (6)$$

Differentiable Heaviside function



PDE density filter / Vector case

Actual design variable

- Precursor variable, $\begin{bmatrix}\bar{\xi}\\ \bar{\eta}\end{bmatrix}$, Filtered variable, $\begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix}$, Projected variable, $\begin{bmatrix}\xi\\ \eta\end{bmatrix}$
- Helmholtz filtering

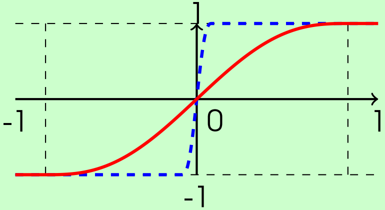
$\left(\text{Filter radius}\right)^2 \mathbf{I}$

$$-\mathbf{R}_v \nabla^2 \begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix} + \begin{bmatrix}\tilde{\xi}\\ \tilde{\eta}\end{bmatrix} = \begin{bmatrix}\bar{\xi}\\ \bar{\eta}\end{bmatrix}, \tag{5}$$

- Heaviside projection

$$\begin{bmatrix}\xi(\mathbf{x})\\ \eta(\mathbf{x})\end{bmatrix} = \begin{bmatrix}2\tilde{H}(\tilde{\xi}(\mathbf{x}))-1\\ 2\tilde{H}(\tilde{\eta}(\mathbf{x}))-1\end{bmatrix}. \tag{6}$$

Differentiable Heaviside function



Isoparametric projection method

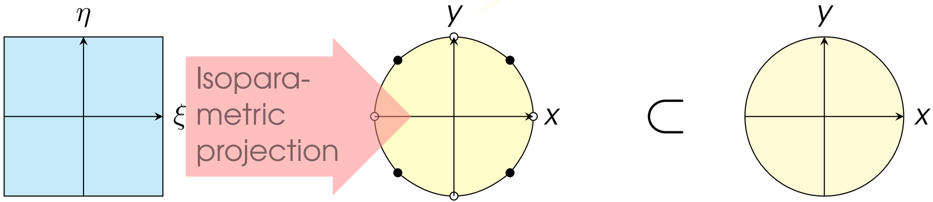
- Vectorial design variable, $\begin{bmatrix} \xi \\ \eta \end{bmatrix}$ (Projected variable)

  $-1 \leqslant \xi \leqslant 1, -1 \leqslant \eta \leqslant 1,$
- Isoparametric projection,

$$\vartheta = \begin{bmatrix} c \\ s \end{bmatrix} = \mathbf{N}\left(\xi, \eta\right) \quad (7)$$

Serendipity element
Second order eight node quadrilateral element

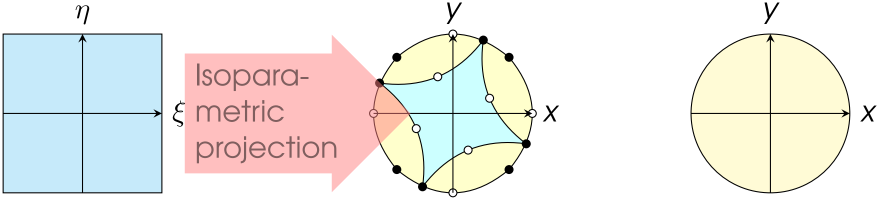
- Orientation vector, $c^2 + s^2 \leqslant 1$



$-1 \leqslant \xi \leqslant 1, \quad -1 \leqslant \eta \leqslant 1$ Shape function: **N** $\subset$ $c^2 + s^2 \leqslant 1$

Transform box constraints to circular constraints

Isoparametric projection method for discrete angle set

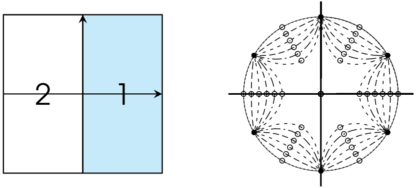


$-1 \leqslant \xi \leqslant 1, \quad -1 \leqslant \eta \leqslant 1$

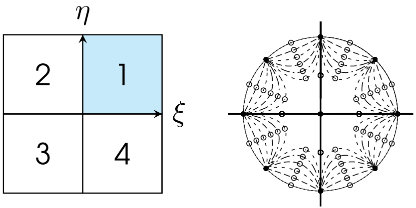
Shape function: **N**

$c^2 + s^2 \leqslant 1$

6 options / 2 elements



8 options / 4 elements



Basic idea

- $\mathbf{C}_{ijkl}$ has 4th order pronominal of $\cos(\theta)$ and $\sin(\theta)$.
- These are the element of the orientation vector, i.e. $\mathbf{e} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$.
- We also have $\vartheta = \begin{bmatrix} c \\ s \end{bmatrix}$, we can replace

  $\cos(\theta)$ and $\sin(\theta)$ by c and s, respectively

Material interpolation

Solid part (Anisotropic)

Void part

Void part

$$\mathbf{C}(\rho, \vartheta) = \mathbf{C}_v + \rho^p \left(\mathbf{C}_i + \underbrace{\hat{\mathbf{T}}^{-1}(\tilde{\vartheta}) \cdot (\mathbf{C}_u - \mathbf{C}_i) \cdot \hat{\mathbf{T}}'(\tilde{\vartheta})}_{\text{Rotated Tensor}} - \mathbf{C}_v \right). \quad (8)$$

Isotropic component of $\mathbf{C}_u$

$$\hat{\mathbf{T}}^{-1}(\vartheta)\mathbf{C}\hat{\mathbf{T}}'(\vartheta) = \hat{\mathbf{C}}(\vartheta) = \underbrace{\begin{bmatrix} \hat{C}_{11}(c,s) & \hat{C}_{12}(c,s) & \hat{C}_{16}(c,s) \\ \hat{C}_{12}(c,s) & \hat{C}_{22}(c,s) & \hat{C}_{26}(c,s) \\ \hat{C}_{16}(c,s) & \hat{C}_{26}(c,s) & \hat{C}_{66}(c,s) \end{bmatrix}}_{\text{4th order Polynomial functions of } c \text{ and } s} \quad (9)$$

E depends on $\sqrt{(c^2 + s^2)}$

Material interpolation

Rotation tensor can be written as

$$\mathbf{T}(\vartheta) = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}, \qquad \mathbf{T}'(\vartheta) = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}. \qquad (10)$$

therefore, elements of $\hat{\mathbf{C}}(\vartheta)$ are

$$\begin{aligned}
\hat{C}_{11} &= C_{11}c^4 - 4C_{16}c^3s + (2C_{12} + 4C_{66})c^2s^2 - 4C_{26}cs^3 + C_{22}s^4 \\
\hat{C}_{12} &= C_{12}c^4 + 2(C_{16} - C_{26})c^3s + (C_{11} + C_{22} - 4C_{66})c^2s^2 - 2(C_{16} - C_{26})cs^3 + C_{12}s^4 \\
\hat{C}_{16} &= C_{16}c^4 + (C_{11} - C_{12} - 2C_{66})c^3s - 3(C_{16} - C_{26})c^2s^2 + (C_{12} - C_{22} + 2C_{66})cs^3 - C_{26}s^4 \\
\hat{C}_{22} &= C_{22}c^4 + 4C_{26}c^3s + (2C_{12} + 4C_{66})c^2s^2 + 4C_{16}cs^3 + C_{11}s^4 \\
\hat{C}_{26} &= C_{26}c^4 + (C_{12} - C_{22} + 2C_{66})c^3s + 3(C_{16} - C_{26})c^2s^2 + (C_{11} - C_{12} - 2C_{66})cs^3 - C_{16}s^4 \\
\hat{C}_{66} &= 2(C_{16} - C_{26})c^3s + (C_{11} - 2C_{12} + C_{22} - 2C_{66})c^2s^2 - 2(C_{16} - C_{26})cs^3 + C_{66}(c^4 + s^4).
\end{aligned}$$
(11)

4th order polynomials, no trigonometric functions

Mapping to $-\pi/2 < \theta < \pi/2$

Variable $\vartheta = \begin{bmatrix} c \\ s \end{bmatrix}$ covers whole circle, $-\pi < \theta < \pi$, but since $\mathbf{C}_{ijkl}$ is non-directional, we want to map $-\pi < \theta < \pi$ into $-\pi/2 < \theta < \pi/2$

$$
\begin{aligned}
\cos^4(\theta/2) &= (1+\cos(\theta))^2 \Big/ 4 & \cos^3(\theta/2)\sin(\theta/2) &= \sin(\theta)\,(1+\cos(\theta)) \Big/ 4 \\
\cos^2(\theta/2)\sin^2(\theta/2) &= (1-\cos(\theta))(1+\cos(\theta)) \Big/ 4 & \cos(\theta/2)\sin^3(\theta/2) &= \sin(\theta)\,(1+\cos(\theta)) \Big/ 4 \\
\sin^4(\theta/2) &= (1-\cos(\theta))^2 \Big/ 4\,, & &\text{(Half angle formulas)}
\end{aligned}
$$

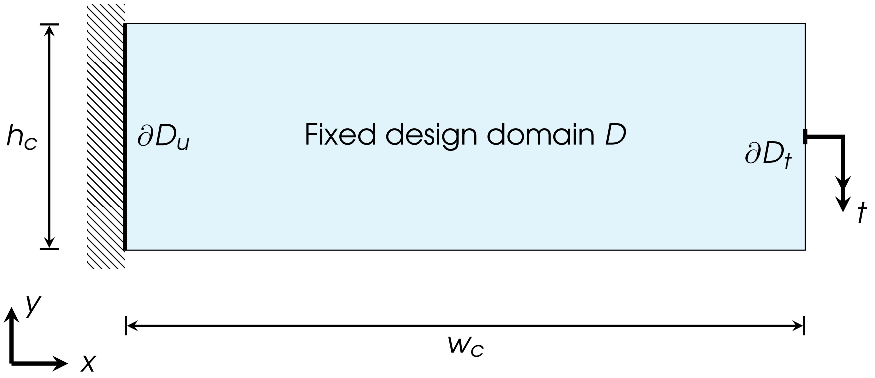
(12)

and therefore

$$
\begin{aligned}
\hat{C}_{11} &= \left\{(1+c)^2 C_{11} - (1-c)^2 C_{22} - 4\,(1+c)\,sC_{16} - 4\,(1-c)\,sC_{26} + 2\left(1-c^2\right)(2C_{66}+C_{12})\right\} \Big/ 4 \\
\hat{C}_{12} &= \left\{\left(1-c^2\right)(C_{11}+C_{22}) + 2\left(1+c^2\right) C_{12} + 4cs\,(C_{16}-C_{26}) + 4\left(c^2-1\right) C_{66}\right\} \Big/ 4 \\
\hat{C}_{16} &= \left\{s\,(1+c)\,C_{11} - s\,(1-c)\,C_{22} - \left(2-2c-4c^2\right) C_{16} + \left(2+2c-4c^2\right) C_{26} - 2cs\,(C_{12}+2C_{66})\right\} \Big/ 4 \\
\hat{C}_{22} &= \left\{-(1-c)^2 C_{11} + (1+c)^2 C_{22} + 4s\,(1-c)\,C_{16} + 4s\,(1+c)\,C_{26} + 2\left(1-c^2\right)(2C_{66}+C_{12})\right\} \Big/ 4 \\
\hat{C}_{26} &= \left\{s\,(1-c)\,C_{11} - s\,(1+c)\,C_{22} + 2csC_{12} + \left(2+2c-4c^2\right) C_{16} - \left(2-2c-4c^2\right) C_{26} - 4sC_{66}\right\} \Big/ 4 \\
\hat{C}_{66} &= \left\{\left(1-c^2\right)(C_{22}+C_{11}) - 2\left(1-c^2\right) C_{12} + 4cs\,(C_{16}-C_{26}) + 4c^2 C_{66}\right\} \Big/ 4\,.
\end{aligned}
\tag{13}
$$

2nd order polynomial, no trigonometric functions!!

Numerical examples / Short cantilever

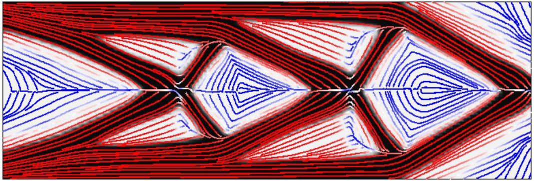


Height $h_c = 1$, **Width** $h_c = 3$, **Mesh size** $d = 0.02$, **Filter size** $d = 0.1d$,

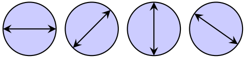
Short cantilever / Result 1/2

Volume fraction = 0.5

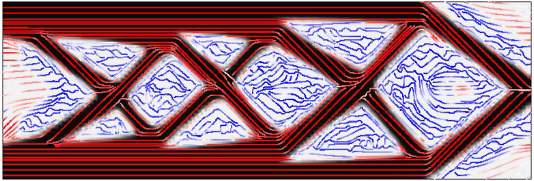
Continuous, C=4.17



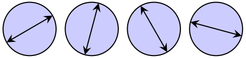
Discrete 4 options C=4.98



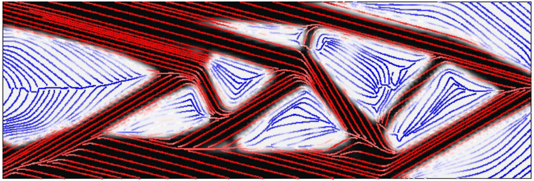
$(0, \pi/4, \pi/2, 3\pi/4)$



Discrete 4 options C=5.52



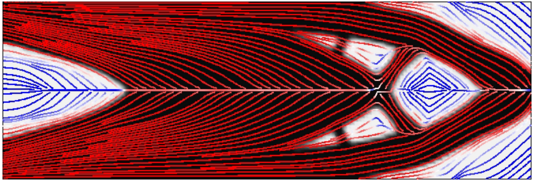
$(0, \pi/4, \pi/2, 3\pi/4) + \pi/6$



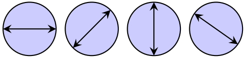
Short cantilever / Result 2/2

Volume fraction = 0.75

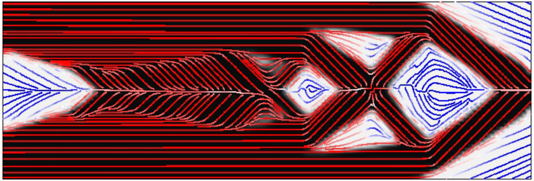
Continuous, C=3.09



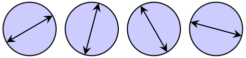
Discrete 4 options C=3.78



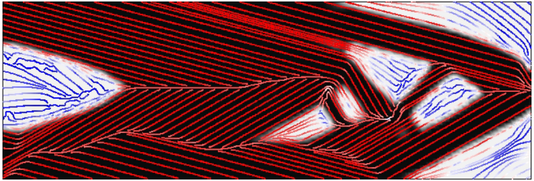
$(0, \pi/4, \pi/2, 3\pi/4)$



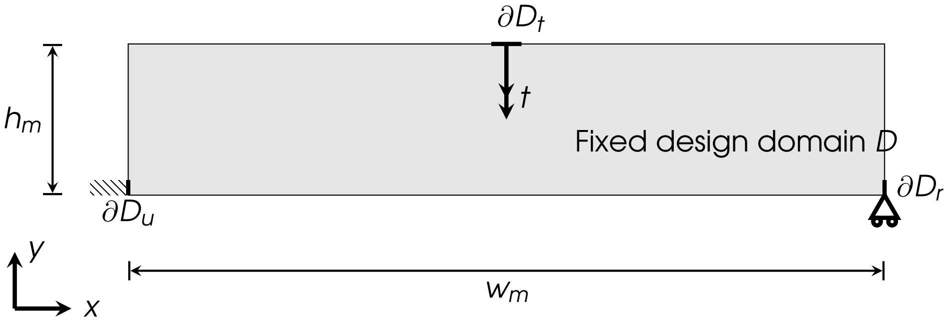
Discrete 4 options C=4.12



$(0, \pi/4, \pi/2, 3\pi/4) + \pi/6$



Numerical examples / MBB beam



Height $h_c = 1$, **Width** $h_c = 5$, **Mesh size** $d = 0.02$, **Filter size** $d = 0.1d$,

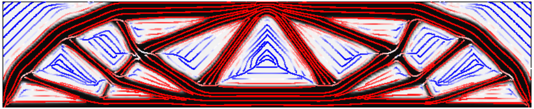
MBB beam / Result

Isotropic, Volume fraction 0.25

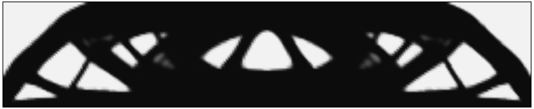
Anisotropic, Volume fraction 0.25

Isotropic, Volume fraction 0.5

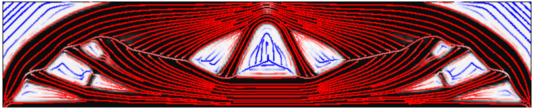
Anisotropic, Volume fraction 0.5



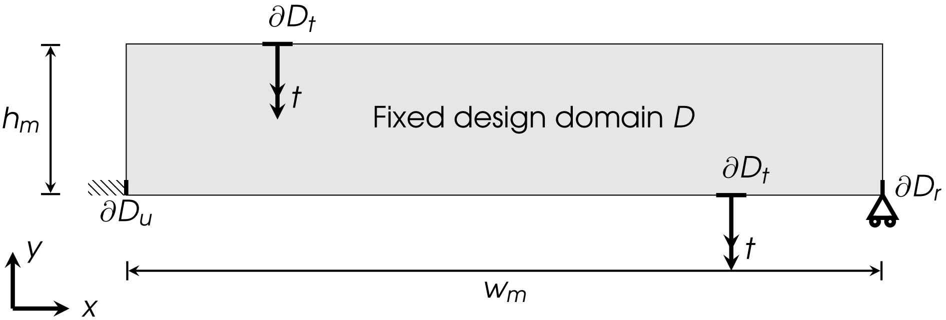
Isotropic, Volume fraction 0.75



Anisotropic, Volume fraction 0.75



Numerical examples / Multiload beam

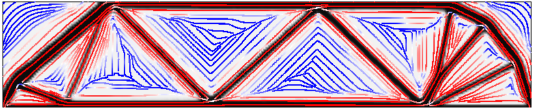


Height $h_c = 1$, **Width** $h_c = 5$, **Mesh size** $d = 0.02$, **Filter size** $d = 0.1d$,

Multiload beam / Result, Multiload case

Isotropic, Volume fraction 0.25

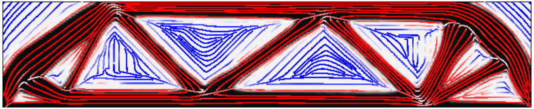
Anisotropic, Volume fraction 0.25



<4>

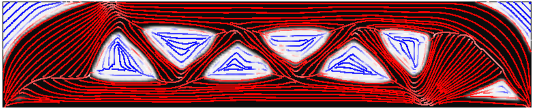
Isotropic, Volume fraction 0.5

Anisotropic, Volume fraction 0.5

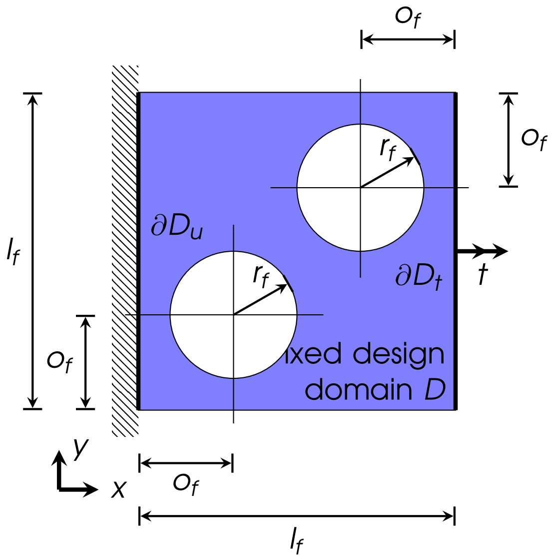


Isotropic, Volume fraction 0.75

Anisotropic, Volume fraction 0.75



Numerical examples / Holed panel problem



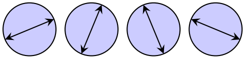
Side length $l_f = 2$

Hole radius $r_f = 0.4$

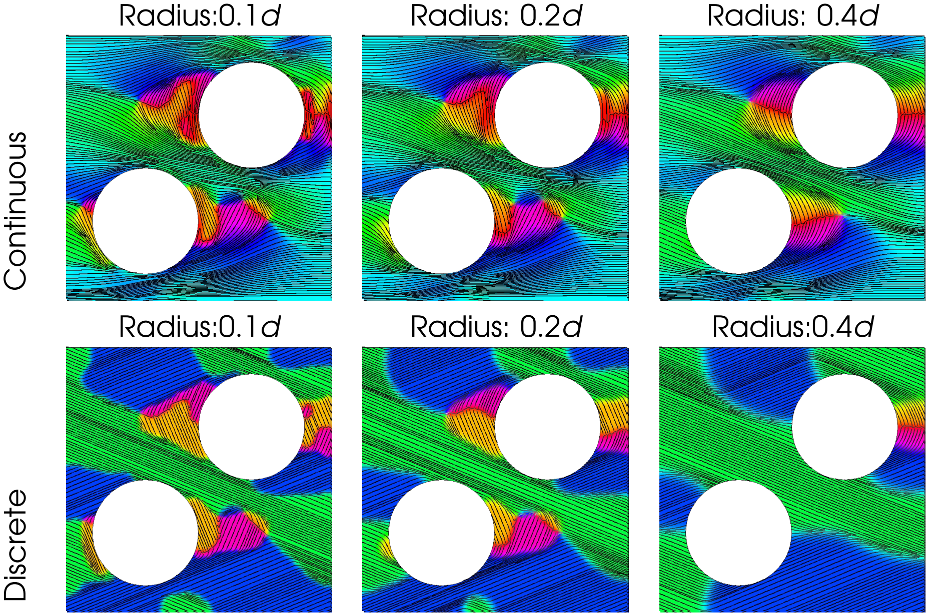
Hole offset $o_f = 0.6$

Mesh size $d = 0.02$

Angle set $(0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3}{4}\pi) + \frac{\pi}{8}$



Holed panel problem / Results



Future work

Toolpath design

- Toolpath generation algorithm based on PDE is being developed

3D design

1. 3D design of density + Layer by layer 2D orientation design
   - For current composite printers
2. Orientation design on 3D surfaces
   -
3.
4. Full 3D orientation design
   - For Full 3D composite printer

Open platform composite printer is desired!!

- Open driver development, open G-code specification (or at least SDK)
- Open hardware based research platform

Conclusion

- A method for simultaneous optimization of topology and orientation is proposed.
- Capable for both continuous design and discrete design
- Comparison to orientation angle design
  - Compatible with density filtering
  - Less local optima issue
- Comparison to DMO
  - Continuous design
  - Design variable is 2 per design point in 2D problem regardless to number of options
  - DMO offers more degree of freedom in material choice
- Comparison to FMO
  - Physically feasible design

General topology optimization method with continuous and discrete orientation design using isoparametric projection
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