Title and Abstract

Nicholas Edelen (MIT)

Lecture 1: *Convexity estimates for free-boundary mean curvature flow*

Abstract of Lecture 1: We prove the convexity estimates of Huisken-Sinestrari for mean-convex, free-boundary mean curvature flows, and use this to show that type-II singularities are smoothly modelled on self-translators having free-boundary in a plane. We allow for any barrier hypersurface that is embedded with bounded geometry.

Lecture 2: *The free-boundary Brakke flow*

Abstract of Lecture 2: We extend Brakke's weak notion of mean curvature flow to have a free-boundary condition, and using toy examples we show why this extension is necessary. Contrary to the classical free-boundary flow considered in the first talk, for which the barrier is "invisible," our notion allows for the flow to "pop" or break up upon tangential contact with the barrier. We adapt Ilmanen's elliptic regularization and White's a priori curvature estimates to prove existence of the free-boundary Brakke flow, and smooth short-time existence.

Lami Kim (Tokyo Institute of Technology)

On the mean curvature flow of grain boundaries

Abstract : Suppose that $\Gamma_0 \subset \mathbb{R}^{n+1}$ is a closed countably n-rectifiable set whose complement $\mathbb{R}^{n+1} \setminus \Gamma_0$ consists of more than one connected component. Assume that the n-dimensional Hausdorff measure of Γ_0 is finite or grows at most exponentially near infinity. Under these assumptions, we prove a global-in-time existence of mean curvature flow in the sense of Brakke starting from Γ_0. There exists a finite family of open sets which move continuously with respect to the Lebesgue measure, and whose boundaries coincide with the space-time support of the mean curvature flow. This is a joint work with Y. Tonegawa.

Keita Kunikawa (Tohoku University)

Convergence of generalized Lagrangian mean curvature flow in Fano manifolds

Abstract : In this talk, we generalize several results for the Hamiltonian stability and the mean curvature flow of Lagrangian submanifolds in a Kahler-Einstein manifolds to more general Kahler manifolds including Fano manifolds by using the methodology proposed by T. Behrndt. We first consider a weighted measure on a Lagrangian manifold in such a Kahler manifold and investigate the variational problem of the Lagrangian for the weighted volume under Hamiltonian deformations. We call a stationary point and a local minimizer of the weighted volume f-minimal and Hamiltonian f-stable. We show such examples naturally appear in toric Fano manifolds. Moreover, we consider the generalized Lagrangian mean curvature flow which is introduced by Behrndt and also by Smoczyk-Wang. We generalize the result by H. Li, and show that if the initial Lagrangian is a small Hamiltonian deformation of an f-minimal and Hamiltonian f-stable Lagrangian, then the generalized MCF converges to an f-minimal one.

Haozhao Li (University of Science and Technology of China)

Lecture 1 : *The extension problem of mean curvature flow*

Abstract of Lecture 1 : In this talk, I will show that the mean curvature blows up at the first finite singular time for a closed smooth embedded mean curvature flow in $\mathbb{R}^3$. This is joint work with Bing Wang.

Lecture 2 : *Regularity scales and convergence of the Calabi flow*

Abstract of Lecture 2 : We introduce regularity scales to study the behavior of Calabi flow. Based on the estimates of regularity scales, we obtain convergence theorems of the Calabi flow with the long time existence assumption. This is joint work with Bing Wang and Kai Zheng.

Felix Schulze (University College London)

Lecture 1: *Existence of Brakke flow solutions from surface clusters via elliptic regularisation*

Abstract of Lecture 1: We consider clusters of n-dimensional surfaces in any codimension where away from a closed set with zero $(n-1)$-dimensional Hausdorff measure, along the smooth boundaries three sheets meet under an equal angle. We show that under a topological condition, there exists a Brakke flow starting from such a cluster, which attains the initial cluster in C^∞ away from the junctions and in C^1 at the triple junctions. For curves (i.e. networks) in any codimension the assumption of equal angles initially is not necessary, as long as they are positive. The proof uses elliptic regularisation and a recent regularity result for mean curvature flow with triple edges. This is joint work with B. White.

Lecture 2: *Optimal isoperimetric inequalities for surfaces in any codimension in Cartan-Hadamard manifolds*

Abstract of Lecture 2: Let (M^n, g) be simply connected, complete, with non-positive sectional curvatures, and Σ a 2-dimensional surface in M^n. Let S be an area minimising 3-current such that $\partial S = \Sigma$. We use a weak mean curvature flow, obtained via elliptic regularisation, starting from Σ, to show that S satisfies the optimal Euclidean isoperimetric inequality: $|S| \leq 1/(6\sqrt{\pi})|\Sigma|^{3/2}$. We also obtain the optimal estimate in case the sectional curvatures of M are bounded from above by $\kappa < 0$ and characterise the case of equality. The proof follows from an almost monotonicity of a suitable isoperimetric difference along the approximating flows in one dimension higher.

Knut Smoczyk (University of Hannover)

Mean curvature flow of maps between Riemannian manifolds, Part 1 and Part 2

Abstract of Lecture 1 and Lecture 2: This talk consists of two parts. In the first part I will introduce mean curvature flow and in particular the flow by mean curvature of a graph generated by maps between Riemannian manifolds. In the second part we discuss more specific results, like the evolution of length and area decreasing maps or area preserving maps between Riemannian manifolds. In the two-dimensional weakly area decreasing case we give a complete classification of the longtime behavior of the flow. The results are joint work with Andreas Savas-Halilaj.

Keisuke Takasao (Kyoto University)

Phase field method and monotonicity formula for the volume preserving mean curvature flow

Abstract: In this talk, we show the global existence of the weak solution for the volume preserving mean curvature flow via the reaction diffusion equation with a non-local term studied by Golovaty. We also show the monotonicity formula for the problem.

Hikaru Yamamoto (Tokyo University of Science)

Ricci-mean curvature flows and its Gauss maps

Abstract: First, I introduce a Ricci-mean curvature flow. A Ricci-mean curvature flow is a coupled equation of a mean curvature flow and a Ricci flow. The ambient metric is evolving under the Ricci flow and a submanifold is moving in this ambient space along the mean curvature flow. Recently, Ricci-mean curvature flows have been appeared in some contexts. In this talk, I will give a generalization of a theorem due to E. Ruh and J. Vilms. They proved that the Gauss map of a minimal submanifold in a Euclidean space is a harmonic map. Then, our generalization is a time dependent version of that theorem. It says that the Gauss maps of a Ricci-mean curvature flow is a vertically harmonic map heat flow. This is also a generalization of a result due to M.-T. Wang for a mean curvature flow in a Euclidean space. I will also give some applications of this theorem and its variant. This talk is based on a joint work with N. Koike.