The Green function of the interior transmission problem for the diffusion equation and its applications

Gen Nakamura
(Hokkaido University, Japan)

Consider the interior transmission problem arising in inverse boundary value problems for the diffusion equation with discontinuous diffusion coeffcients. We prove the unique solvability of the interior transmission problem by constructing its Green function. First, we construct a local parametrix for the interior transmission problem near the boundary in the Laplace domain, by using the theory of pseudo-diferential operators with a large parameter. Second, by carefully analyzing the analyticity of the local parametrix in the Laplace domain and estimating it there, a local parametrix for the original parabolic interior transmission problem is obtained via the inverse Laplace transform. Finally, using a partitionof unity, we patch all the local parametrices and the fundamental solution of the diffusion equation to generate a global parametrix for the parabolic interior transmission problem, and then compensate it to get the Green function by the Levi method. The uniqueness of the Green function is justified by using the duality argument, and then the unique solvability of the interior transmission problem is concluded. We would like to emphasize that the Green function for the parabolic interior transmission problem is constructed for the first time in this study.

As an application of the unique solvability of the interior transmission problem, we will give a linear sampling method for the diffusion equation to identify an unknown inclusion inside a known background.