Superconductivity in Quasicrystals

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Collaborators

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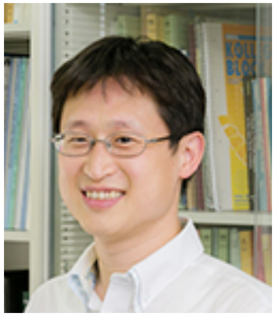
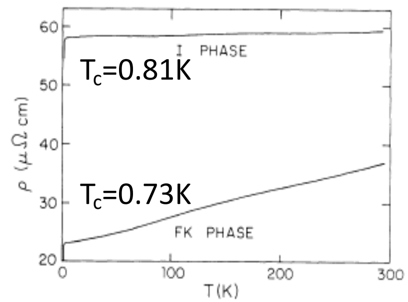


Superconducting quasicrystal?

Al-Zn-Mg

Wong *et al*., PRB **35**, 2494 (1987)
Graebner and Chen, PRB **58**, 1945 (1987)

- No magnetization data.
- Later identified as approximant.



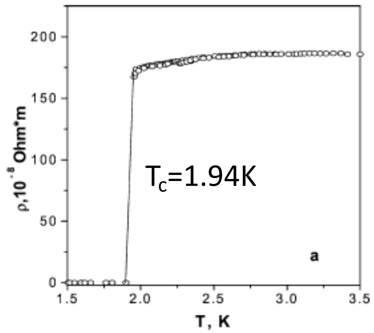
Al-Cu-Mg, Al-Cu-Li

Wagner *et al*., PRB **38**, 7436 (1988)

- No magnetization data.

Ti-Zr-Ni

Azhazha *et al*, Phys. Lett. A **303**, 87 (2002)



- No magnetization data.
- Possible mixture of other phase.

For the discovery of a new superconductor, we would have

- Zero resistivity
- Meissner effect
- Identification of crystal structure
- Reproducibility

by K. Kitazawa

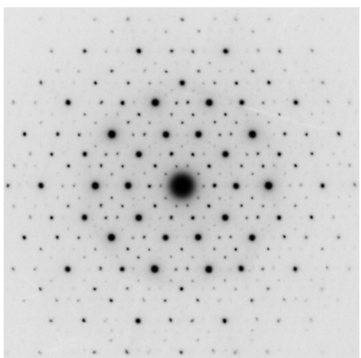
DOI: 10.1038/s41467-017-02667-x OPEN

Discovery of superconductivity in quasicrystal

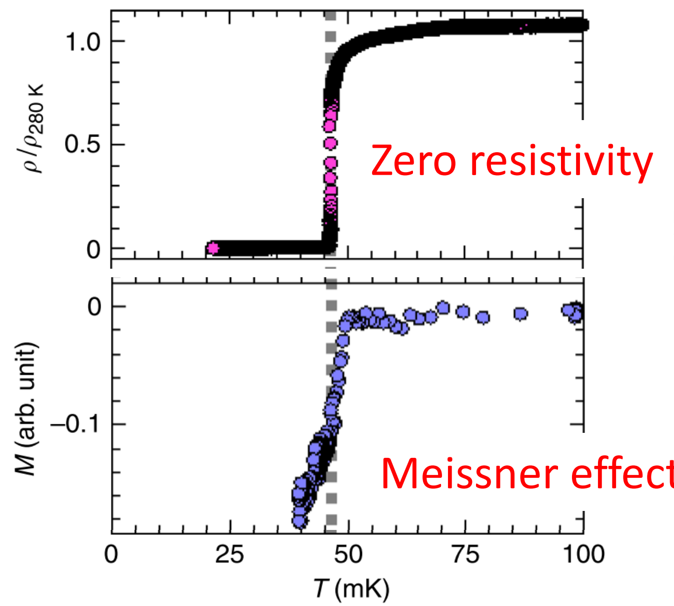
K. Kamiya[1,5], T. Takeuchi[2], N. Kabeya[3], N. Wada[1], T. Ishimasa[4], A. Ochiai[3], K. Deguchi[1], K. Imura[1] & N.K. Sato[1]

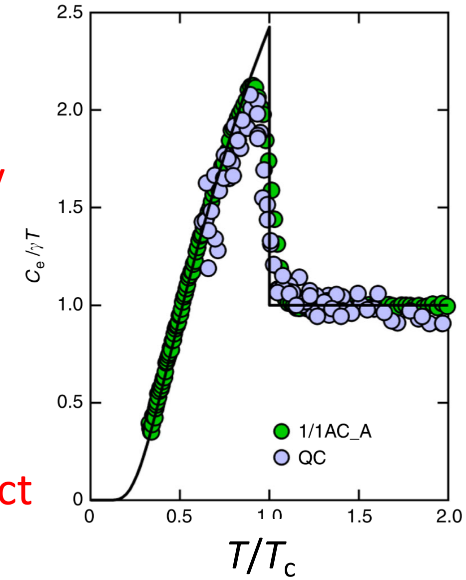
Al-Zn-Mg alloy

T_c=50mK



Structure identification





Bulk property

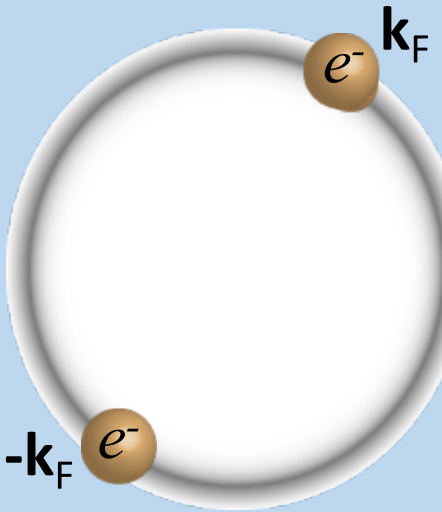
First example of electronic long-range order in QC

Top 10 Breakthroughs of 2018 in Physics World

What's the issue?

Standard understanding of superconductivity

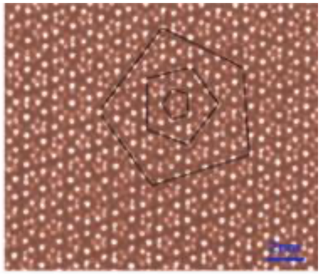
- Presence of Fermi surface
- Cooper pair
  = (2 el. with $\mathbf{k}_F$ and $-\mathbf{k}_F$)
- Many properties calculated
  in momentum space.



Quasicrystal: No momentum space, no Fermi surface

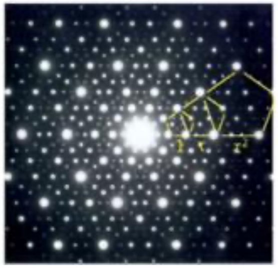
How can we understand a superconducting QC?

Note: Two different reciprocal spaces



$\rho(\boldsymbol{r}) = \langle c_{\boldsymbol{r}}^{\dagger} c_{\boldsymbol{r}} \rangle$ *Local density*

Figures from https://www.kek.jp/ja/newsroom/2011/12/08/1200/



$$S(\boldsymbol{q}) = \int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d\boldsymbol{r}$$

q: F. T. of *absolute* coordinate **r**

In periodic systems, Fermi surface is defined by a peak in

$$A(\boldsymbol{k}, \omega = E_F) = -\frac{1}{\pi} \mathrm{Im} G(\boldsymbol{k}, \omega = E_F)$$

$$\text{F. T. of } G(\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, t) = -i \langle T c_{\boldsymbol{r}_1}(t) c_{\boldsymbol{r}_2}^{\dagger}(0) \rangle$$

k: F. T. of *relative* coordinate

This is not well defined in QC.

What's the issue?

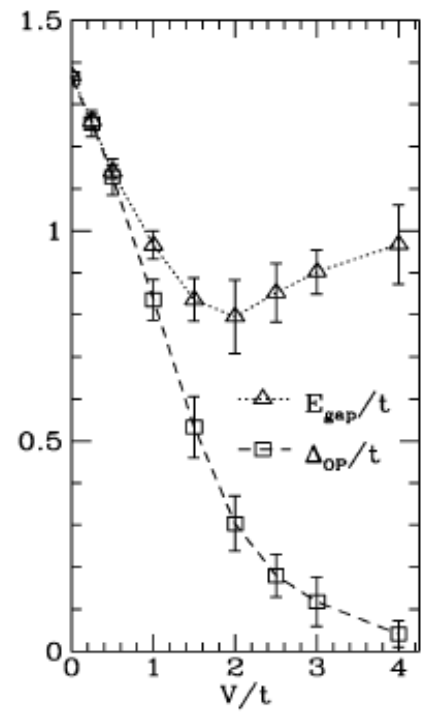
Anderson's theorem

P. Anderson, J. Phys. Chem. Solids **11**, 26 (1959)

s-wave superconductivity is robust against weak (nonmagnetic) disorder

But, strong disorder can destroy SC!

Figure from A. Ghosal, M. Randeria, and N. Trivedi, PRL **81**, 3940 (1998).



Normal state: metal → Anderson insulator

What about quasicrystal?

Normal state: critical wave function

What's the issue?

Quasicrystal

Self-similarity (fractal)

Superconductivity

Macroscopic quantum state

Novel SC properties?

Fractal superconductivity!

How to address the issues?

- No momentum space
- Nonuniform (but not random)

DFT for approximants?

M. Saito, T. Sekikawa, and Y. Ono,
Phys. Status Solidi B 2000108 (2020):
Conductivity and specific heat

Simplified model for QC?

Essence:

- Quasiperiodicity
- Pairing attraction

Our approach

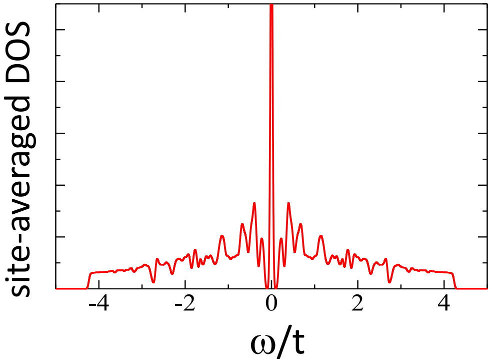
cf. 1D: M. Tezuka and A. M. Garcia-Garcia, PRA **82**, 043613 (2010).

Model of quasiperiodicity

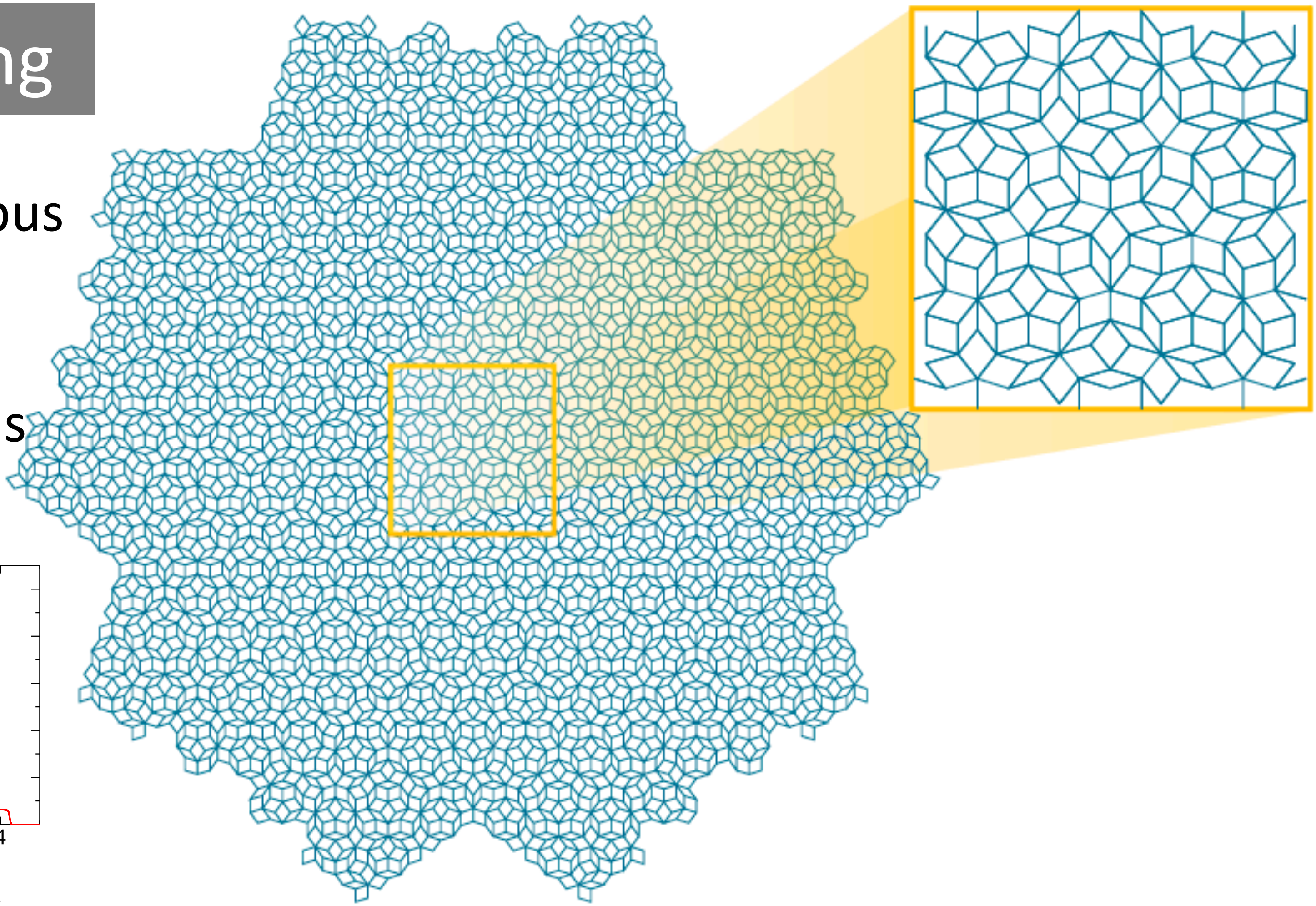
Penrose tiling

Vertex of rhombus
→ lattice point

Edge of rhombus
→ hopping t



"Bandwidth" = $8.46t$



Five-fold rotational symmetry

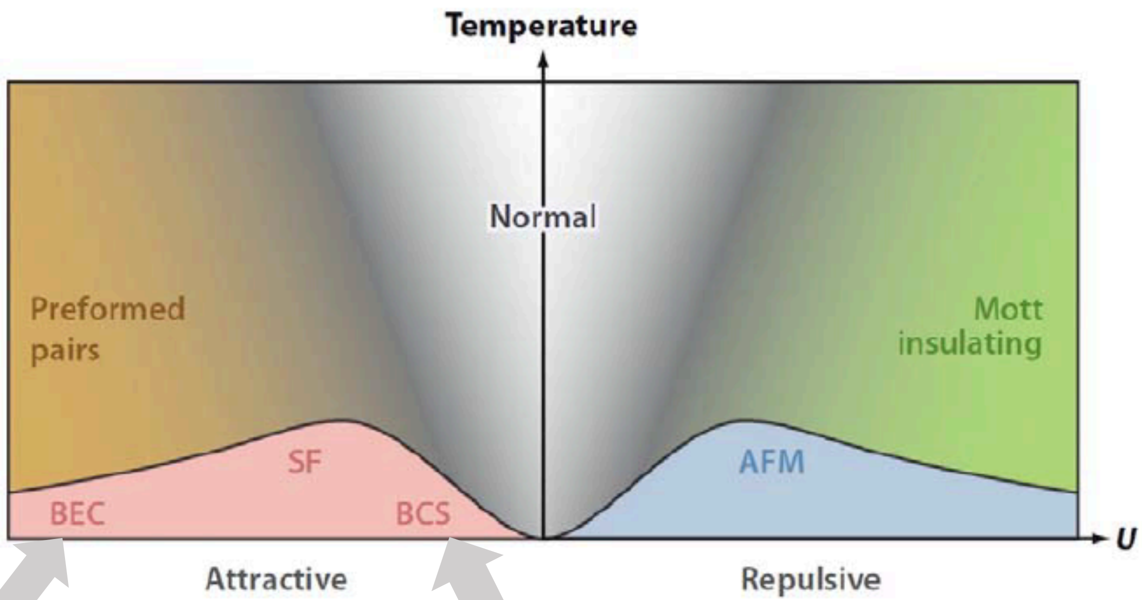
Model with pairing attraction

Attractive Hubbard model

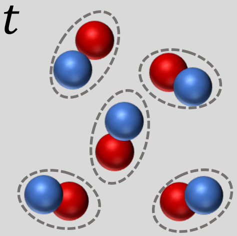
$$H = -t\sum_{\langle ij\rangle\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i\sigma} n_{i\sigma} + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$U < 0$

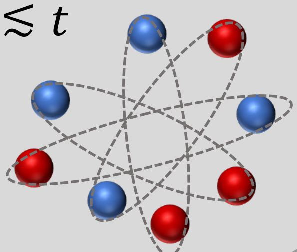
Cubic lattice
Half filling



Strong attraction
$|U| \gg t$



Weak attraction
$|U| \lesssim t$

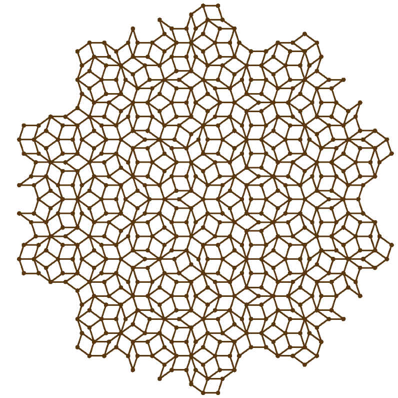


- SC at low T for any U<0.
- BCS-BEC crossover with U.

Attractive Hubbard model on Penrose tiling

$$H = -t \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} - \mu \sum_{i\sigma} n_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

on



Inhomogeneity → Real-space approaches

Bogoliubov-de Gennes theory (BdG)

- Static mean field (one-body approx., weak *U*)
- Large size ~ 1 million sites : Y. Nagai, JPSJ **89**, 074703 (2020)

Real-space dynamical mean-field theory (RDMFT)

- Dynamical mean field (many-body physics, weak-to-strong *U*)
- < 10,000 sites

A. Georges *et al.*, RMP **68**, 13 (1996)
M. Potthoff and W. Nolting, PRB **59**, 2549 (1999)

Bogoliubov - de Gennes theory (BdG)

Eigenenergy
Eigenstates

$$n_{i\sigma} = \langle c_{i\sigma}^{\dagger} c_{i\sigma} \rangle$$

$$\Delta_i = U \langle c_{i\uparrow} c_{i\downarrow} \rangle$$

Site-dependent local quantities

$$\hat{H}_{BdG} = \begin{pmatrix} Un_{1\downarrow} - \mu & -t & \cdots & \Delta_1 & 0 & \cdots \\ -t & Un_{2\downarrow} - \mu & \cdots & 0 & \Delta_2 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ \Delta_1 & 0 & \cdots & -Un_{1\uparrow} + \mu & t & \cdots \\ 0 & \Delta_2 & \cdots & t & -Un_{2\uparrow} + \mu & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

: 2N x 2N matrix

Real-space dynamical mean-field theory (RDMFT)

Site-dependent impurity problem

$$\hat{g}_0(i\omega_n)^{-1} = \hat{G}_{ii}(i\omega_n)^{-1} + \hat{\Sigma}_i(i\omega_n)$$

Exact diag.

$$\hat{\Sigma}_i(i\omega_n) = \begin{pmatrix} \Sigma_i^{nor}(i\omega_n) & \Sigma_i^{ano}(i\omega_n) \\ \Sigma_i^{ano}(i\omega_n) & -\Sigma_i^{nor}(-i\omega_n) \end{pmatrix}$$

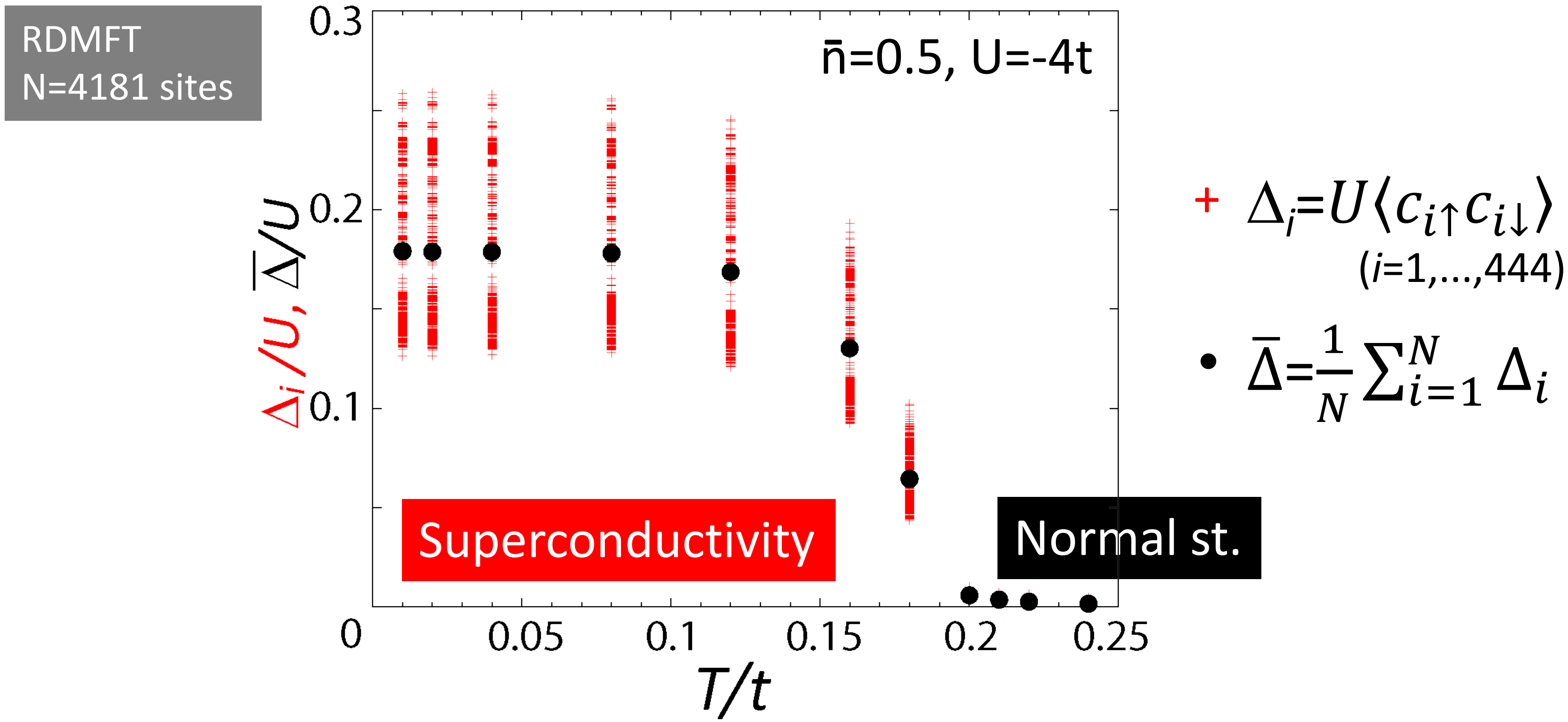
Site- & energy-dependent local self-energy

$$\hat{G}(i\omega_n)^{-1} = \begin{pmatrix} i\omega_n + \mu - \Sigma_1^{nor} & -t & \cdots & -\Sigma_1^{ano} & 0 & \cdots \\ -t & i\omega_n + \mu - \Sigma_2^{nor} & \cdots & 0 & -\Sigma_2^{ano} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ -\Sigma_1^{ano} & 0 & \cdots & i\omega_n - \mu + \Sigma_1^{nor} & t & \cdots \\ 0 & -\Sigma_2^{ano} & \cdots & t & i\omega_n - \mu + \Sigma_2^{nor} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

: 2N x 2N matrix

- *Geometry of the Penrose lattice comes in the one-body part.*
- *Nonlocal correlations are neglected.*

Local SC order parameter

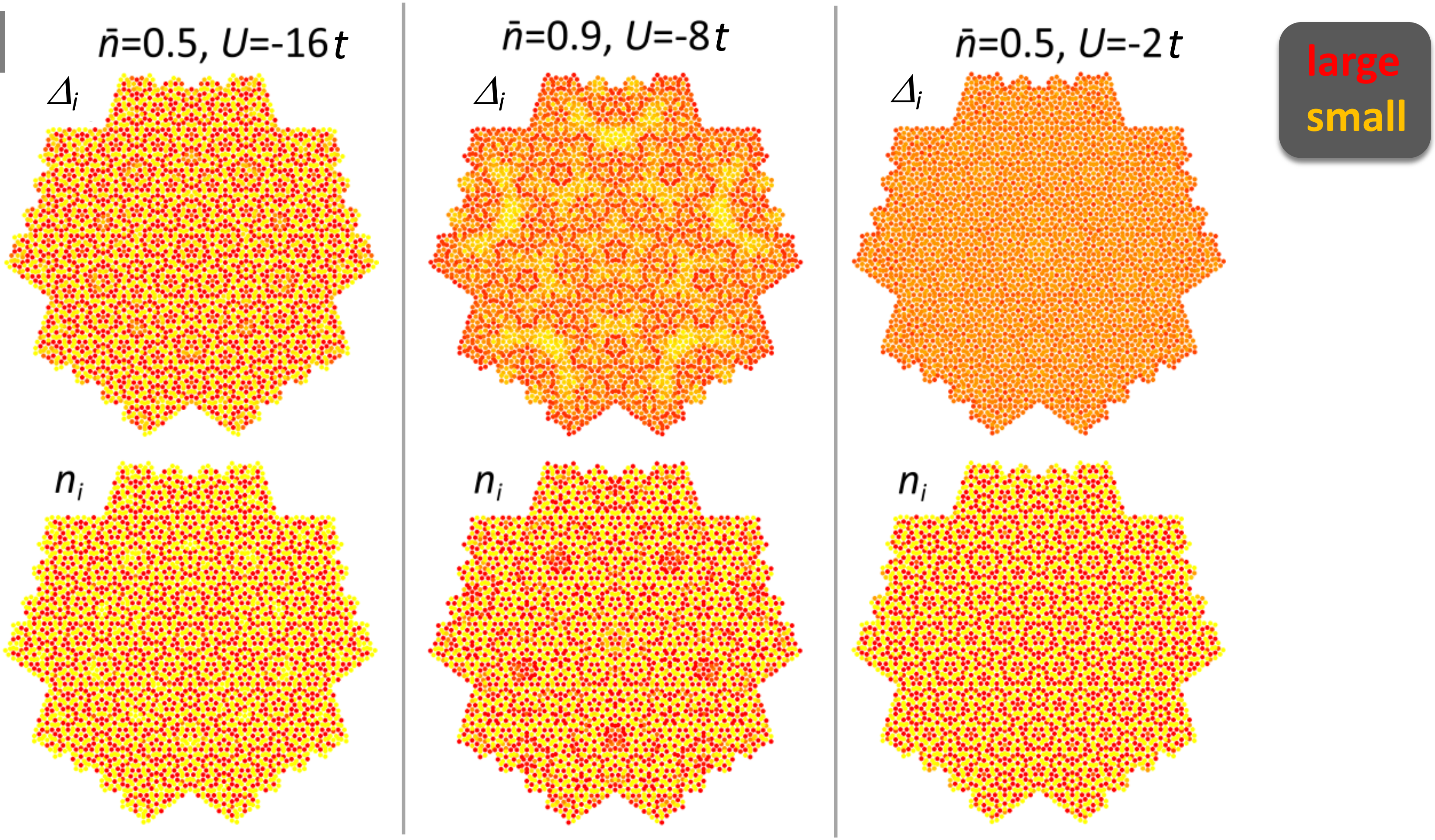


- Superconductivity occurs at low *T*.
- Transition occurs *simultaneously* at every sites.

SS, N. Takemori, A. Koga, and R. Arita, PRB **95**, 024509 (2017).

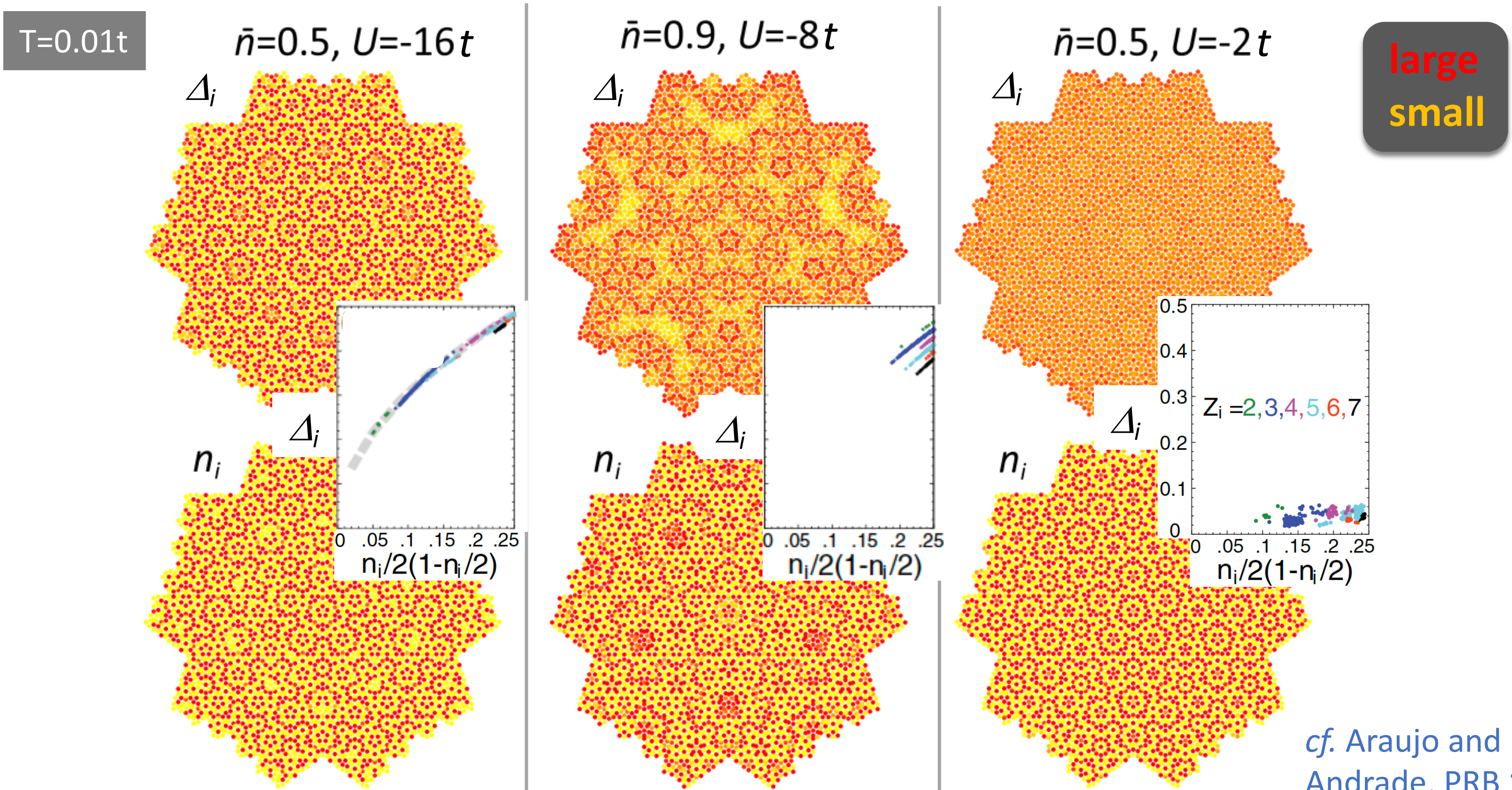
Three different superconducting states

T=0.01t



Δ_i	Order similar to n_i	Order different from n_i	No clear pattern

Three different superconducting states



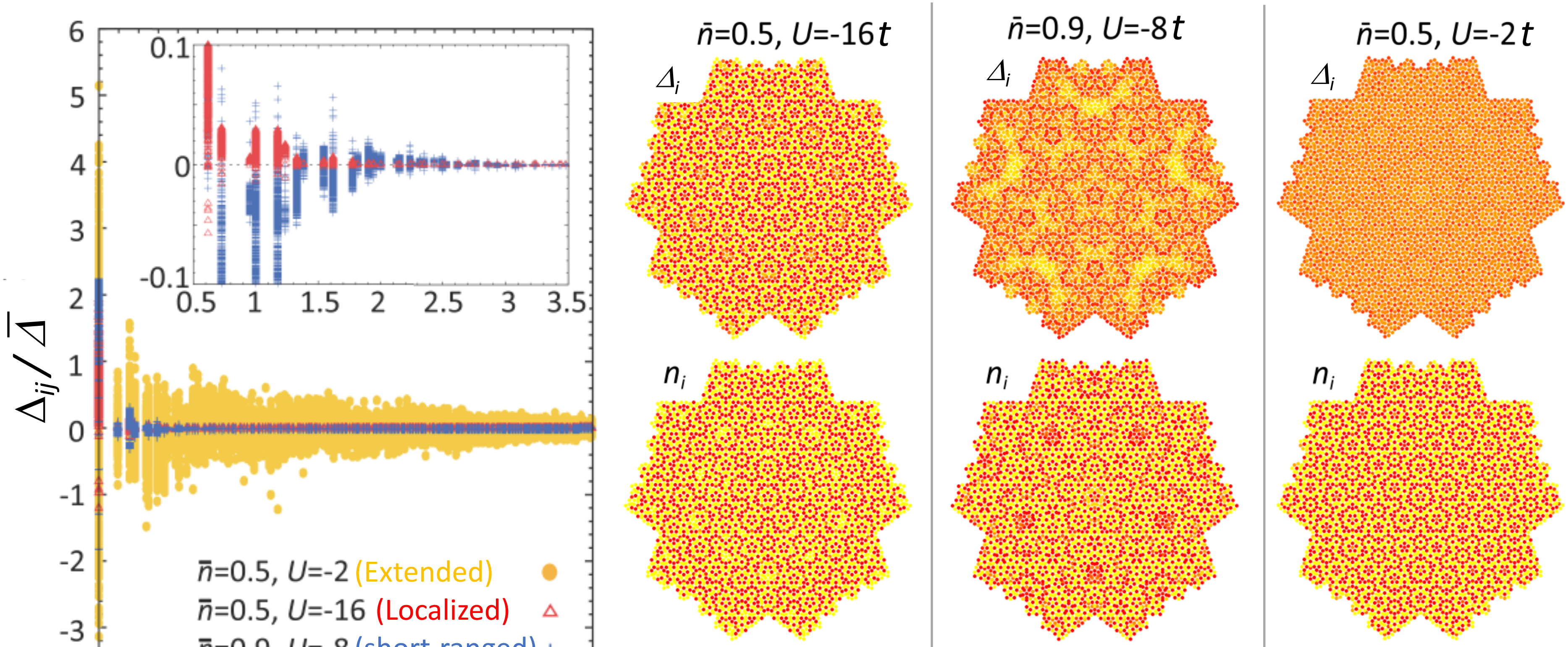
cf. Araujo and Andrade, PRB **100**, 014510 (2019)

Δ_i	Order similar to n_i	Order different from n_i	No clear pattern
	→ Determined by n_i	→ Determined by Z_i	→ Determined by LDOS(?)

SS, N. Takemori, A. Koga, and R. Arita, PRB **95**, 024509 (2017).

Spatial extension of Cooper pairs

$\Delta_{ij}=\langle c_{i\uparrow}c_{j\downarrow}\rangle$: Off-site SC order parameter



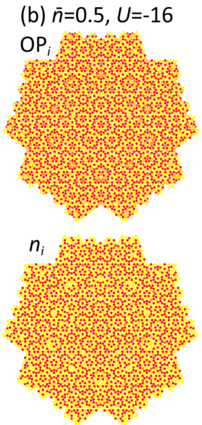
Red (n_i) **Blue (Z_i)** **Yellow (LDOS)**

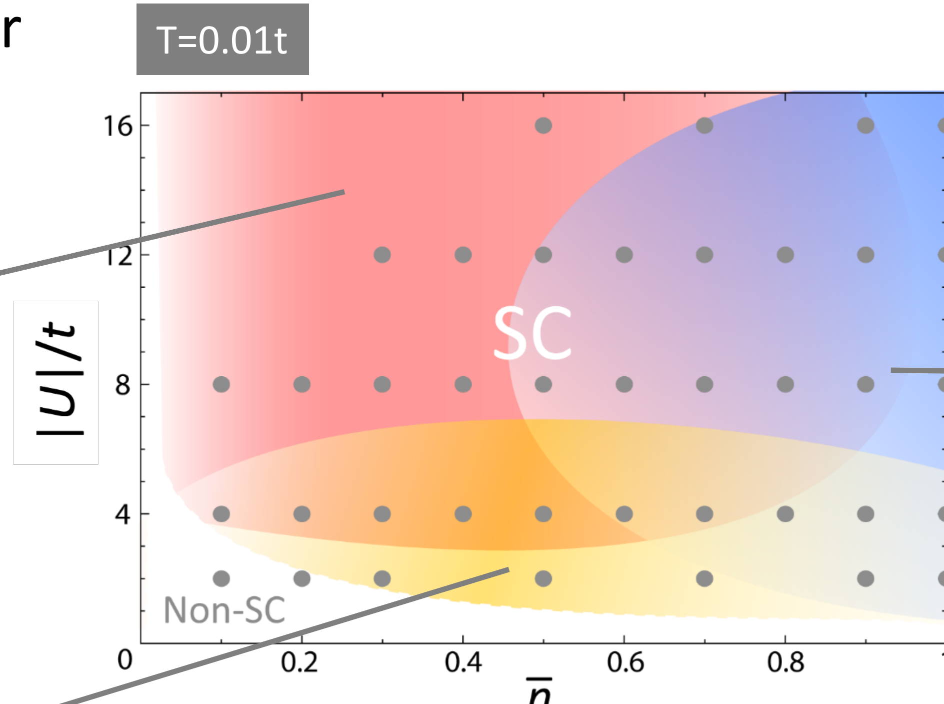
Cooper pairs are

Localized **Short-ranged** **Extended**

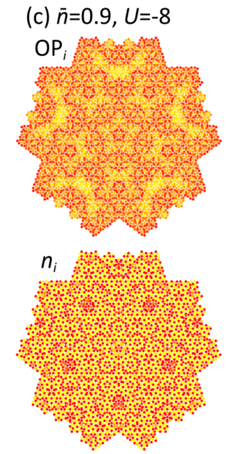
Crossover of three different SC states

Localized pair

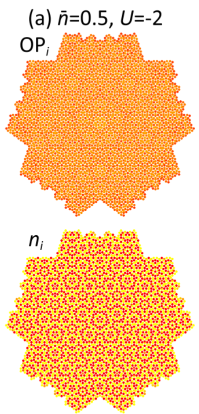




Short-ranged pair



Extended pair



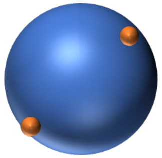
cf. Cooper instability in quasiperiodic system

Y. Zhang *et al.*, arXiv:2002.06485

$$|\Psi_{\rm A}\rangle = \sum_{mn}^{\tilde{\varepsilon}_{m,n}>0} a_{mn}\hat{c}^{\dagger}_{m\uparrow}\hat{c}^{\dagger}_{n\downarrow}|{\rm FS}\rangle$$

$\langle\Psi_A|\hat{H}|\Psi_A\rangle < 0$ for any U<0

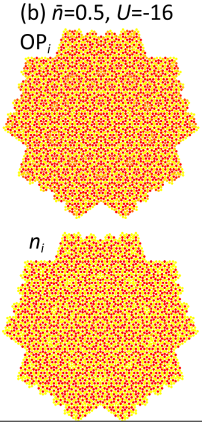
$$\Delta \propto \exp\left[\frac{1}{\alpha U}\right]$$

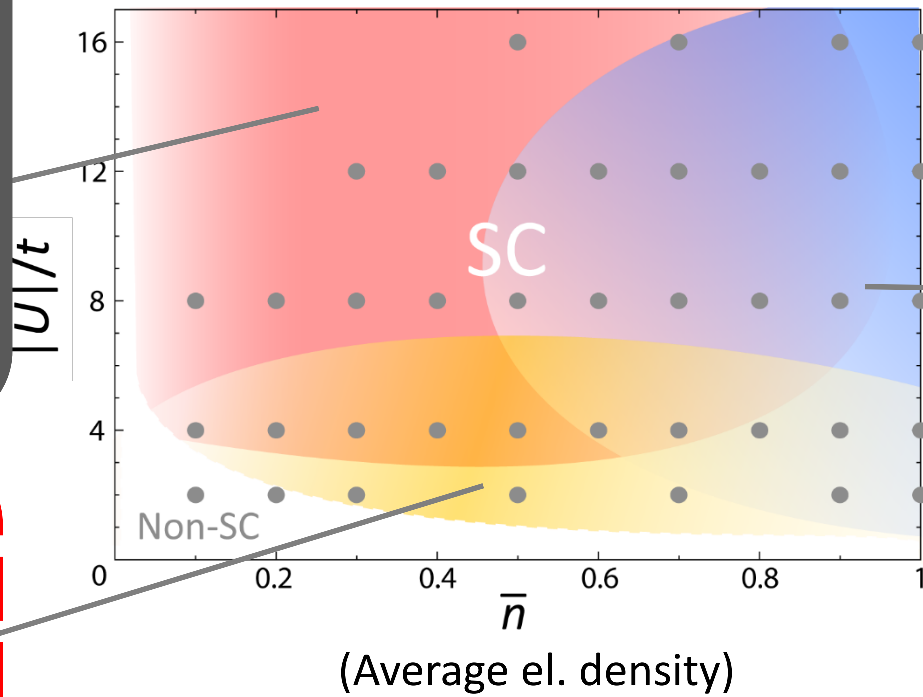


Crossover of three different SC states

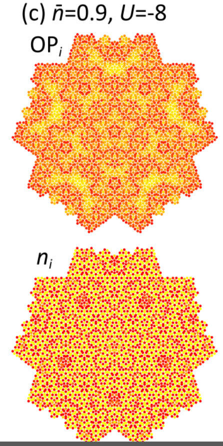
BEC: Lattice structure may be irrelevant

Localized pair

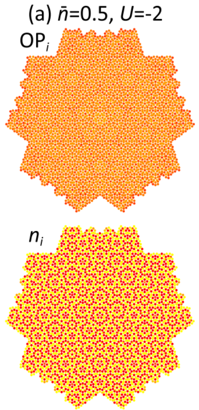




Short-ranged pair



Extended pair



Extended without Fermi surface!
What's happening?

Unusual pairing in momentum space (1)

Cooper pair on periodic lattice: $c_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}$

➡ $c_{\mathbf{k}\uparrow}c_{\mathbf{k}'\downarrow}$ for aperiodic lattice

FT of relative coordinate *i-j*

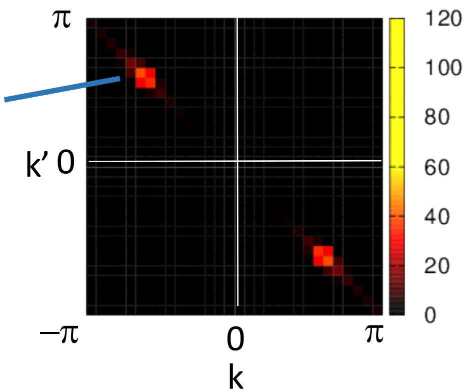
FT of *i* and *j*, respectively

Square lattice

$|\langle c_{\mathbf{k}\uparrow}c_{\mathbf{k}'\downarrow}\rangle|$ for $k_x=k_y=k$ and $k_x'=k_y'=k'$

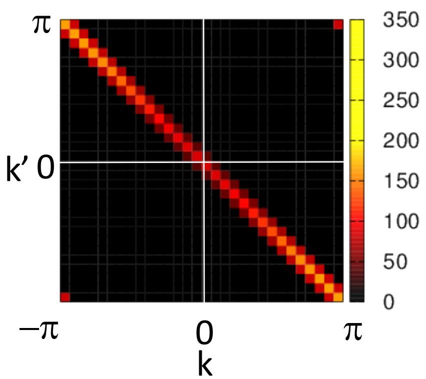
BCS **(U=-t, n=0.5)**

Fermi momentum



Finite only along k'=-k

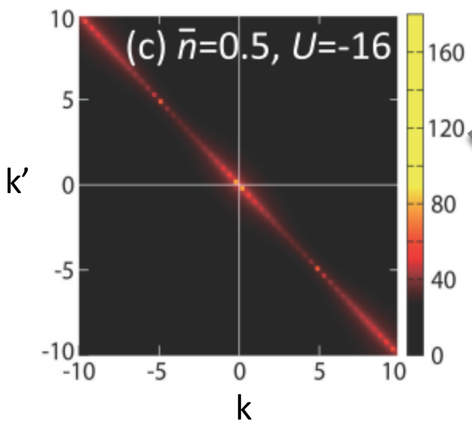
BEC **(U=-16t, n=0.5)**



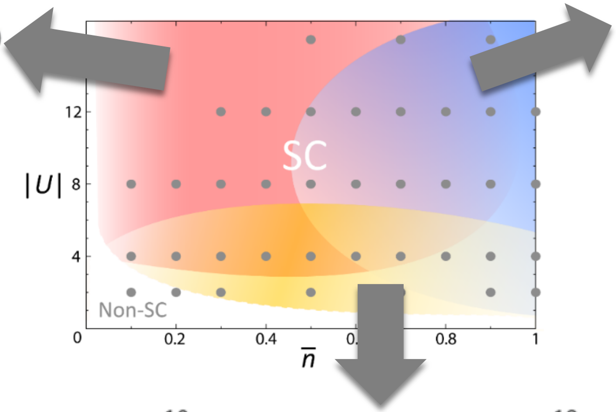
Unusual pairing in momentum space (2)

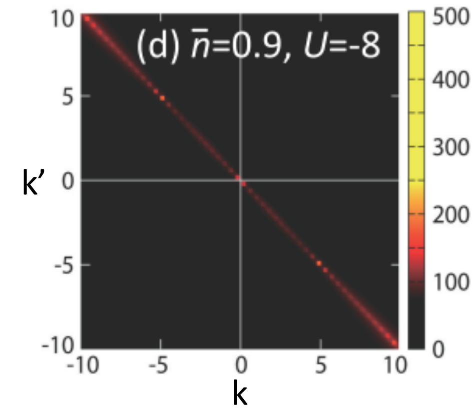
$|\langle c_{\mathbf{k}\uparrow} c_{\mathbf{k}'\downarrow} \rangle|$ for $k_x = k_y = k$ and $k_x' = k_y' = k'$

Penrose lattice

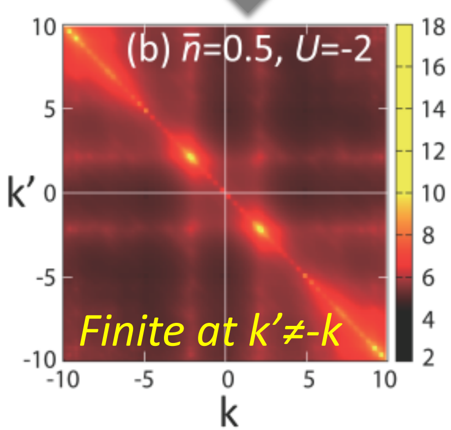


Localized pair
→BEC-like





Short-ranged pair
→BEC-like



Extended pair

Different from both
BCS and BEC characteristics

SS, N. Takemori, A. Koga, and R. Arita, PRB **95**, 024509 (2017).

What about the property?

Universal properties in BCS theory

$$\frac{2E_g^0}{T_c} \cong 3.52$$

$E_g(T)$: Energy gap
$E_g^0 = E_g(0)$

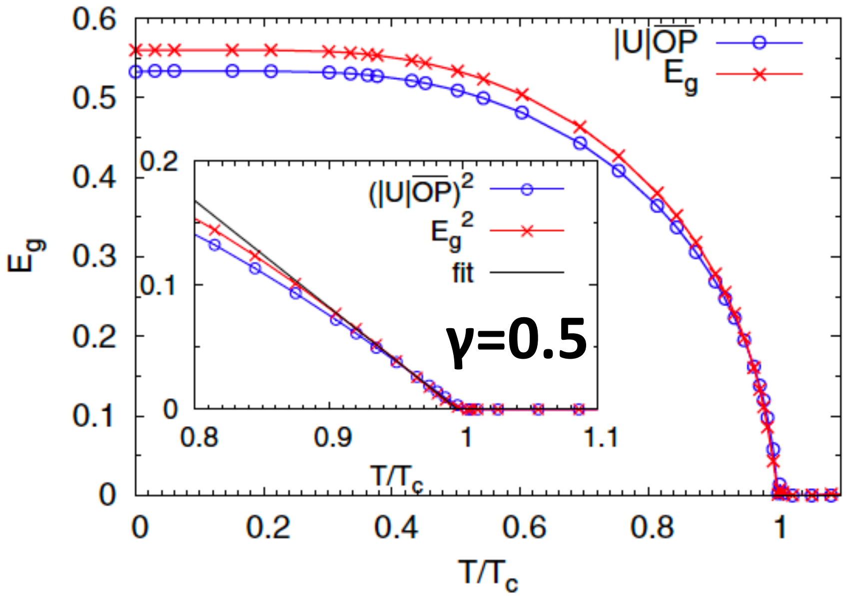
$$\frac{E_g(T)}{E_g^0} \cong A_1\left(1 - \frac{T}{T_c}\right)^{\gamma}, A_1 \cong 1.74, \gamma = \frac{1}{2}$$

$\frac{\Delta C_e}{C_{en}} \cong 1.43$: Jump of specific heat

Does these relations hold in quasiperiodic SC?

The gap & T_c

BdG
U=-3t



cf. Amorphous SC

amorphous metal	T_c [K]	$2\Delta_0$ [meV]	$\frac{2\Delta_0}{kT_c}$	λ
Bi	6,1	2,42	4,60	2,2 - 2,46
Ga	8,4	3,32	4,60	1,94 - 2,25
$Sn_{0,9}Cu_{0,1}$	6,76	2,6	4,46	1,84
$Pb_{0,9}Cu_{0,1}$	6,5	2,66	4,75	2,0
$Pb_{0,75}Bi_{0,25}$	6,9	2,96	4,98	2,76
$In_{0,8}Sb_{0,2}$	5,6	2,13	4,40	1,69
$Tl_{0,9}Te_{0,1}$	4,2	1,67	4,6	1,70

Strong coupling SC

Bergmann, Phys. Rep. **27**, 159 (1976)

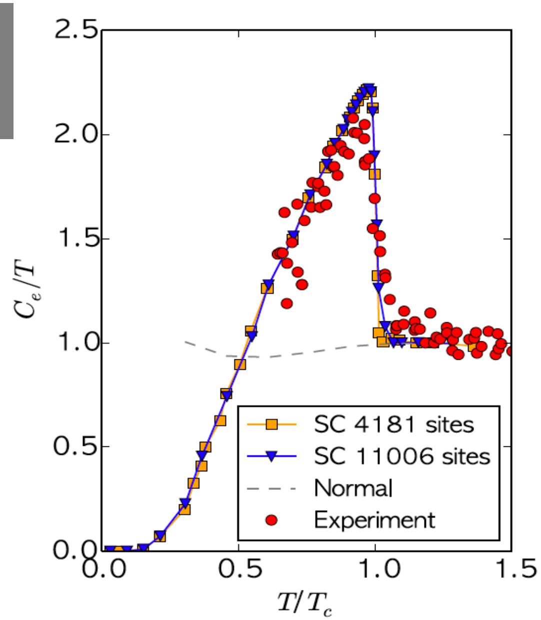
	Penrose				Square		BCS
	1591	4181	11 006	Ext	2500	10 000	
$\frac{2E_g^0}{T_c}$	3.35	3.38	3.38	3.38	3.46	3.45	3.52
A_1	1.61	1.63	1.69	1.70	1.70	1.70	1.74

$2E_g^0/T_c$: Small but substantial shift to a lower value ⬌ Amorphous SC

A_1: No significant change

Jump of specific heat

BdG
U=-3t



$$S = 2\sum_\alpha \left\{ \ln(1+e^{-\beta E_\alpha}) + \frac{\beta E_\alpha}{e^{\beta E_\alpha}+1} \right\}$$

$$C_e = T\frac{dS}{dT}$$

Experiment: Kamiya *et al*., Nature Commun. **9**, 154 (2018)

	Penrose				Square		BCS
	1591	4181	11 006	Ext	2500	10 000	
$\frac{\Delta C}{C_{en}}$	1.13	1.21	1.21	1.21	1.40	1.39	1.43

～15 % smaller Jump

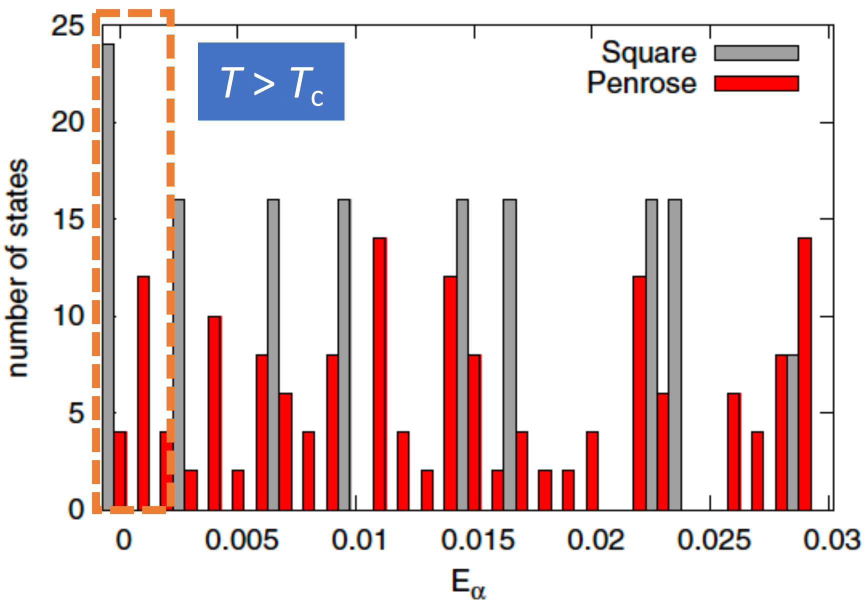
N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

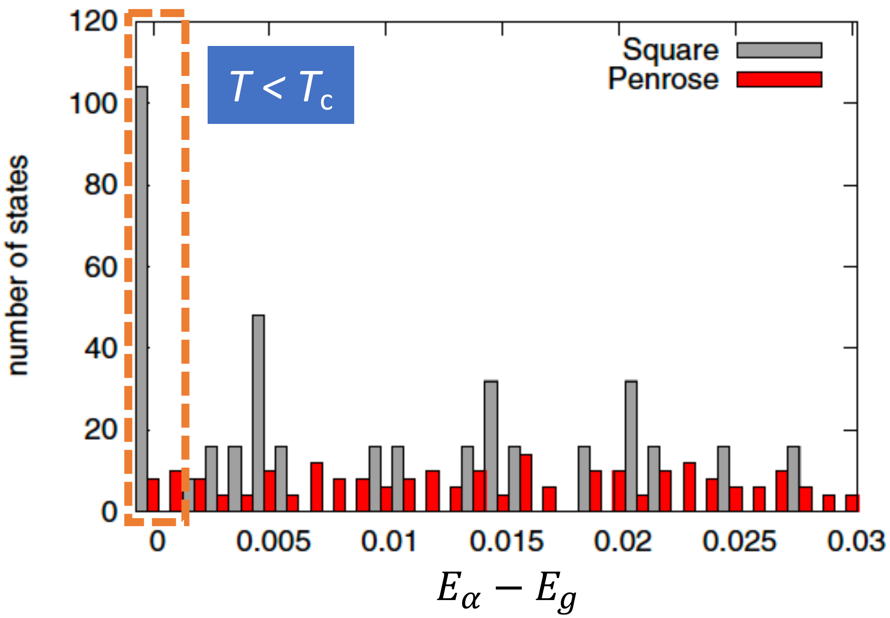
Absence of coherence peak

BdG
U=-3t

$$C_e = 2\beta \sum_{\alpha} \left(-\frac{\partial f(E_\alpha)}{\partial E_\alpha} \right) \left(E_\alpha^2 + \frac{\beta}{2} \frac{\partial E_\alpha^2}{\partial \beta} \right)$$

Only around E_F



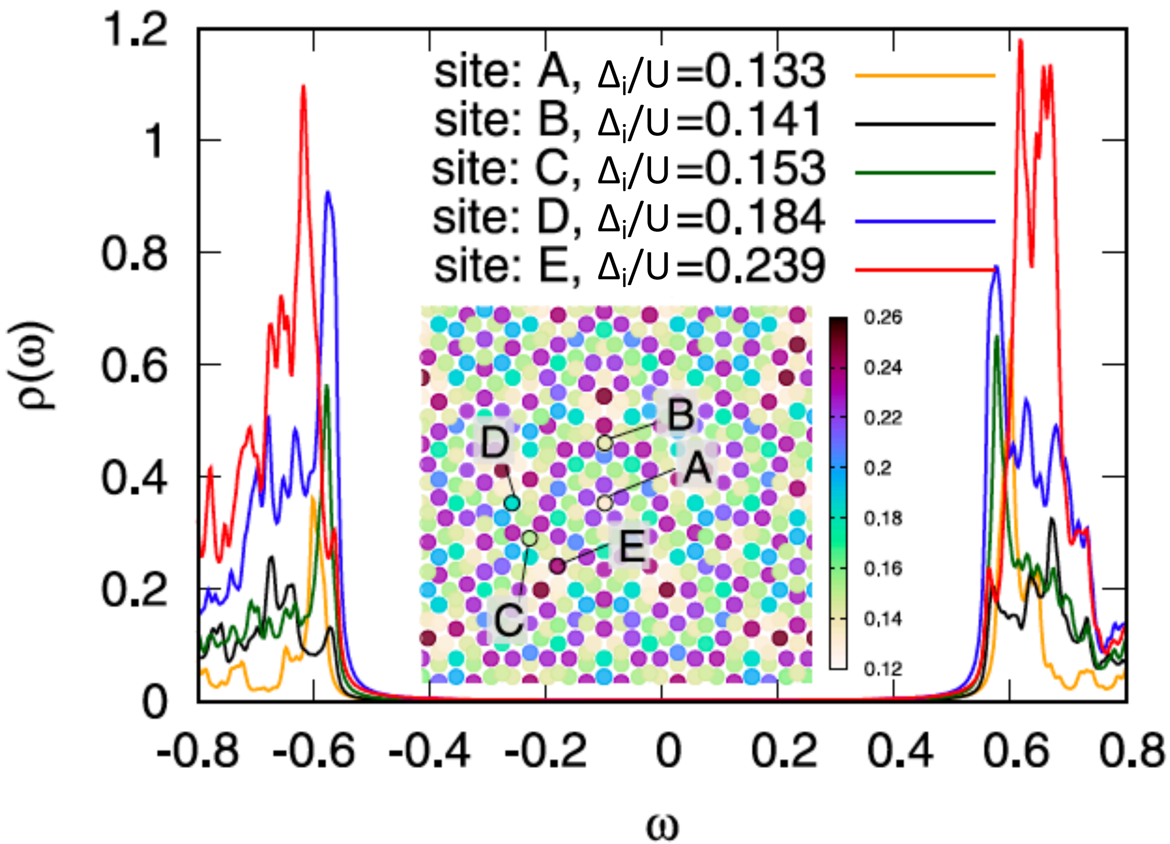


Absence of Fermi surface → Absence of coherence peak

→ Smaller jump

Local density of states

BdG
U=-3t



- The spectral gap is much more uniform than $\langle c_{i\uparrow} c_{i\downarrow} \rangle$.
- The weight of spectral peaks significantly depends on sites.

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

I-*V* characteristics (1)

Normal metal (periodic) | Super-conductor

$$I(V) \propto \int_{-\infty}^{\infty} \rho(E)[f(E) - f(E+eV)]dE$$

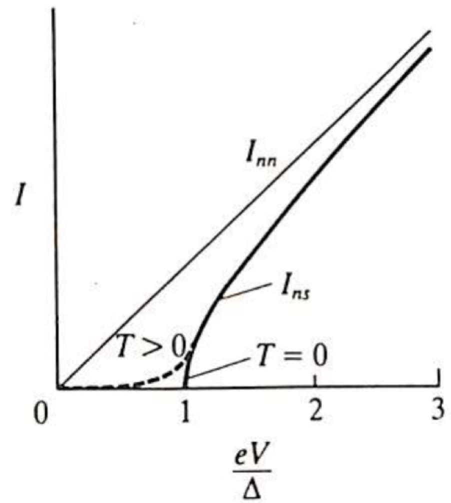
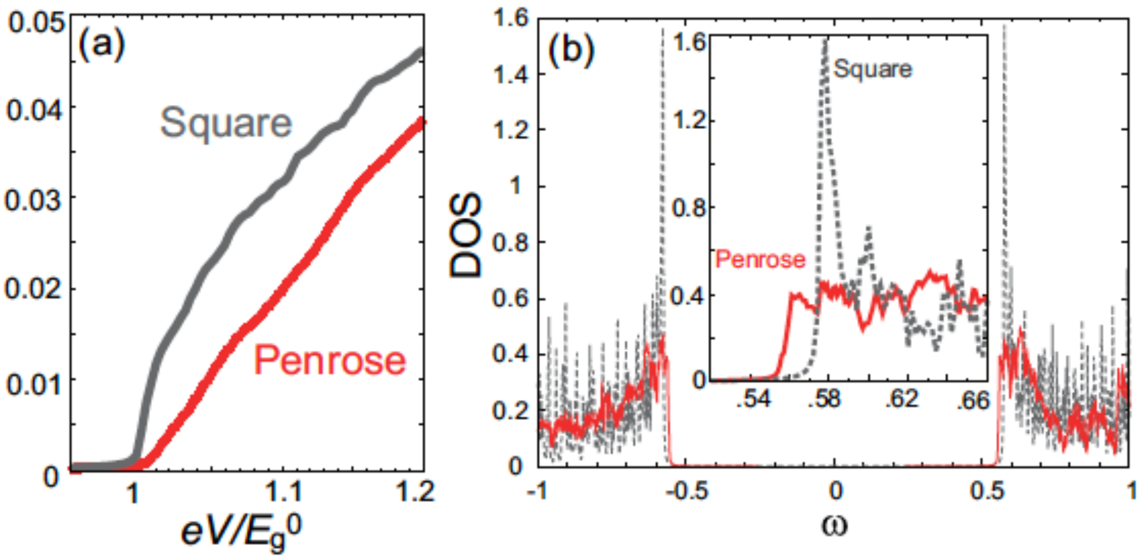


Fig. from M. Tinkham, INTRODUCTION TO SUPERCONDUCTIVITY

T = 0

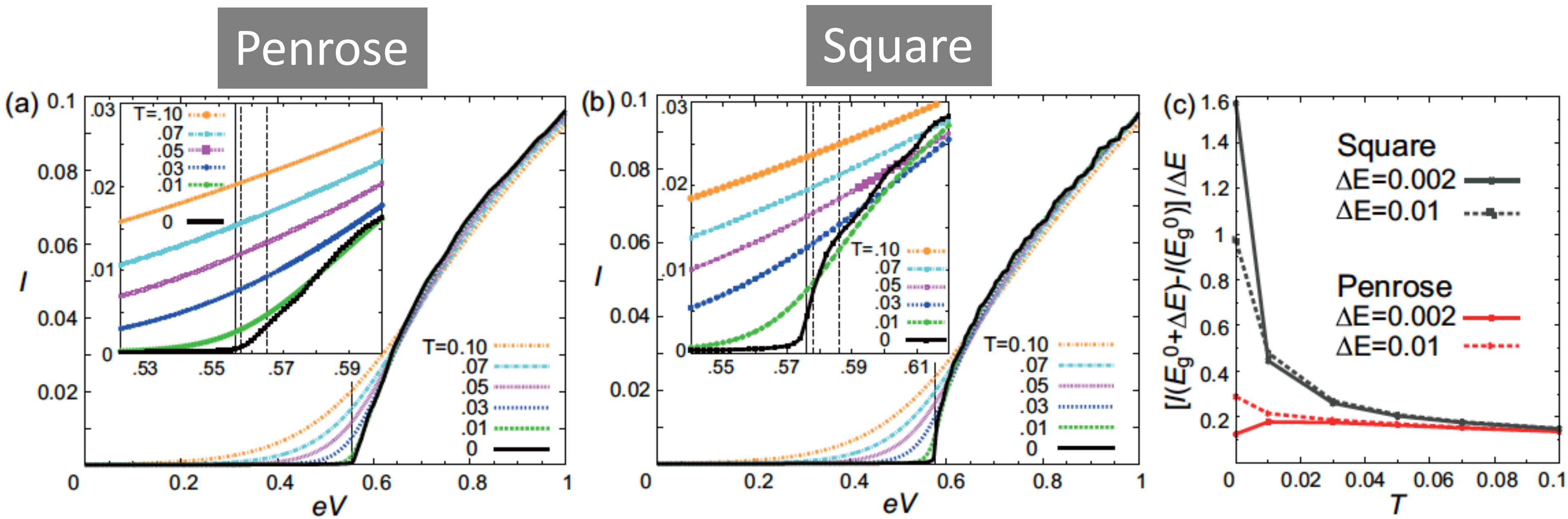


Finite gradient at the threshold voltage ⇐ Absence of coherence peak

N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

I-V characteristics (2)

Any signature at finite T?



Weakly T-dependent slope signals quasiperiodic SC.

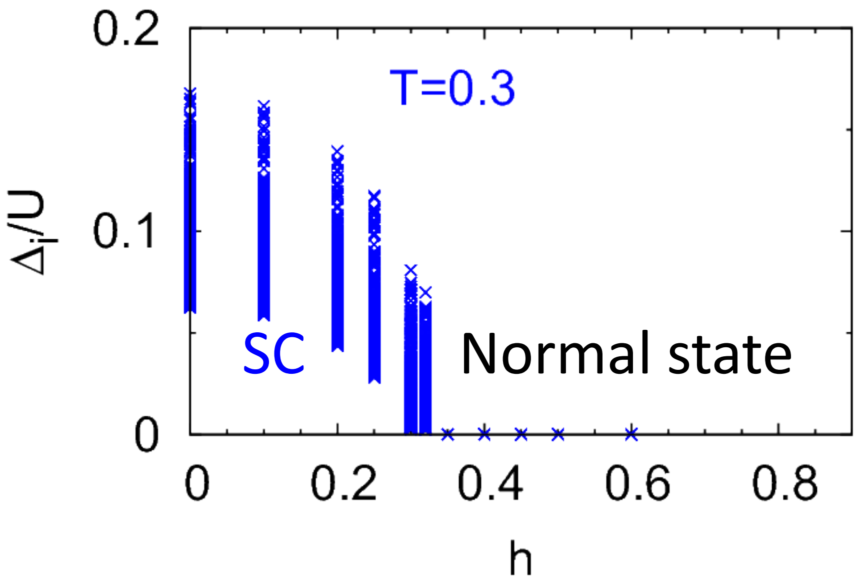
N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).

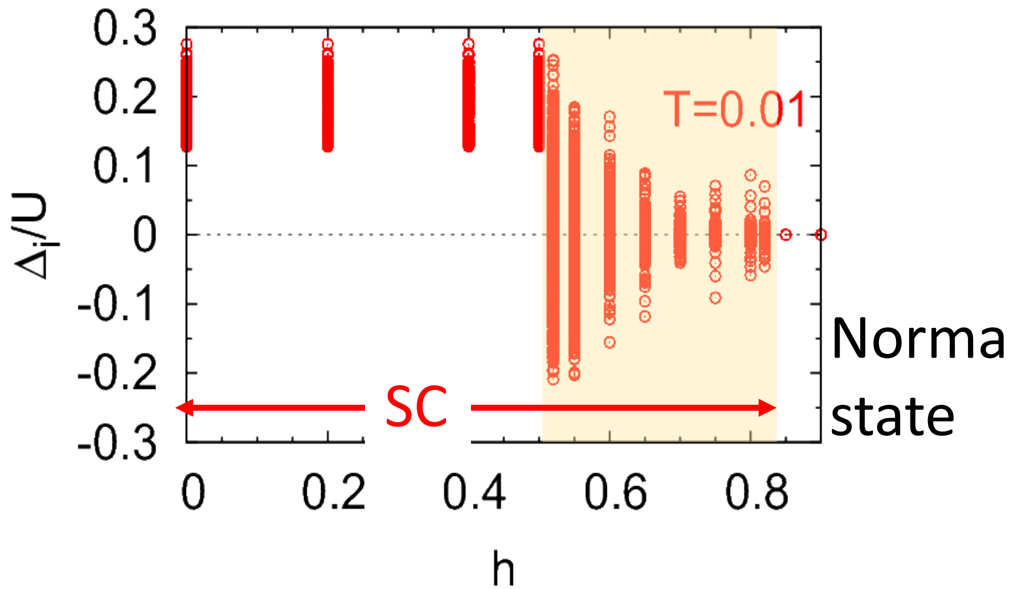
Effect of magnetic field

BdG
11006 sites
$\bar{n}$=0.5
t=1, U=-3

Only Zeeman effect, no orbital motion : Magnetic field parallel to plane

T_c (h=0)=0.34

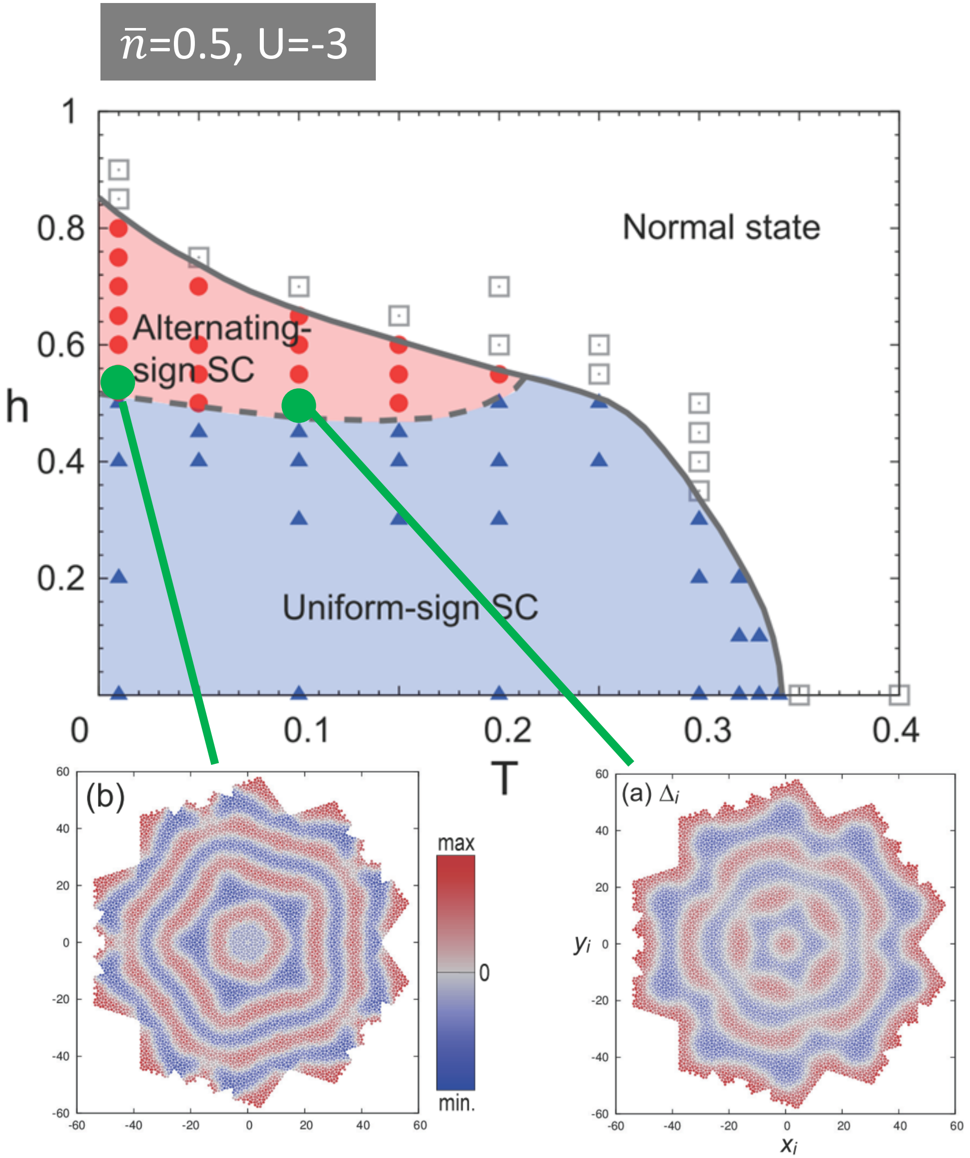




- 1^{st} order transition.
- $\Delta_i \geqq 0$.

- Strange behavior before H_c .
- Both positive and negative Δ_i .

FFLO-like state in quasiperiodic systems



FFLO in periodic systems

Fulde and Ferrell, PR **135**, A550 (1964).
Larkin and Ovchinnikov, ZETF **47**, 1136 (1964).

$$\langle c_{\mathbf{k}+\mathbf{q}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$$

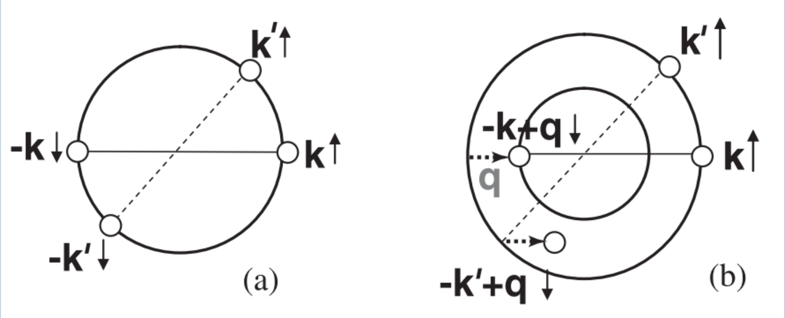


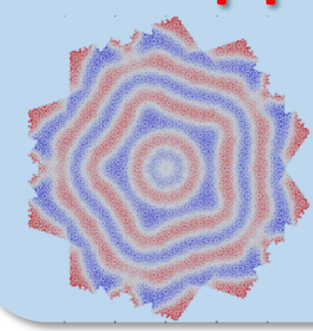
Fig. from Matsuda and Shimahara, JPSJ **76**, 051005 (2007).

Even without Fermi surface, the sign change occurs!

Impurities
(random potential)

Quasiperiodic
potential

FFLO-like states



Electrons self-organize into a pattern compatible with the quasiperiodicity!

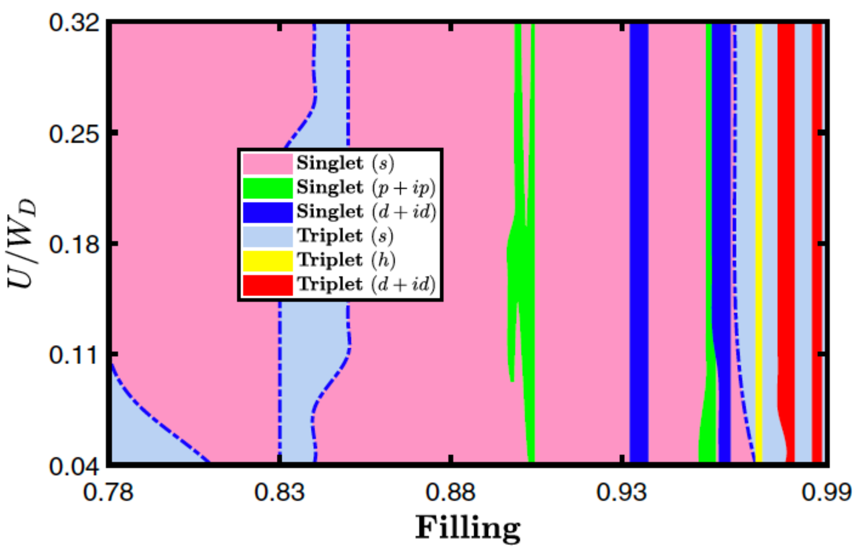
Are any other exotic pairings possible?

PHYSICAL REVIEW LETTERS 125, 017002 (2020)

Kohn-Luttinger Mechanism Driven Exotic Topological Superconductivity on the Penrose Lattice

Ye Cao,[1] Yongyou Zhang,[1] Yu-Bo Liu,[1] Cheng-Cheng Liu,[1] Wei-Qiang Chen,[2,3,4] and Fan Yang[1,*]

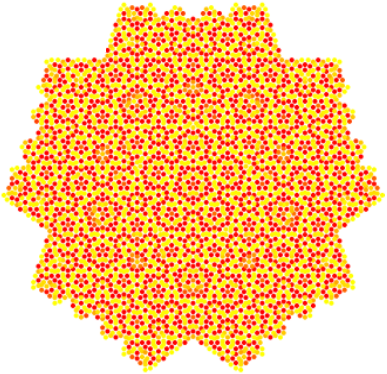
- Anisotropic and/or spin-triplet pairings
- Superconductivity from repulsive U
- Kohn-Luttinger mechanism without Fermi surface
- Topological superconductivity
- Spontaneous vortices without magnetic field



Angular momentum of absolute coordinate, NOT a relative one of a pair

Both spin-singlet and triplet are possible for each orbital pairing.

Summary

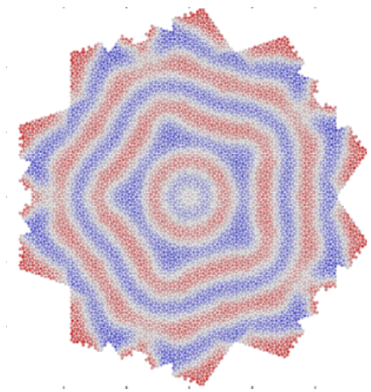


- Order parameter shows various spatial patterns, depending on the spatial extent of Cooper pairs.
- Weak-coupling SC with unusual extended Cooper pairs

  SS, N. Takemori, A. Koga and R. Arita, PRB **95**, 024509 (2017).

- Deviation from the BCS universal properties

  N. Takemori, R. Arita and SS, PRB **102**, 115108 (2020).



- FFLO-like SC at high magnetic field

  SS and R. Arita, Phys. Rev. Research **1**, 022002(R) (2019).

- Anisotropic SC

  Y. Cao *et al*., PRL **125**, 017002 (2020).

- Topological SC

  I. C. Fulga *et al*., PRL **116**, 257002 (2016).
  R. Ghadimi *et al*., arXiv:2006.06952

QC can be a novel platform of exotic SC!