WAIS2020@Tokyo University of Science

Improved CRT-RSA Secret Key Recovery Method from Sliding Window Leakage

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Our Contribution

1. Formularize **the exact bit recovery rate** from Square & Multiply sequence on Sliding Window method

2. **Propose the new method** for recovering CRT-RSA secret keys from Square & Multiply sequences

3. **Experiment** of proposed method

Agenda

1. Background
   Threat of Side-Channel Attacks

2. Previous Result [BBG+17]
   Previous Key Recovery Method

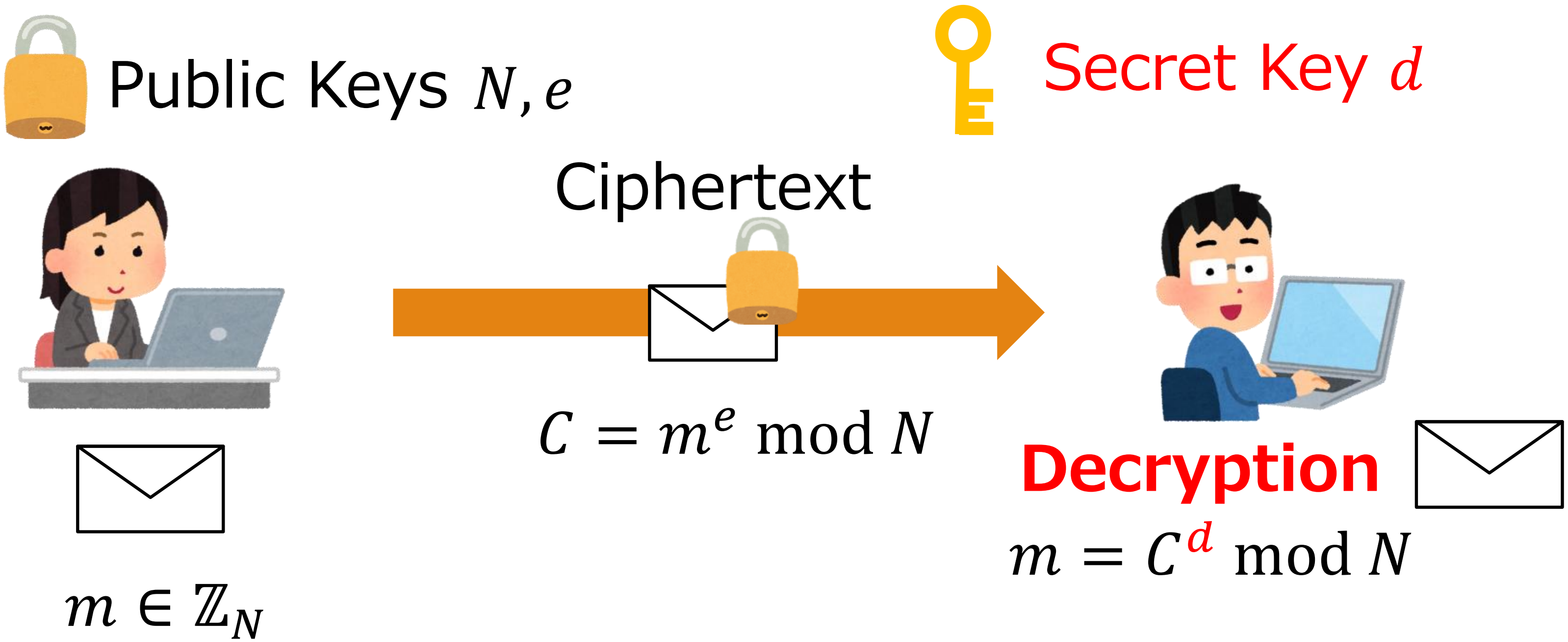
3. Our Result
   New Key Recovery Method

4. Conclusion

RSA Encryption Scheme [RSA78]

p, q : distinct $n/2$-bit prime numbers

$N = pq, ed \equiv 1 \bmod (p-1)(q-1)$.



CRT-RSA Scheme (PKCS#1)

Faster Decryption than the standard RSA

Secret Keys

$d \in \mathbb{Z}^*_{(p-1)(q-1)}$

smaller →

Chinese Remainder Theorem (CRT) decomposition

Secret Keys

$(d_p, d_q) \in \mathbb{Z}^*_{p-1} \times \mathbb{Z}^*_{q-1}$

$d_p \equiv d \bmod p-1$

$d_q \equiv d \bmod q-1$

Decryption

$m = C^d \bmod N$

faster →

$m_1 = C^{d_p} \bmod p$

$m_2 = C^{d_q} \bmod q$

We deal half bits.

↓ **CRT**

m

Sliding Window Method

We calculate c^d by **squaring** and **multiplication**.

Squaring: $X \to X^2$ **M**ultiplication: $X \to aX$

Input: c, d, and **window size w**
Output: c^d

We set c^0 as initial value.

We read bits of d from the most significant bit.
-If bit is "0", **S**quaring
-If bit is "1", we read w-bits.

w-bits ➡ $\begin{cases} w & \textbf{S}\text{quarings} \\ 1 & \textbf{M}\text{ultiplication} \end{cases}$

Threat of Side Channel Attacks

CRT-RSA scheme implemented by SW-method is known to be vulnerable against **side channel attacks**! [BBG+17]

SMSMSSM

⟹Secret keys ($\boldsymbol{d_p}, \boldsymbol{d_q}$)

*Difficult in larger w

Cache Access of decryption

c^{d_p}, c^{d_q}

[BBG+17] Bernstein et al. “Sliding Right into Disaster: Left-to-Right Sliding Windows Leak.” CHES 2017.

Our Motivation from [BBG+17]

	Success Rate
$w = 4$, 1024-bit CRT-RSA	≈100%
$w = 5$, 2048-bit CRT-RSA	8.6%

In $w = 5$, we only recover 8.6% CRT-RSA secret keys because of **too many candidates**.

By decreasing the number of candidates,
we may recover more CRT-RSA secret keys.

Question
How do we **decrease** the number of candidates?

Our New Method

We extract more information as bits.

SSMSSSSMSSM…

↓

SSMSSSSMSSM… **+ x101x1x1…**

↓

11010111…

We recover **more** secret keys when $w = 5$.

Technique in [BBG+17]

Proposed key recovery algorithm from **S&M sequences**

They use **Branch and Bound** Strategy.

-Compute some candidate bits
-Prune branches

Iteration

*If there is no pruning, final candidates **always** include **the correct secret keys**.

How do they compute candidate bits sequentially?

How do they prune branch?

Computing Candidate Bits [HS09]

We can compute candidate bits using **mathematical relationship** in CRT-RSA.

$$p[i] + q[i] \equiv c_1 \bmod 2,$$

$$d_p[i + \tau(k_p)] + p[i] \equiv c_2 \bmod 2,$$

$$d_q[i + \tau(k_q)] + q[i] \equiv c_3 \bmod 2.$$

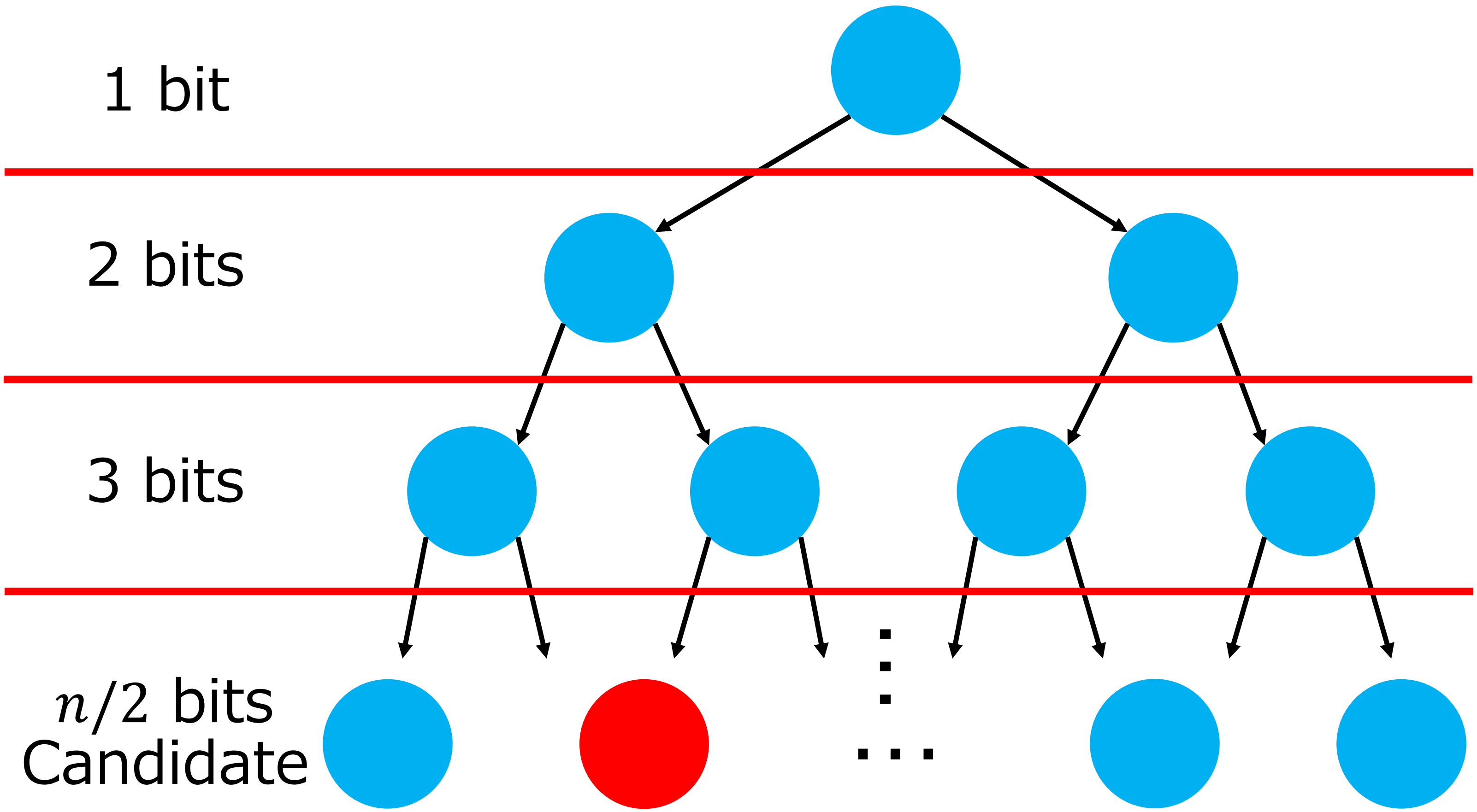
Candidate bits | Known

This simultaneous equations have one degree of freedom.

We obtain **two candidates**.

[HS09] Heninger and Shacham. "Reconstructing RSA Private Keys from Random Key Bits." CRYPTO 2009.

Visualization of Calculation



There are exponential candidates in tree.
How do we prune branch?

Pruning in [BBG+17]

Method A	Method B*
SSMSSSSMSSM⋯	**SSMSSSSMSSM⋯**
↓ We partially recover bits.	↓ [HS09]
x10xx1x1⋯	
↓ [HS09]	
11010111⋯	**11010111⋯**

*They showed that we can recover CRT-RSA secret keys in polynomial time by method B when $w \leq 4$.

Method A

Given sequences

d_p **SMSSSSM**

d_q **SSMSSSS**

Recover bits **partially***

d_p 1 0 0 ? 1

d_q ? 1 0 0 ?

Calculated Bits

d_p 1 0 1 1 0

d_q 0 1 1 0 1

Compare on **bits**
We remain leaves only when there is no mismatch.

*[Vre18] proposed the optimal bit recovery method.

[Vre18] van Vredendaal. "Exploiting Mathematical Structures in Cryptography." Eindhoven University of Technology, 2018.

Optimal Bit Recovery [Vre18]

"Optimal" means we recover **all common bits** when we consider all candidates.

Example **SSMSSM**, $w = 2$

All Candidates	0	**1**	0	**1**	→	x **1** x **1**
	1	**1**	0	**1**		
	1	**1**	1	**1**		

*Recovered bits are always correct.

Method B

Given sequences

d_p **SMSSSSM**

d_q **SSMSSSS**

Calculated Bits

d_p 1 0 1 1 0

d_q 0 1 1 0 1

Convert into S&M sequences

d_p **SSSMSMS**

d_q **SSSMSSM**

Compare on **S&M sequences**

We remain leaves only when there is no mismatch.

Experimental Results of [BBG+17]

- Implement with Depth first search (DFS)
- If we generate 1,000,000 leaves, return failure.
- They measure success rate on 500,000 trials.

	Method A	Method B
$w = 4$, 1024-bit CRT-RSA	28%	≈100%
$w = 5$, 2048-bit CRT-RSA	0%	8.6%

Method B recovers more CRT-RSA secret keys

Our Key Idea

Method A and B can be combined!

Method A

SSMSSSSMSSM···

↓ [Vre18]

x10xx1x1···

↓ [HS09]

11010111···

Combine Here!!!

Method B

SSMSSSSMSSM···

↓ [HS09]

11010111···

Our New Method

We extract more information on bits.
Then, we decrease candidates!

SSMSSSSMSSM…

↓ We partially recover bits by [Vre18].
We determine more bits with high accuracy.
*These bits are not always correct.

SSMSSSSMSSM… + **x101x1x1…**

↓ [KSI14]

11010111…

[KSI14] Kunihiro et al. “Recovering RSA Secret Keys from Noisy Key Bits with Erasures and Errors. ” IEICE Trans. Fundamentals. E97-A, 1273—1284, 2014.

Explicit Form of the Bit Recovery Rate

When $w \geq 2$,

$$\frac{2}{w+1} + \frac{\sum_{k=0}^{w-1} f_w(k) g(k)}{2(w+1)} + \frac{2^w - 1}{2^{w-1}(2^{w-1}+1)} \frac{1}{3(w+1)}$$

$$f_w(k) = \frac{2}{3 \cdot 2^k}\left(1 - \frac{1}{2^{w-k}}\right)\left(1 - \frac{2}{2^{w-k}}\right)$$

$$g(k) = 2\left(1 - \frac{2^k}{2^{k+2} - 1}\right)\prod_{j=1}^{k} \frac{2^{j-1}}{2^{j+1} - 1}$$

w	Theoretical
3	60.95%
4	49.81%
5	41.92%

Experimental Results

w	Theoretical (%)	2048-bit CRT-RSA 100 times (%)
3	60.95	60.80
4	49.81	49.96
5	41.92	41.84
6	36.09	36.19
7	31.65	31.76

Our analysis matches with the experiment.

Hidden Information

w	Theoretical (%)	2048-bit CRT-RSA 100 times (%)
4	49.81	49.96

We **cannot** recover secret keys when $w = 4$, because less than 50% bits are recovered [PPS12].

However, we **can** recover secret keys when $w = 4$, by method B of [BBG+17].

*Using S&M sequences directly

There are more information in unrecovered bits!

How do we extract more information?

[PPS12] Paterson et al. "A Coding-Theoretic Approach to Recovering Noisy RSA Keys." Asiacrypt 2012.

Extracting Hidden Information

We extract information as **the proportion of "1"**.

Example SSMSSM, $w = 2$

All Candidates	Recovered bits
0 1 0 1	
1 1 0 1	x 1 x 1
1 1 1 1	
# of candidates: 3	# of candidates: 4

# of candidates: 3 < # of candidates: 4

There are some bits as almost "1".

*We check this heuristically by Monte-Calro approach.

Thus, we determine more bits with high accuracy!!!

Our New Method **(Again)**

We extract more information on bits.
Then, we decrease candidates!

SSMSSSSMSSM···

↓ We partially recover bits by [Vre18].
We determine more bits with high accuracy.
*These bits are not always correct.

SSMSSSSMSSM··· **+** **x101x1x1···**

↓ [KSI14]

11010111···

[KSI14] Kunihiro et al. “Recovering RSA Secret Keys from Noisy Key Bits with Erasures and Errors. ” IEICE Trans. Fundamentals. E97-A, 1273—1284, 2014.

Our New Method: Step 1

SSMSSSSMSSM⋯

We partially recover bits by [Vre18].

*Recovered bits are <u>always</u> correct.

SSMSSSSMSSM⋯ + x10xx1x1⋯

We choose sufficient candidates randomly, from <u>all</u> candidates.

We determined bits as “1”, when the proportion of “1” is more than $1-\varepsilon$.

*These bits are <u>not always</u> correct.

SSMSSSSMSSM⋯ + x101x1x1⋯

Our New Method: Step 2

SSMSSSSMSSM··· + x101x1x1···

Pruning in Method A, B of [BBG+17]

- Method A: Recovered Bits
- Method B: S&M Sequences

+

[KSI14] on additional determined bits

- Calculate $\lfloor 1/\varepsilon \rfloor$-bits
- If there are more than one mismatches, we discard a leaf.

11010111···

Experiment

We perform experiment on 2048-bit CRT-RSA, $w = 5$.

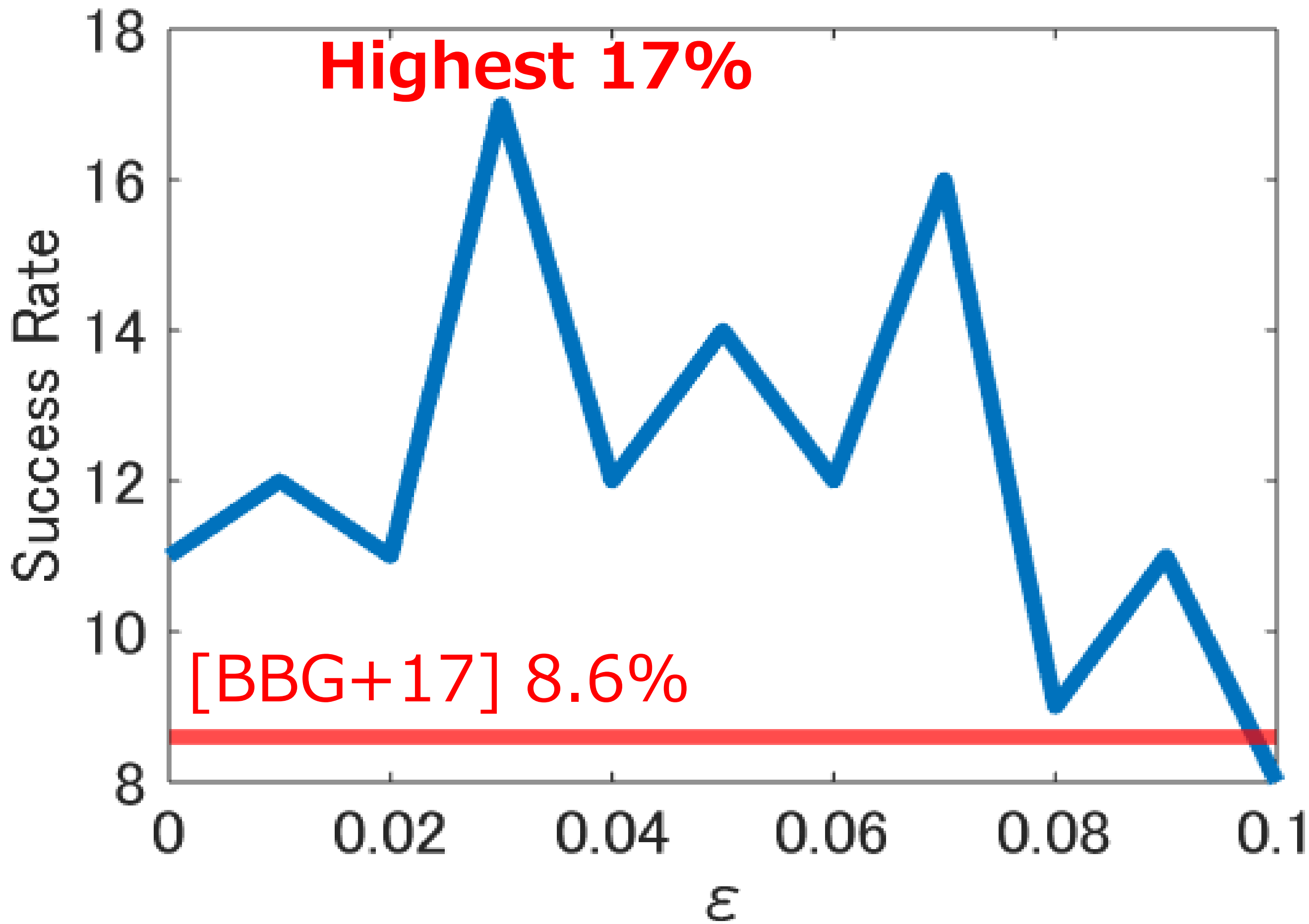
We set the parameter $0 \leq \varepsilon \leq 0.1$ at 0.01 intervals.

In each ε,

- We determine additional bits based on 1,000 samples.
- Implement by Depth first search (DFS)
- If we generate L leaves, return failure.
- We generate 100 CRT-RSA keys randomly, and measure success rate.

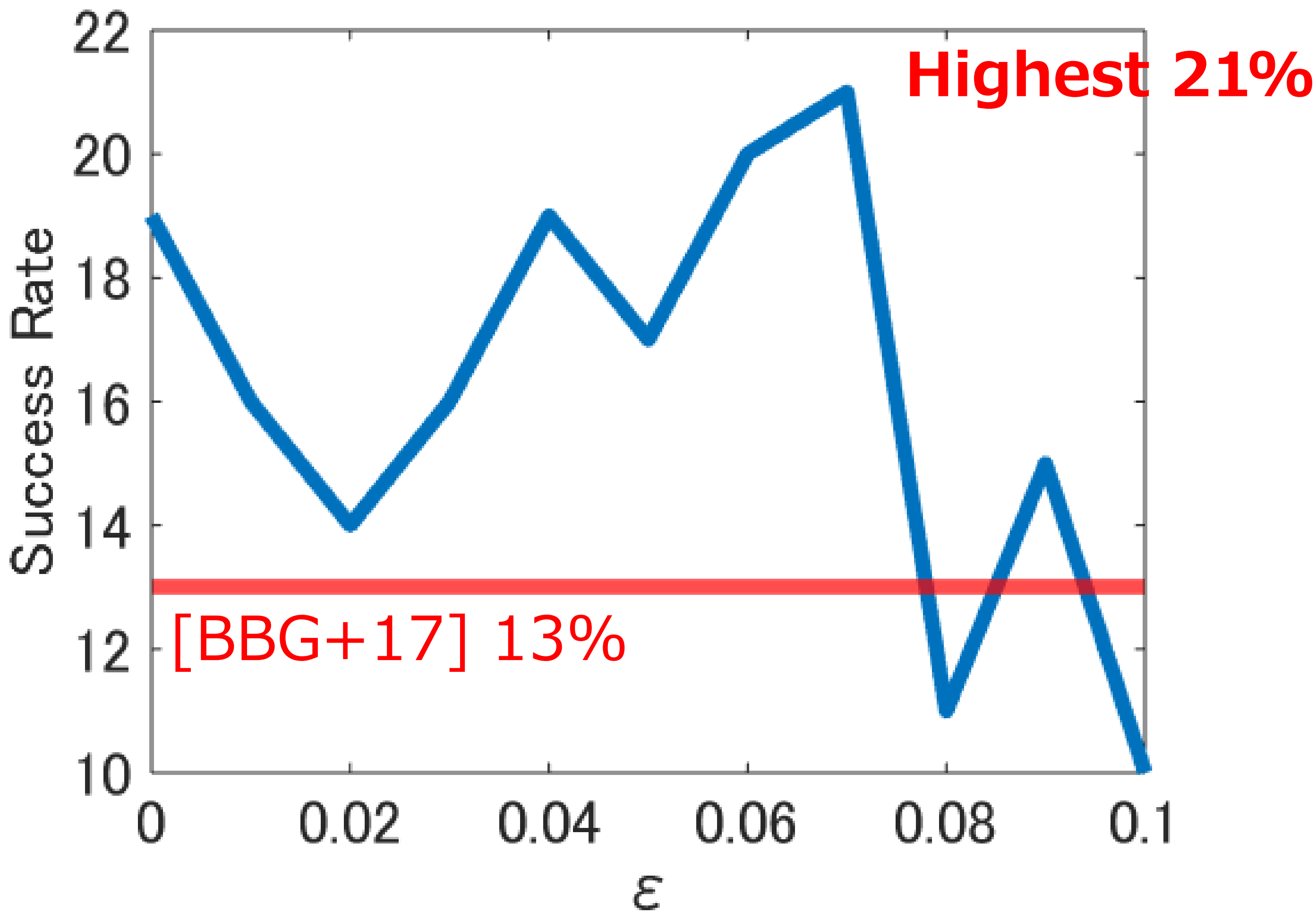
Experimental Result ($L = 1{,}000{,}000$)

Our method recovers more secret keys!



Experimental Result ($L = 2{,}000{,}000$)

Our method recovers more secret keys!



Conclusion

1. Formularize **the exact bit recovery rate** from Square & Multiply sequence on Sliding Window method

2. **Propose the new method** for recovering CRT-RSA secret keys from Square & Multiply sequences

3. **Experiment** of proposed method

L	[BBG+17]	[Ours]
1,000,000	8.6%	17%
2,000,000	13%	21%