ELASTIC WAVES IN SHADOW

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Abstract.

In this talk, we study the diffraction of elastic waves with high frequency.

We assume that the elastic materiel is isotropic.

At first, we explain about P and S waves. P wave is longitudinal wave, that is, the wave whose direction of propagation is parallel with that of displacement. S wave is transversal wave, that is, the wave whose direction of propagation is perpendicular with that of displacement. Concerning the velocity of propagation, P wave is faster than S wave. In general, elastic wave in isotropic materiel is the superposition of P and S waves.

Let us consider the liberty of the direction of displacement for P and S waves with respect to that of propagation. Since the direction of displacement of P wave is parallel with that of propagation, P wave does not have the liberty. On the other hand, S wave has the liberty, because the direction of displacement of S wave is perpendicular with that of propagation. Therefore, the polarization becomes problem with respect to S wave.

The conversion of mode is the phenomenon at which the kind of wave change. This phenomenon occurs in general, when elastic waves arrive at the obstacle.

Assume that elastic materiel contain a strictly covex obstacle. We fix an incident point at the boundary of the obstacle. When S wave with high frequency arrives at this point from certain direction, The conversion of mode occurs and P wave appears at the tangential direction with respect to the obstacle. This P wave does the diffraction. Then, by the conversion of mode, S wave appears from the boundary of the obstacle to the interior of the elastic materiel. We call the waves on the boundary generated by the diffraction the surface waves. In this case, the surface waves are the superposition of P and S waves. Assume that the S wave which arrives at the incident point is polarized. Then, near this point, we have the following conclusions.

1. The surface wave is polarized.
2. The direction of the polarization of the surface wave depends on that of S wave which arrives at the incident point.

Equation.

$$\begin{cases} Lu = 0 & \text{in } \mathbf{R} \times \Omega \\ u\,|_{\mathbf{R}\times\partial\Omega} = g \end{cases} \tag{1}$$

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

where

$$L = \partial_t^2 - A(\partial_x) \quad \text{in} \quad \mathbf{R}_t \times \mathbf{R}_x^3 ,$$
$$A(\partial_x) = \mu\triangle + (\lambda+\mu)\text{grad div} \quad \text{in} \quad \mathbf{R}_x^3 ,$$
$$\lambda > 0 , \quad \mu > 0 \ : \text{constants},$$
$$\Omega \subset \mathbf{R}_x^3 \ : \text{exterior domain whose boundary is smooth.}$$

Notations.
$z \in \partial\Omega$: fixed point.
$N = \mathbf{R} \times \partial\Omega$
$\triangle_{\partial\Omega}$: Laplacian on $\partial\Omega$.
$\sigma(\triangle_{\partial\Omega})$: principal symbole of $\triangle_{\partial\Omega}$.
$c_1 = (\lambda+2\mu)^{1/2}$, $\quad c_2 = \mu^{1/2}$
$q_n \in C^\infty(T^*N , \mathbf{R})$: functions defined by $\quad q_n(\tau,\beta) = -\tau^2 - c_n^2\sigma(\triangle_{\partial\Omega})(\beta)$;
$n = 1 , 2 \quad$ with $\tau \in \mathbf{R}$, $\quad \beta \in T^*(\partial\Omega)$.
$\pi : T^*N \to N \quad$: projection.
$\rho \in T^*N\backslash 0$: fixed point satisfying $q_1(\rho) = 0$, $\quad \pi(\rho) = (0,z)$.
γ : curve in T^*N defined by $\gamma(s) = \exp sH_{q_1}(\rho)$.
$\partial/\partial n$: derivation along the normal vector of $\partial\Omega$.
$m \in \mathbf{R}$: fixed number satisfying $1 \leqq m < 3$.
$WF_A(*)$: Analytic wave front set.
$WF_G^k(*)$: Gevrey k wave front set.

Assumptions.
(H.1) $\partial\Omega$ is analytic.
(H.2) $\mathbf{R}^3\backslash\Omega$ is strictly convex.

Statement of Theorem.

Theorem. *Assume that (H.1) and (H.2) are satisfied. Let* $0 < s_0 < s_1$ *with* s_1 *small enough,* $\omega \subset T^*N\backslash 0$ *small conic neighborhood of* ρ , $\omega \cap \gamma([s_0 , s_1]) = \phi$, $WF_A(g) \subset \omega$ *and* u *be the outgoing solution of (1). Then, we have following* (i) *and* (ii).
(i) $\gamma([s_0 , s_1]) \cap WF_G^3((\partial u/\partial n) \mid_N) = \phi$.
(ii) $\gamma([s_0 , s_1]) \cap WF_G^m((\partial u/\partial n) \mid_N) = \phi$ or $\gamma([s_0 , s_1]) \subset WF_G^m((\partial u/\partial n) \mid_N)$.

The propagation of surface wave is described by (ii) of Theorem. The polarization of surface wave is formulated by the algebra of unilateral operators.