# EVOLUTION EQUATIONS WITH SIMPLEST NONLINEAR ORNSTEIN-UHLENBECK OPERATORS

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This is the joint work with Philippe Clément (Technical University of Delft).

The Cauchy problem for the nonlinear heat equation with the Ornstein-Uhlenbeck operator is given by

$$(\mathrm{NLH})_{p,\nu} \quad \begin{cases} \dfrac{\partial u}{\partial t} - \mathrm{div}(|\nabla u|^{p-2}\nabla u) + \nu|\nabla u|^{p-2}(x\cdot\nabla)u + |u|^{q-2}u = 0 \ \text{ on } \mathbf{R}^N \times \mathbf{R}_+, \\ u(x,0) = u_0(x), \ \ x \in \mathbf{R}^N. \end{cases}$$

More generally, we consider the Cauchy problem for the complex Ginzburg-Landau equation with the Ornstein-Uhlenbeck operator:

$$(\mathrm{CGL})_{p,\nu} \quad \begin{cases} \dfrac{\partial u}{\partial t} - (\lambda + i\alpha)e^{\frac{\nu|x|^2}{2}}\mathrm{div}(e^{-\frac{\nu|x|^2}{2}}|\nabla u|^{p-2}\nabla u) + (\kappa + i\beta)|u|^{q-2}u - \gamma u = 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \mathbf{R}^N \times \mathbf{R}_+, \\ u(x,0) = u_0(x), \ \ x \in \mathbf{R}^N. \end{cases}$$

Here $\lambda, \kappa > 0$, $\nu \geq 0$, $\alpha, \beta, \gamma \in \mathbf{R}$ and $p, q \geq 2$ are constants; $i = \sqrt{-1}$, and $u$ is a complex-valued unknown function; note that

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) + \nu|\nabla u|^{p-2}(x\cdot\nabla)u = -e^{\frac{\nu|x|^2}{2}}\mathrm{div}(e^{-\frac{\nu|x|^2}{2}}|\nabla u|^{p-2}\nabla u).$$

In particular, $(\mathrm{CGL})_{2,0}$ is the usual complex Ginzburg-Landau equation and recently studied very extensively (see e.g. Levermore-Oliver [**3**] and Ginibre-Velo [**1**], [**2**]). However, one cannot apply the well-known semilinear theory to $(\mathrm{CGL})_{p,\nu}$ with $p > 2$ and $\nu > 0$. Therefore there seems to be no preceding work on $(\mathrm{CGL})_{p,\nu}$. We shall present the existence result of strong solutions together with smoothing effect. The proof depends on the abstract perturbation theory for Cauchy problems of evolution equations with subdifferential operators in complex Hilbert space, developed recently by Okazawa-Yokota [**4**].

## References


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