

# The Geometric Optics for the Kirchhoff-type equations

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## Abstract

We study the nonlinear first-order symmetric hyperbolic integro-differential system on  $\mathbb{R}_t \times \mathbb{R}_x^d$

$$\frac{\partial U}{\partial t} - \sum_{j=1}^d A_j(s(t)) \frac{\partial U}{\partial x_j} + s'(t) B(s(t)) U = G(t, x), \quad (0.1)$$

where  $A_j(s)$ ,  $B(s)$ , are  $N \times N$  matrices with suitably smooth elements for  $s \in \mathbb{R}$  in a neighbourhood of 0. The function  $s(t)$  is given by

$$s(t) = \int_{\mathbb{R}_x^d} \langle S U(t, x), \overline{U(t, x)} \rangle dx, \quad s'(t) := ds(t)/dt,$$

where  $S = \{s_{ij}\}$  is a constant self-adjoint matrix.

The main purpose is to present a method for finding small-amplitude, high-frequency wave solutions of the Kirchhoff-type hyperbolic systems (0.1) of non-local quasilinear partial differential equations.

From the geometric optics point of view the basic feature of the Kirchhoff equation is a *globally existing phase function*. Indeed, if the initial data for the phase function is a linear function, then the eikonal equation has solution defined for all  $t \geq 0$ . This allows to consider *semi-global* weakly nonlinear geometric optics approximation, that is an approximation which is applicable for every given time-interval  $[0, T]$ ,  $T > 0$ . The transport equation is  $\mathbb{R}$ -linear and nonlocal. It has solution existing in general on the bounded interval  $[0, T]$ .

We construct the formal solutions of (0.1) with expansions of the form

$$u(\varepsilon, t, x) = u_0(t, x) + U(\varepsilon, t, x, \phi(t, x)/\varepsilon),$$

where  $u_0(t, x)$  is a given solution to the equation,  $\phi$  is a phase function, while  $U(\varepsilon, t, x, \theta)$  is periodic in  $\theta$  and is constructed as an asymptotic series

$$U(\varepsilon, t, x, \theta) \sim \sum_{j=1}^{\infty} \varepsilon^j U_j(t, x, \theta).$$

To justify nonlinear geometric optics for the system (0.1) we establish results on a global solvability and on a global stability with respect to the right-hand side  $G$  and the initial data given at  $t = 0$ .