The Geometric Optics for the Kirchhoff-type equations

Karen Yagdjian

Institute of Mathematics, National Acad. Sci. of Armenia, Yerevan, 375019, Armenia

Abstract

We study the nonlinear first-order symmetric hyperbolic integro-differential system on $\mathbb{R}_t \times \mathbb{R}^d_x$

$$\frac{\partial U}{\partial t} - \sum_{j=1}^{d} A_j(s(t)) \frac{\partial U}{\partial x_j} + s'(t) B(s(t)) U = G(t,x)\,, \tag{0.1}$$

where $A_j(s)$, $B(s)$, are $N \times N$ matrices with suitably smooth elements for $s \in \mathbb{R}$ in a neighbourhood of 0. The function $s(t)$ is given by

$$s(t) = \int_{\mathbb{R}^d_x} < SU(t,x), \overline{U(t,x)} > dx\,, \qquad s'(t) := ds(t)/dt\,,$$

where $S = \{s_{ij}\}$ is a constant self-adjoint matrix.

The main purpose is to present a method for finding small-amplitude, high-frequency wave solutions of the Kirchhoff-type hyperbolic systems (0.1) of non-local quasilinear partial differential equations.

¿From the geometric optics point of view the basic feature of the Kirchhoff equation is a *globally existing phase function*. Indeed, if the initial data for the phase function is a linear function, then the eikonal equation has solution defined for all $t \geq 0$. This allows to consider *semi-global* weakly nonlinear geometric optics approximation, that is an approximation which is applicable for every given time-interval $[0,T]$, $T > 0$. The transport equation is $\mathbb{R}$-linear and nonlocal. It has solution existing in general on the bounded interval $[0,T]$.

We construct the formal solutions of (0.1) with expansions of the form

$$u(\varepsilon,t,x) = u_0(t,x) + U(\varepsilon,t,x,\phi(t,x)/\varepsilon)\,,$$

where $u_0(t,x)$ is a given solution to the equation, ϕ is a phase function, while $U(\varepsilon,t,x,\theta)$ is periodic in θ and is constructed as an asymptotic series

$$U(\varepsilon,t,x,\theta) \sim \sum_{j=1}^{\infty} \varepsilon^j U_j(t,x,\theta)\,.$$

To justify nonlinear geometric optics for the system (0.1) we establish results on a global solvability and on a global stability with respect to the right-hand side G and the initial data given at $t = 0$.