Precise asymptotic formulas for semilinear eigenvalue problems

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We consider the following nonlinear two-parameter problem

$$\begin{aligned} -u''(x) + \lambda u(x)^q &= \mu u(x)^p, \;\; x \in I = (0,1), \\ u(x) &> 0, \;\; x \in I, \\ u(0) &= u(1) = 0, \end{aligned} \tag{0.1}$$

where $1 < q < p$ and $\lambda, \mu > 0$ are parameters.

The purpose of this talk is to establish the asymptotic formulas for the eigencurve $\mu = \mu(\lambda)$ with the exact second term as $\lambda \to \infty$ by using a variational method. We also establish the critical relationship between p and q from a point of view of the decaying rate of the second term of $\mu(\lambda)$.

The study of two-parameter eigenvalue problems began with the oscillation theory and has been investigated by many authors. We refer to Atkinson [1], Binding and Browne [2], Cantrell [3], Faierman [4], Faierman and Rauch [5], Shibata [6, 7, 8], Sleeman [9], Turyn [10], Volkmer [11] and the references therein. One of the main problems in this area is to analyze the structure of the solution set $\{(\lambda, \mu, u)\}$ of (1.1) and the effective approach to this problem is to study the structure of the set $S_{\lambda,\mu} := \{(\lambda, \mu, \|u\|_{p+1})\} \subset \mathbf{R}^3$ for large λ. In Shibata [8], by using a standard variational framework, the variational eigencurve $\mu = \mu(\lambda)$ was defined to analyze $S_{\lambda,\mu}$ and a simple asymptotic formula for $\mu(\lambda)$ as $\lambda \to \infty$ was established to understand the first term of $\mu(\lambda)$ as $\lambda \to \infty$. However, the remainder estimate of $\mu(\lambda)$ has not been obtained. The purpose here is to obtain *the exact second term* of $\mu(\lambda)$ as $\lambda \to \infty$. We emphasize that the second term depends deeply on the relationship between p and q. Finally, it should be mentioned that the asymptotic behavior of such eigencurve is also effected by the variational framework (cf. [6, 7]).

Notations and Definitions

Let $H_0^1(I)$ be the usual real Sobolev space. $\|u\|_r$ denotes the usual L^r-norm. For $u \in H_0^1(I)$

$$E_\lambda(u) := \frac{1}{2}\|u'\|_2^2 + \frac{1}{q+1}\lambda\|u\|_{q+1}^{q+1},$$

$$M_\gamma := \{u \in H_0^1(I) : \|u\|_{p+1} = \gamma\},$$

where $\gamma > 0$ is a *fixed constant.* For a given $\lambda > 0$, we call $\mu(\lambda)$ the *variational eigenvalue* when the following conditions are satisfied:

$$(\lambda, \mu(\lambda), u_\lambda) \in \mathbf{R}_+ \times \mathbf{R}_+ \times M_\gamma \ \text{ satisfies } \ (0.1).$$

$$E_\lambda(u_\lambda) = \inf_{u \in M_\gamma} E_\lambda(u).$$

Then $\mu(\lambda)$ is obtained as a Lagrange multiplier and is represented explicitly as follows:

$$\mu(\lambda) = \frac{\|u_\lambda'\|_2^2 + \lambda\|u_\lambda\|_{q+1}^{q+1}}{\gamma^{p+1}}.$$

We discuss the asymptotic behavior of $\mu(\lambda)$ as $\lambda \to \infty$ precisely.

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