反応拡散方程式の時間大域解の存在・非存在について

五十嵐 威文（日本大学大学院理工学研究科数学専攻）

次の初期値問題 (IVP) を考える。

$$(\text{IVP}) \begin{cases} u_t - \Delta u = k(x,t)u + h(x,t)u^p & \text{in } \mathbf{R}^n \times (0,\infty), \\ u(x,0) = a(x) \geq 0 & \text{in } \mathbf{R}^n. \end{cases}$$

但し，$k \in C(\mathbf{R}^n \times [0,\infty))$, $k \in C^1(\mathbf{R}^n)$, $h \in C(\mathbf{R}^n \times [0,\infty))$, $h(x,t) \geq 0$, $a \in C^2(\mathbf{R}^n) \cap L^\infty(\mathbf{R}^n)$, $p > 1$ とする。

反応拡散方程式の時間大域解の存在や非存在は，多くの研究者によって研究されてきているが，特に $k(x,t) \equiv 0$，$h(x,t) \equiv 1$ の場合は Fujita [1], Hayakawa [2], Kobayashi-Sirao-Tanaka [4] の結果が，$k(x,t) \equiv 0$，$h(x,t) = h(t)$ の場合は Meier [5] の結果が，$k(x,t) \equiv 0$, $h(x,t) = h(x)$ の場合は Pinsky [6] の結果が，$k(x,t) \equiv 0$，$h(x,t) = |x|^\sigma t^q$, $(\sigma > -2,\ q \geq 0)$ の場合は Qi [7] の結果が，$k(x,t) \equiv k(x)$，$h(x,t) \equiv 1$ の場合は Zhang [8] の結果がそれぞれ知られている。

本講演では，(IVP) に対し，次の Fujita [1] の拡張の結果が得られたことを報告する。

Theorem 1

Suppose that

$$k(x,t) \geq 0, \tag{1}$$

or that k is a function depending only on x; that is, $k(x,t) = k(x)$, and for $b_1 > 2$ and a given number $a > 0$,

$$-\frac{a}{1+|x|^{b_1}} \leq k(x) \leq 0. \tag{2}$$

Assume that for constant $c > 0$ and any given number $a_1 > 1$,

$$h(x,t) \geq c(t+a_1)^q (\log(t+a_1))^r, \quad q > -1,\ r \in \mathbf{R}.$$

1. *If $p < 1 + (2q+2)/n$, then* (IVP) *does not have any global positive solution for any nontrivial initial data $a(x)$.*
2. *If $p = 1 + (2q+2)/n$ and any $r > 0$, then* (IVP) *does not have any global positive solution for any nontrivial initial data $a(x)$.*

Theorem 2

Let k be a function depending only on x; that is, $k(x,t) = k(x)$. Suppose that for $b_1 > 2$ and a sufficiently small $\delta > 0$,

$$0 \leq k(x) \leq \frac{\delta}{1+|x|^{b_1}}, \tag{3}$$

or that for $b_1 > 2$ and a given number $a > 0$,

$$-\frac{a}{1+|x|^{b_1}} \leq k(x) \leq 0. \tag{4}$$

Assume that for constant $c > 0$ and any given number $a_1 > 1$,

$$h(x,t) \leq c(t+a_1)^q (\log(t+a_1))^r, \quad q > -1,\ r \in \mathbf{R}.$$

1. *If $p > 1 + (2q+2)/n$, then* (IVP) *has global positive solutions for sufficiently small initial data $a(x)$.*

2. *If $p = 1 + (2q+2)/n$ and any $r < -1$, then* (IVP) *has global positive solutions for sufficiently small initial data $a(x)$.*

References

[1] H.Fujita, On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$, *J. Fac. Sci. Univ. Tokyo Sect. A Math.* **16** (1966), 109-124.

[2] K.Hayakawa, On nonexistence of global solutions of some semilinear parabolic differential equations, *Proc. Japan Acad.* **49** (1973), 503-505.

[3] T.Igarashi, Existence and nonexistence of global solutions for $u_t - \Delta u = k(x,t)u + h(x,t)u^p$ in $\mathbf{R}^n$, *Advances in Mathematical Sciences and Applications* Vol.14, No.2 (2004), to appear.

[4] K.Kobayashi, T.Sirao and H.Tanaka, On the growing up problem for semilinear heat equations, *J. Math. Soc. Japan* **29** (1977), 407-424.

[5] P.Meier, On the critical exponent for reaction-diffusion equations, *Arch. Rational Mech. Anal.* **109** (1990), 63-71.

[6] R.G.Pinsky, Existence and nonexistence of global solutions for $u_t = \Delta u + a(x)u^p$ in $\mathbf{R}^n$, *J. Differential Equations* **133** (1997), 152-177.

[7] Y.-W.Qi, The critical exponents of parabolic equations and blow-up in $\mathbf{R}^n$, *Proc. Roy. Soc. Edinburgh Sect.* **128A** (1998), 123-136.

[8] Qi S.Zhang, The quantizing effect of potentials on the critical number of reaction-diffusion equations, *J. Diff. Eq.* **170** (2001), 188-214.

[9] Qi S.Zhang, An optimal parabolic esitimate and its applications in prescribing scalar curvature on some open manifolds with *Ricci* ≥ 0, *Math Ann.* **316** (2000), 703-731.

[10] Qi S.Zhang, Semilinear parabolic problems on manifolds and applications to the non-compact Yamabe problem, *Electron. J. Differential Equations* **2000**, No.46 (2000), 1-30.

[11] Qi S.Zhang, On a parabolic equation with a singular lower order term, , *Indiana Univ. Math. J.* **46**, No.3 (1997), 989-1020.