NONLINEAR PSEUDODIFFERENTIAL EQUATIONS ON A SEGMENT

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Our aim in the present talk is to study the global existence and large time asymptotic behavior of solutions to the initial-boundary value problem for the nonlinear Whitham equation on a segment $[0, a]$

$$(0.1) \qquad \begin{cases} u_t + u\, u_x + \mathbb{K}u = 0, \ t > 0, x \in (0, a)\,, \\ \qquad u(x, 0) = u_0(x), \ x \in (0, a)\,, \end{cases}$$

where the pseudodifferential operator $\mathbb{K}u$ on a segment $[0, a]$ is defined by

$$(0.2) \qquad \begin{aligned} \mathbb{K}u \ = \ & \theta_a(x) \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} e^{px} K(p) \\ & \times \left(\widehat{u}(p, t) - \frac{u(0, t) - e^{-pa} u(a, t)}{p} \right) dp, \end{aligned}$$

$$\theta_a(x) = \begin{cases} 1, 0 \leqslant x \leqslant a, \\ 0, x \notin [0, a]\,. \end{cases}$$

and $K(p) = C_\alpha p^\alpha$, $\alpha \in \left(\frac{3}{2}, 2 \right)$, C_α will be chosen below.

There are many open natural questions which we need to solve in this respect. First of them is how many boundary data should we pose on the initial-boundary value problem with pseudodifferential operator $\mathbb{K}$ for it's correct solvability.

Let us start with the following linear nonlocal initial-boundary value problem

$$(0.3) \qquad \begin{cases} u_t + \mathbb{K}u = f(x, t), \ t > 0, \ x \in (0, a)\,, \\ \quad u(x, 0) = u_0(x), \ x \in (0, a)\,, \\ \quad \partial_x^j u(0, t) = h_{0j}(t), \ j = 1, ..., m, \\ \quad \partial_x^l u(a, t) = h_{al}(t), \ l = 1, ..., n, \end{cases}$$

where the pseudodifferential operator $\mathbb{K}u$ on a segment $[0, a]$ we define by the inverse Laplace transformation

$$\begin{aligned} \mathbb{K}u \ = \ & \theta_a(x) \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} e^{px} K(p) \\ & \times \left(\widehat{u}(p, t) - \sum_{j=1}^{[\alpha]} \frac{\partial_x^{j-1} u(0, t) - e^{-pa} \partial_x^{j-1} u(a, t)}{p^j} \right) dp. \end{aligned}$$

Note that we get the usual differential operator, when the symbol $K(p)$ is a polynomial.