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Propagation of Singularities for semilinear Wave Equations with nonlinearity satisfying null condition

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We consider the propagation of singularities for the following semilinear wave equation,

$$\Box u = f(u)\{(\partial_t u)^2 - |\nabla u|^2\} + g(u),$$

where

$$x = (t,\ x_1, \cdots, x_{n-1}) \in \mathbf{R}^n, \quad \Box \equiv (\partial^2/\partial_t^2) - \Delta, \quad \Delta = \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2}, \quad f,\ g \in C^\infty.$$

Definition 1. We say that a subset K of $\mathbf{R}_x^n \times (\mathbf{R}_\xi^n \backslash \{0\})$ is a conic set if $(x,\ \xi) \in K$ implies that $(x,\ t\xi) \in K$ for any $t > 0$.

Definition 2. We call u is in $H_{ml}^\tau(x_0,\ \xi_0)$ if there exists a smooth function $\phi(x)$ supported near x_0 with $\phi(x_0) = 1$ and a conic neighborhood K of ξ_0 in $\mathbf{R}^n \backslash \{0\}$ such that

$$\langle \xi \rangle^\tau \chi_K(\xi) |\widehat{\phi u}(\xi)| \in L^2(\mathbf{R}^n)$$

where $\chi_K(\xi)$ is the characteristic function of K and $\langle \xi \rangle = (1 + \sum \xi_i^2)^{1/2}$.
If Γ is a closed conic set in $\mathbf{R}_x^n \times (\mathbf{R}_\xi^n \backslash \{0\})$ (that is, conic in the ξ variables), we shall say that $u \in H_{ml}^\tau(\Gamma)$ if $u \in H_{ml}^\tau(x,\ \xi)$ for all $(x,\ \xi) \in \Gamma$.

Definition 3. Let $p(x, \xi)$ is a characteristic polynomial of differential operator P. The curves $x(s),\ \xi(s)$ are bicharacteristics if

$$\frac{dx_j}{ds} = \frac{\partial p}{\partial \xi_j}(x(s),\ \xi(s)), \quad \frac{d\xi_j}{ds} = -\frac{\partial p}{\partial x_j}(x(s),\ \xi(s)) \qquad (j = 1,\ \cdots,\ n.)$$

Since $\displaystyle\sum_{j=1}^{n} \Big(\frac{\partial p}{\partial \xi_j} \frac{\partial}{\partial x_j} - \frac{\partial p}{\partial x_j} \frac{\partial}{\partial \xi_j} \Big) p = 0$ we see that p is constant on each of these curves

; one on which p
vanishes is called a null-bicharacteristic of p.

Theorem 4. Suppose that U is an neighborhood of x_0 and $f,\ g$ is C^∞. Suppose that $u \in H^s(U),\ s > n/2$, satisfies

$$\Box u = f(u)\{(\partial_t u)^2 - |\nabla u|^2\} + g(u),$$

Let Γ denote a null bicharacteristic for $\Box$ and suppose that

$$u \in H^r_{ml}(x_0,\ \xi_0) \quad \text{for some point } (x_0,\ \xi_0) \text{ on } \Gamma,$$

then $u \in H^r_{ml}(\Gamma)$ for $n/2 < s \leqq r \leqq 2s - n/2$.

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