abstract:

Applications of Banach-valued Besov spaces to Evolution Equations in Banach spaces

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It is well known that Besov spaces are very usefull in studying many problems. I report here their applications to the evolution equation:

(EE) $$du/dt + A(t)u = f(t),\ a < t < b,$$

where $-A(t)$ is the generator of a semigroup of linear operators in a Banach space X. We consider only ' parabolic type'. Namely, we assume that

(A1) $-A(t)$ is a linear operator with dense domain, and there exist constants $\kappa > \pi/2$ and C_0 such that the resolvent set of $-A(t)$ contains the sector $\Sigma_\kappa := \{\lambda \in \mathbb{C}; |\arg\lambda| \le \kappa\}$ for any $t \in I := [a, b]$ and $\|\lambda(\lambda + A(t))^{-1}\|_{X\to X} \le C_0$ holds for any $\lambda \in \Sigma_\kappa$ and any $t \in I$.

Therefore,$-A(t)$ generates an analytic semi-group $\{e^{-\tau A(t)}; \tau \ge 0\}$ on X.

Case 1. $A(t) = A$ is independent of t. Crandall-Pazy, 1969, proved that $F(t) := \displaystyle\int_a^t e^{-(t-s)A} f(s)ds$ is strongly differentiable and satisfies (EE) if the modulus of continuity $\omega(h : f)$ of f is integrable near 0 with the measure dh/h. Furthermoere, Baillon, 1980, showed that if F is differantiable for every continuous function f then X has a spacial property or A is bounded. We prove that F is strongly differantiable and satisfies (EE) if f belongs to $B^0_{\infty,1}(I;X)_{loc} \cap L^1(I;X), I := (a, b)$ (J.Math.Soc. Japan, 1990).

Case 2. The domain $\mathcal{D}(A(t))$ of $A(t)$ is independent of t, which we write by Y. Tanabe, 1960, has constructed the evolutin operator $U(t, s)$ to (EE) when $A(t)$ is Hölder continuous $\mathcal{L}(Y, X)$-valued function. We have improved his result, that is, we have constructed it under the assumption that the modulas of continuity $\omega(h)$ of $A(t)$ as an $\mathcal{L}(Y, X)$-valued function is integrable near 0 with dh/h(Osaka J. Math. 2001). We also showed that $F(t) := \int_a^t U(t, s)f(s)ds$ is strongly differentiable and satisfies (EE) if f satisfies the same conditon as in Case I.

Case 3. The domain $\mathcal{D}(A(t)^{1/m})$ is independent of t, where m is some positive integer m greater than 1. We put $Y = \mathcal{D}(A(t)^{1/m})$. Assuming that $A(t)^{1/m}$ is Hölder continuous with a exponent θ greater than $1 - 1/m$ as an $\mathcal{L}(Y, X)$-valued function, T. Kato, 1961, has constructed the evolution operator. We recently improved his result. Our assumption is '$A(t)$ belongs to $B^{1-1/m}_{\infty,1}(I;\mathcal{L}(Y,X))$'. We also have the same result for F as in Case 2.

Case 3'. A. Yagi (1988) and Acquistapace-Terreni (1986) have constructed the evolution operator to (EE) under the following assumptions:

(A1') There exist constants $\kappa > \pi/2$ and C_0 such that the resolvent set of $-A(t)$ contains the sector Σ_κ for any $t \in I$ and $\|(\lambda - A(t))^{-1}\|_{X\to X} \le C_0/(|\lambda|+1)$ holds for any $-\lambda \in \Sigma_\kappa$ and any $t \in I$.

(A3') For some $0 < \theta, \eta \le 1$ with $\theta + \eta > 1$, $A(t)(\lambda - A(t)^{-1}\{A(t)^{-1} - A(s)^{-1}\}\|_{X\to X} \le K|t-s|^\eta/(|\lambda|+1)^\theta$ holds for any $s, t \in I$ and $\lambda \le 0$. K is a constant.

We have improved this result, that is, we proved the following:

Assume (A1) and the following hypotheses (A2), (A3):

(A2) There exist a number $\theta \in (0,1)$ and a Banach space Y continuously imbedded in X such that the domain $\mathcal{D}(A(t)^\theta) = Y$ for any $t \in I$ and $A(\cdot)^\theta \in \mathcal{C}(I;\mathcal{L}(Y,X))$.

(A3) 0 belongs to the resolvet set of $A(t)$ for any $t \in I$ and $A(t)^{-1} \in B^{1-\theta}_{\infty,1}(I;\mathcal{L}(X,Y))$.

Then, there exists the evolution operator to (EE).

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