Asymptotic formulas for eigenvalues of nonlinear Sturm-Liouville problems

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In this talk, we are concerned with the following nonlinear Sturm-Liouville problem

$$-u''(t) + f(u(t)) = \lambda u(t), \quad t \in I := (0,1), \tag{1}$$

$$u(t) > 0, \quad t \in I, \tag{2}$$

$$u(0) = u(1) = 0, \tag{3}$$

where $\lambda > 0$ is an eigenvalue parameter.

We assume that $f(u)$ satisfies at least the following conditions (A.1)–(A.3).

(A.1) $f(u)$ is a function of C^1 for $u \ge 0$ satisfying $f(0) = f'(0) = 0$.
(A.2) $g(u) := f(u)/u$ is strictly increasing for $u \ge 0$ ($g(0) := 0$).
(A.3) $g(u) \to \infty$ as $u \to \infty$.

The typical example of $f(u)$ which satisfies (A.1)–(A.3) is $f(u) = u^p$ ($p > 1$).

Under the conditions (A.1)–(A.3), we know that for each given $\alpha > 0$, there exists a unique solution $(\lambda, u) = (\lambda(\alpha), u_\alpha) \in \mathbf{R}_+ \times C^2(\bar{I})$ with $\|u_\alpha\|_2 = \alpha$. The set $\{(\lambda(\alpha), u_\alpha), \alpha > 0\}$ gives all solutions and is an unbounded curve of class C^1 in $\mathbf{R}_+ \times L^2(I)$ emanating from $(\pi^2, 0)$.

The objective of this talk is to study precisely the global behavior of this bifurcation branch in $\mathbf{R}_+ \times L^2(I)$. To this end, we establish several types of precise asymptotic formulas for $\lambda(\alpha)$ as $\alpha \to \infty$.