Global DIV-CURL Lemma in 3D bounded domains

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We consider a global version of the Div-Curl lemma for vector fields in a bounded domain $\Omega \subset \mathbb{R}^3$ with the smooth boundary $\partial\Omega$. Suppose that $\{u_j\}_{j=1}^\infty$ and $\{v_j\}_{j=1}^\infty$ converge to u and v weakly in $L^r(\Omega)$ and $L^{r'}(\Omega)$, respectively, where $1 < r < \infty$ with $1/r + 1/r' = 1$. Assume also that $\{\text{div } u_j\}_{j=1}^\infty$ is bounded in $L^q(\Omega)$ for $q > \max\{1, 3r/(3+r)\}$ and that $\{\text{rot } v_j\}_{j=1}^\infty$ is bounded in $L^s(\Omega)$ for $s > \max\{1, 3r'/(3+r')\}$, respectively. If either $\{u_j \cdot \nu|_{\partial\Omega}\}_{j=1}^\infty$ is bounded in $W^{1-1/q,q}(\partial\Omega)$, or $\{v_j \times \nu|_{\partial\Omega}\}_{j=1}^\infty$ is bounded in $W^{1-1/s,s}(\partial\Omega)$ (ν: unit outward normal to $\partial\Omega$), then it holds that $\int_\Omega u_j \cdot v_j dx \to \int_\Omega u \cdot v dx$. As an immediate consequence, we prove the well-known Div-Curl lemma for any open set in $\mathbb{R}^3$. The Helmholtz-Weyl decomposition for $L^r(\Omega)$ plays an essential role for the proof.