Resolvent problem for the complex Ginzburg-Landau equation with non-homogeneous term in L^p

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Let Ω be a bounded or unbounded domain in $\mathbb{R}^N$ with compact C^2-boundary $\partial\Omega$. For a given function $f : \Omega \to \mathbb{C}$, we consider the following resolvent problem:

$$(\mathbf{RCGL}) \qquad \begin{cases} (\xi + i\eta)u - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^{q-2}u = f & \text{on } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $i = \sqrt{-1}$, ξ, λ, $\kappa \in \mathbb{R}_+ := (0, \infty)$, η, α, $\beta \in \mathbb{R}$, and p, $q \geq 2$ are constants and u is complex valued unknown function. (**RCGL**) is associated with the initial-boundary value problem for the complex Ginzburg-Landau equation (**CGL**) (see [**3**]). In particular, if "$f \in L^q(\Omega) \cap L^2(\Omega)$" then there exists a unique strong solution to (**RCGL**) (see [**2**]); for the case where "$f \in L^2(\Omega)$" see [**1**]. In this talk we generalize the results obtained by [**1**] and [**2**]. Namely, we establish the existence and uniqueness of strong solutions to (**RCGL**) for "$f \in L^p(\Omega) \cap L^2(\Omega)$" under the restriction on p and q:

$$(*) \qquad \begin{cases} 2 \leq q < 2 + \dfrac{2}{N}p & (N \geq 2), \\ 2 \leq q < 2 + p & (N = 1). \end{cases}$$

Definition. A function u is said to be a *strong solution* to (**RCGL**) if
(a) $u \in H^2(\Omega) \cap H_0^1(\Omega) \cap L^p(\Omega)$;
(b) u satisfies the equation in (**RCGL**) formulated in $L^2(\Omega)$.

Main Theorem. *Let $N \in \mathbb{N}$, ξ, λ, $\kappa \in \mathbb{R}_+$, η, α, $\beta \in \mathbb{R}$ and let condition $(*)$ be satisfied. Assume that*

$$\frac{|\alpha|}{\lambda} \leq \frac{1}{c_p}, \qquad \frac{|\beta|}{\kappa} > \frac{1}{c_q} \qquad \left(c_s := \frac{s-2}{2\sqrt{s-1}} \right).$$

Then for $f \in L^p(\Omega) \cap L^2(\Omega)$ and $\xi \geq 1 + CL^\theta$ there exists a unique strong solution u to (**RCGL**)*, where $L := (1/p)\,\|f\|_{L^p}^p$, $C := C(\beta, \kappa, \lambda) > 0$ and $\theta := \theta(p, q, N) > 0$ are constants.*

When $p = q$, this theorem gives the result established by [**2**]. On the other hand, the case where $p = 2$ yields the result obtained by [**1**].

References

[1] 堀 哲郎, *Resolvent problem for the complex Ginzburg-Landau equation with non-homogeneous term in L^2*, 修士論文 (2008).

[2] N. Okazawa, *Semilinear elliptic problems associated with the complex Ginzburg-Landau equation*, Partial Differential Equations and Functional Analysis, Oper. Theory Adv. Appl, vol. 168, 169–187, Birkhäuser, Basel, 2006.

[3] N. Okazawa and T. Yokota, *Monotonicity method applied to the complex Ginzburg-Landau and related equations*, J. Math. Anal. Appl. **267** (2002), 247–263.