Gradient flows: from Hilbert spaces to metric spaces; an introduction

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Abstract

The goal of this lecture is to give an introduction to the theory of gradient flows in metric spaces which is developed in the first part of the recent book of L. Ambrosio, N. Gigli and G. Savaré [1]. Starting from convex gradient flows in Hilbert spaces we arrive at a generalization which includes many interesting gradient flows in metric spaces of probabilities in $\mathbb{R}^N$, equipped with the L^2-Wasserstein metric. This new approach initiated by Otto, Jordan, Kinderlehrer, Brenier and others has proved to be very fruitful in various fields of Analysis.

In this lecture I shall concentrate on the construction of the flow by an approach "à la Crandall-Liggett". Even if it does not provide optimal rate of convergence, it has the advantage of being simpler than the approach in [1]. An elementary proof of the triangle inequality for the Wasserstein metric will be discussed (joint work with W. Desch [2]).

References

[1] L. Ambrosio, N. Gigli and G. Savaré, "Gradient flows in Metric Spaces and in the Space of Probability Measures", Lectures in Mathematics, ETH Zürich, 2nd ed. 2008 Birkhäuser Verlag.

[2] Ph. Clément and W. Desch, *An elementary proof of the triangle inequality for the Wasserstein metric*, Proc. Amer. Math. Soc. **136** (2008), 333-339.