非線形Sturm-Liouville問題の分岐曲線の漸近挙動と逆問題

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We consider the following nonlinear Sturm-Liouville problem

$$\begin{aligned} -u''(t) + f(u(t)) &= \lambda u(t), \quad t \in I, && (0.1)\\ u(t) &> 0, \quad t \in I, && (0.2)\\ u(0) &= u(1) = 0, && (0.3) \end{aligned}$$

where $I := (0,1)$ and $\lambda > 0$ is a parameter.

We assume that $f(u)$ satisfies the following conditions (A.1)–(A.3):

(**A.1**) $f(u)$ is a function of C^1 for $u \geq 0$ satisfying $f(0) = f'(0) = 0$.

(**A.2**) $f(u)/u$ is strictly increasing for $u \geq 0$.

(**A.3**) $f(u)/u \to \infty$ as $u \to \infty$.

The typical examples of f which satisfy (A.1)–(A.3) are as follows.

$$\begin{aligned} f(u) &= u^p \qquad (p > 1),\\ f(u) &= u^p \log(u+1) \qquad (p > 1),\\ f(u) &= u^p \cdot \left(1 - \frac{1}{1+u^2}\right) \qquad (p > 1),\\ f(u) &= u^p e^u \qquad (p > 1). \end{aligned}$$

(1) Let $1 \leq q < \infty$ be fixed. For any given $\alpha > 0$, there exists a unique solution pair of (1.1)–(1.3) $(\lambda, u) = (\lambda(q,\alpha), u_\alpha) \in \mathbf{R}_+ \times C^2(\bar{I})$ such that $\|u_\alpha\|_q = \alpha$.

(2) The set $\{(\lambda(q,\alpha), u_\alpha) : \alpha > 0\}$ gives all solutions of (1.1)–(1.3), which is an unbounded C^1-bifurcation curve emanating from $(\pi^2, 0)$ in $\mathbf{R}_+ \times L^q(I)$ and $\lambda(q,\alpha)$ is C^1 and strictly increasing for $\alpha > 0$.

In this talk, we first establish the asymptotic expansion formulas for $\lambda(q,\alpha)$ as $\alpha \to \infty$ and $\alpha \to 0$. Secondly, based on these formulas, we introduce the framework of inverse bifurcation problem and introduce some recent results how to determine the unknown nonlinear term f from the L^q-bifurcation curve $\lambda(q,\alpha)$.

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