Radial fingering in a Hele-Shaw cell with the effect of viscous normal stress

Hisasi Tani

Hele-Shaw problem, which is about a time evolution of an interface between two fluids in a Hele-Shaw cell, has been intensively studied for the reason of its relation with a wide variety of phenomena such as the growth of a snow crystal and the enhanced oil recovery [1, 2]. Under the assumptions that (i) both fluids are incompressible, and (ii) the flows obey the Darcy's law, the problem to be solved is a Laplace equation with appropriate boundary conditions, traditionally the kinematic boundary condition and the Young--Laplace law [1-3, 6]. The Young--Laplace law expresses that pressure of fluids is discontinuous at the interface, whose difference is proportional to a curvature of the interface. However, its validity has been discussed in some previous studies for the case of the rectangular geometry [4] and the radial one [5]. Especially in [5], it was suggested the applicability of boundary condition including the effect of the viscous normal stress (VNS), instead of the Young--Laplace law. On the other hand, their results [5] are limited in the linear analysis, while the nonlinearity may plays an essential role on the instability of the interface.

Under such backgrounds, we derive modified weakly nonlinear results including the VNS effects, by following the weakly nonlinear analysis due to Miranda [7]. Then we numerically analyze the derived equation and discuss the effects of VNS on the instability and the behavior of the radially growing interface.

[1] P. Saffman and G. Taylor, *The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid*, Proc. R. Soc. Lond. A (1958) **245** 312
[2] P. Pelce, *Dynamics of Curved Fronts*, Academic Press (1988)
[3] L. Paterson, Radial fingering in a Hele Shaw cell, J. Fluid Mech. (1981) **113** 513
[4] C. Park and G. Homsy, *Two-phase displacement in Hele-Shaw cells: theory*, J. Fluid Mech. (1984) **139** 291
[5] H. Kim *et al*., *Viscous potential flow analysis of radial fingering in a Hele-Shaw ce ll*, Phys. Fluids (2009) **21** 074106
[6] J. Miranda and M. Widom, *Radial fingering in a Hele-Shaw cell: a weakly nonlinea r analysis*, Physica D (1998) **120** 315