Shape and structure control for elastic bodies with inclusions and cracks

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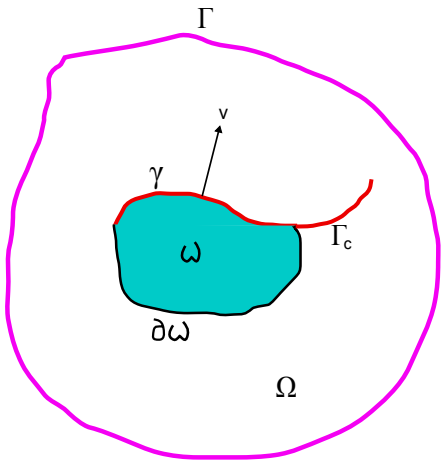
The talk is concerned with a shape and structure control of elastic bodies with rigid inclusions and cracks. It is assumed that rigid inclusions are delaminated, thus cracks are located on the boundary of inclusions as well as outside of the inclusions. We provide the problem formulations and analyze the solution sensitivity with respect to geometrical and structure perturbations in the frame of free boundary models. Inequality type boundary conditions are considered at the crack faces to guarantee a mutual non-penetration between crack faces. Inclusion structure and shape as well as and crack shape are considered as control functions. The cost functional, which is based on the Griffith rupture criterion, characterizes the energy release rate and provides the shape sensitivity with respect to a change of the geometry of the inclusions. We prove an existence of optimal shape and structure in the problems considered.

It is well known that a mutual disposal of rigid inclusions and cracks inside of composite materials is responsible for damage and crack propagation. In particular, one of the objectives in material sciences is an improvement of mutual disposal to prevent a propagation of cracks. The influence of shapes and material properties of inclusions with respect to crack tips is a great mathematical problem. Such properties of inclusions and fibres can be considered as control functions to improve the structure. In this talk, we focus on the shape analysis of cracks and rigid inclusions in the frame of high level models with unknown sets of a contact between crack faces. Our aim is to maximize cost functionals related to the Griffith rupture criterion.

To describe suitably composite materials it is necessary to analyze high level mathematical models of elastic bodies with rigid inclusions and cracks. Rigid inclusions may be delaminated, hence the crack approach with non-penetration conditions is to be used. In such a case, new types of nonlinear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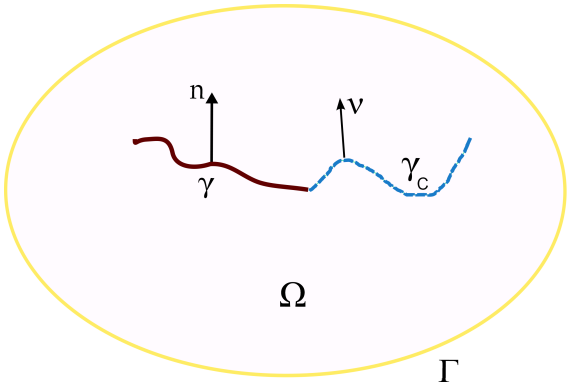
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and nonlocal boundary conditions appear which lead to free boundary value problems being correct from the mechanical standpoint.

The talk provides an analysis of optimal control problems for elastic bodies with a delaminated rigid inclusions and cracks adjacent to the inclusions. The free boundary approach is used to describe an equilibrium state of the bodies with inequality type boundary conditions at the crack faces. Shape and structure control problems are investigated related to the Griffith rupture criterion. In particular, an existence of optimal shapes for rigid inclusions and cracks is proved.

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