Stability issues for standing waves of a quasilinear Schrodinger equation.

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Quasilinear Schrödinger type equations have now been investigated for many years. The interest of these kinds of equations lies on the fact that it can be used to modelize different physical phenomena in many physical situations: self-channeling of a high-power ultra short laser in matter, dissipative quantum mechanics, plasma physics and fluid mechanics in the theory of Heisenberg ferromagnets and magnons and in condensed matter theory. However, little is known about Cauchy problem and the question of global well-posedness is still an open problem in many cases. This is one of the reasons why many efforts have been made to prove existence and stability of particular global solutions such as solitary waves in general and ground states solutions in particular. In this talk, we consider the following quasilinear Schrödinger equation

$$\begin{cases} i\partial_t\phi_t + \kappa\Delta\phi + \phi\Delta|\phi|^2 + |\phi|^{p-1}\phi = 0 & \text{in } (0,\infty)\times\mathbb{R}^N, \\ \phi(0,x) = a_0(x) & \text{in } \mathbb{R}^N. \end{cases} \tag{1}$$

Our aim is to present different kind of results including local existence theory and behavior of particular solutions. More precisely, we will first discuss the local well-possednes of the equations and exhibit the difficulties due to the presence of the quasilinear term. Then, we will introduce the notion of **standing waves**, that is solutions of the form $u_\omega(t,x) = \phi_\omega(x)e^{-i\omega t}$. Here ω is a fixed parameter and note that $u_\omega(t,x)$ satisfies problem (1) if and only if ϕ_ω is a solution of the equation

$$-\Delta u - \kappa u\Delta|u|^2 + \omega u = |u|^{p-1}u, \qquad \text{in } \mathbb{R}^N. \tag{2}$$

Among all of these stationary solutions, the ground states play an important role. We say that a weak solution of (2) is a ground state if it satisfies

$$\mathcal{E}_\omega(u) = m_\omega, \tag{3}$$

where

$$m_\omega = \inf\{\mathcal{E}_\omega(u) : u \text{ is a nontrivial weak solution of (2)}\}.$$

Here, $\mathcal{E}_\omega$ is the action associated with (2) and reads

$$\mathcal{E}_\omega(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 dx + \frac{1}{4}\int_{\mathbb{R}^N} |\nabla |u|^2|^2 dx + \frac{\omega}{2}\int_{\mathbb{R}^N} |u|^2 dx - \frac{1}{p+1}\int_{\mathbb{R}^N} |u|^{p+1} dx.$$

We will then give an existence result for ground states solutions and discuss the orbital stability or instability of these particular solutions. Of course, we will take into account the influence of parameters p of the nonlinear term and κ in front of the quasilinear term.